Laser Physics SK2411 – Session 3 Problems

5.2. Consider a confocal resonator of length $L = 1$ m used for an Ar^+ laser at wavelength $\lambda = 514.5$ nm. Calculate (a) the spot size at the resonator center and on the mirrors; (b) the frequency difference between consecutive longitudinal modes; (c) the number of non-degenerate modes falling within the Doppler-broadened width of the Ar^+ line ($\Delta\nu_0^* = 3.5$ GHz; see Table 2.2).

5.8. A resonator consists of two plane mirrors with a positive lens inserted between them. If the focal length of the lens is f, and L_1, L_2 represent the distances of the lens from the two mirrors, calculate (a) the spot size at the lens position and the spot sizes at each mirror and (b) the conditions under which the cavity is stable.

5.10. A laser operating at $\lambda = 630$ nm has a power gain of 2×10^{-2} per pass, and it is provided with a symmetric resonator consisting of two mirrors each of radius $R = 10$ m and separated by $L = 1$ m. Choose an appropriate mirror-aperture size to suppress TEM_{01} mode operation while allowing TEM_{00} mode operation.

5.13. Consider the $A_1B_1C_1D_1$ matrix in Fig. 5.8d and show that for a stable cavity, we must have $0 < A_1D_1 < 1$ and $-1 < B_1C_1 < 0$. From these results show that $B_1D_1/A_1C_1 < 0$, so that q_1 in Eq. (5.5.6a) is purely imaginary.

Example 5.1. *Number of modes in closed and open resonators.* Consider a He-Ne laser oscillating at the wavelength of $\lambda = 633$ nm with a Doppler-broadened gain linewidth of $\Delta\nu_0^* = 1.7 \times 10^9$ Hz. Assume a resonator length $L = 50$ cm and consider first an open resonator. According to Eq. (5.1.3) the number of longitudinal modes that fall within the laser linewidth is $N_{open} = 2L\Delta\nu_0^*/c \cong 6$. Assume now that the resonator is closed by a cylindrical lateral surface with a cylinder diameter of $2a = 3$ mm. According to Eq. (2.2.16), the number of modes of this closed resonator that fall within the laser linewidth $\Delta\nu_0^*$ is $N_{closed} = 8\pi\nu^2 V\Delta\nu_0^*/c^3$, where $\nu = c/\lambda$ is the laser frequency and $V = \pi a^2 L$ is the resonator volume. From the preceding expressions and data we readily obtain $N_{closed} = (2\pi a/\lambda)^2 N_{open} \cong 1.2 \times 10^9$ modes.

Example 5.5. *Spot sizes for symmetric resonators.* The first case we consider involves a confocal resonator ($g = 0$). From Eqs. (5.5.10a–b) we obtain respectively,

$$w_c = \left(\frac{L\lambda}{\pi}\right)^{1/2} \qquad w_{oc} = \left(\frac{L\lambda}{2\pi}\right)^{1/2} \tag{5.5.11}$$

where the subscript c stands for confocal. Equation (5.5.11) shows that the spot size at the beam waist is, in this case, $\sqrt{2}$ smaller than that at the mirrors (Fig. 5.9a). For the case of a near-plane resonator, i.e., when $R \gg L$, we can write $g = 1 - \varepsilon$, where ε is a small positive quantity. Neglecting higher order terms in ε, Eq. (5.5.10) gives

$$\left(\frac{w_{np}}{w_c}\right) \cong \left(\frac{w_{onp}}{w_c}\right) \cong \left(\frac{1}{2\varepsilon}\right)^{1/4} \tag{5.5.12}$$

where the subscript np stands for near-plane and the spot sizes are normalized to the mirror spot size of a confocal resonator. Equation (5.5.12) shows that, to first order, the two spot sizes are equal; thus the spot size is nearly constant over the length of the resonator (Fig. 5.9b). For the case of a near-concentric resonator, i.e., when $L \cong 2R$, we likewise write $g = -1 + \varepsilon$, where again ε is a small positive quantity. Neglecting terms of higher order in ε, Eq. (5.5.10) gives

$$\left(\frac{w_{nc}}{w_c}\right) = \left(\frac{1}{2\varepsilon}\right)^{1/4} \qquad \left(\frac{w_{onc}}{w_c}\right) = \left(\frac{\varepsilon}{8}\right)^{1/4} \tag{5.5.13}$$

where the subscript nc stands for near-concentric. Equation (5.5.13) shows that the mirror spot size, as a function of ε, is given by the same expression as that for a near-plane resonator. The spot size at the beam waist, however, is now much smaller, and it decreases with decreasing value of ε. Spot size behavior along the resonator is shown in Fig. 5.9c. Numerically, if we take $L = 1$ m and $\lambda = 514$ nm (an argon laser wavelength), we get $w_c \cong 0.4$ mm for a confocal resonator. If we now consider a near-plane resonator, still with $L = 1$ m, $\lambda = 514$ nm and with $R = 10$ m, we obtain $g = 0.9$; from Eq. (5.5.10) we get $w_0 \cong 0.59$ mm and $w \cong 0.61$ mm. Note: Relatively small values of beam spot size are obtained in every case.

Example 5.8. *Diffraction loss of a symmetric resonator.*[6] The diffraction loss per pass for a symmetric two-mirror resonator of finite mirror aperture, as calculated according to the Fox–Li iterative procedure is plotted in Fig. 5.13a (for a TEM_{00} mode) and in Fig. 5.13b (for a TEM_{01} mode) versus the Fresnel number

$$N = \frac{a^2}{L\lambda} \tag{5.5.26}$$

The calculation is performed for a range of symmetric resonators characterized by their corresponding g values. Note that, for a given g value and a given mode (e.g., the TEM_{00} mode), the loss rapidly decreases with an increasing Fresnel number. This is easily understood when, using Eq. (5.5.11), we write the Fresnel number as $N = a^2/\pi w_c^2$ where w_c is the mirror spot size for a confocal resonator of the same length and infinite aperture. Since the mirror spot size does not change strongly with g-value changes (see Example 5.5), the Fresnel number can be interpreted as proportional to the ratio between the area of the mirror cross section (πa^2 for a circular mirror) and the mode cross-section (πw^2) on the mirror. Why loss decreases rapidly when the latter ratio increases is now readily appreciated with the help of Fig. 5.11. Note also that, for a given Fresnel number and g value, the TEM_{00} mode has lower losses than the TEM_{01} mode. The TEM_{00} mode actually has a lower loss than any of the higher order modes. The lowest order mode is thus identified as the lowest loss mode.

11.8. The near-field transverse intensity profile of a Nd:YAG laser beam at $\lambda = 1.064\,\mu$m wavelength is, to a good approximation, Gaussian with a diameter (FWHM) $D \cong 4$ mm. The half-cone beam divergence, measured at the half-maximum point of the far-field intensity distribution, is $\theta_d \cong 3$ mrad. Calculate the corresponding M^2 factor.

11.9. The near-field transverse intensity profile of a pulsed TEA CO_2 laser beam at $\lambda = 10.6\,\mu$m wavelength is, to a good approximation, constant over its 1 cm × 4 cm dimension. The laser is advertised to have an M^2 factor of 16 along both axes. Assuming that the waist is located at the position of the output mirror, calculate the spot size parameters at a distance from this mirror of $z = 3$ m.