3.7. From the condition $f_c(E'_2) \geq f_v(E'_1)$ prove the Bernard–Duraffourg relation $E'_{F_c} - E'_{F_v} \geq h\nu$.

3.8. With the help of Fig. 3.15b calculate for GaAs: The values of E_{F_c} and E_{F_v} at $N = 1.6 \times 10^{18}$ cm^{-3} carrier injection and the overall gain bandwidth at the same injection level.

3.9. In the energy reference system in Fig. 3.9a calculate for GaAs the energies E_2 and E_1 of the upper and lower laser levels for a transition energy exceeding the band gap energy by $0.45kT$.

3.10. With the help of Fig. 3.16 for a bulk GaAs semiconductor, calculate the expected gain at a photon energy exceeding the band gap energy by $0.45kT$ and for a carrier injection of $N = 1.6 \times 10^{18}$ cm^{-3}.

Example 3.6. *Calculation of the absorption coefficient for GaAs.* As an approximation we assume the frequency ν, in the first term on the right-hand side of Eq. (3.2.36), to have the value $\nu \cong E_g/h = 3.43 \times 10^{14}$ Hz, where the energy gap E_g is taken to be 1.424 eV. We also take $m_v = 0.46m_0$ and $m_c = 0.067m_0$, so that $m_r = 0.059\, m_0 = 5.37 \times 10^{-32}$ kg. To calculate the average dipole moment $\mu_{av} = (\mu^2/3)^{1/2}$, we recall that the accurate value of the average electron momentum M_{av} was shown to be such that $M_{av}^2 = 3.38\, m_0 E_g$.[5] The relation between average dipole moment and electron momentum is $M_{av} = m_0\omega|\mu_{av}|/e$,[5] so that

$$\mu_{av} = e(3.38E_g/m_0)^{1/2}/2\pi\nu \cong 0.68 \times 10^{-25}\ \text{C} \times \text{m}.$$

Note that, if we write $r_{av} = \mu_{av}/e$, then $r_{av} \cong 0.426$ nm. Substitution into Eq. (3.2.36) of the values for ν and μ_{av} just calculated and $n = 3.64$ for the refractive index gives $\alpha_0 = 19{,}760(h\nu - E_g)^{1/2}$, where α_0 is expressed in cm^{-1} and the energy in eV. The absorption coefficient as calculated from the latter expression is plotted against $E - E_g$ in Fig. 3.16, where $E = h\nu$. Note that, when $h\nu$ exceeds the energy gap by only 10 meV, the absorption coefficient already reaches a very large value ($\approx 2{,}000$ cm^{-1}).

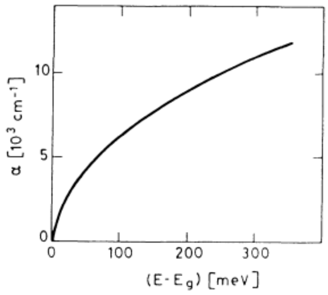

FIG. 3.16. Idealized plot of the absorption coefficient α versus the difference between the photon energy E and gap energy E_g for an intrinsic GaAs bulk semiconductor.

Example 3.7. *Calculation of the transparency density for GaAs.* We first transform Eq. (3.2.40) into the unprimed energy axes in Fig. 3.9a. According to Eq. (3.2.3) we can write $E'_{F_c} = E_g + E_{F_c}$ and $E'_{F_v} = -E_{F_v}$, and Eq. (3.2.40) transforms to $E_{F_c} + E_{F_v} = 0$. From Fig. 3.15a we see that E_{F_c}/kT is a function of (N/N_c); i.e., we can write $E_{F_c}/kT = f(N/N_c)$. Similarly we can write $E_{F_v}/kT = f(N/N_v)$, so that the transparency condition becomes

$$f(N_{tr}/N_c) + f(N_{tr}/N_v) = 0 \tag{3.2.41}$$

To obtain N_{tr} from Eq. (3.2.41) for GaAs, we plot the function $(E_{F_c}/kT) + (E_{F_v}/kT)$ versus N as a dashed line in Fig. 3.15b. At each carrier concentration N, the curve is obtained as the sum of values given by the two continuous curves in the figure. According to Eq. (3.2.41), we can now say that the transparency density N_{tr} is the carrier concentration at which the dashed curve in Fig. 3.15b crosses the zero value of the ordinate. From Fig. 3.15b we obtain $N_{tr} = 1.2 \times 10^{18}$ cm^{-3}.

4.4. A Fabry–Perot interferometer consisting of two identical mirrors, air-spaced by a distance L, is illuminated by a monochromatic em wave of tunable frequency. From a measurement of the transmitted intensity versus the frequency of the input wave we find that the free spectral range of the interferometer is 3×10^9 Hz and its resolution is 60 MHz. Calculate the spacing L of the interferometer, its finesse, and the mirror reflectivity. If the peak transmission is 50% calculate also the mirror loss.

4.8. Show that the power contained in a TEM_{00} Gaussian beam of spot size w is given by $P = (\pi w^2/2)I_p$, where I_p is the peak ($r=0$) intensity of the beam.

4.10. The beam of an Ar laser oscillating in a pure Gaussian TEM_{00} mode at $\lambda = 514.5$ nm with an output power of 1 W is sent to a target at a distance of 100 m from the beam waist. If the spot size at the beam waist is $w_0 = 2$ mm, calculate spot size, radius of curvature of the phase front, and peak intensity at the target position.

4.13. A Gaussian beam of waist spot size w_0 is passed through a solid plate of transparent material of length L and refractive index n. The plate is placed just in front of the beam waist. Using the *ABCD* law of Gaussian beam propagation show that the spot size and radius of curvature of the phase front after the plate are the same as for propagation in a vacuum over a distance $L' = L/n$. According to this result, is the far-field divergence angle affected by the insertion of the plate?

Example 4.3. *Free-spectral range, finesse, and transmission of a Fabry–Perot etalon.* Consider an FP interferometer made of a piece of glass with two plane parallel surfaces coated for high reflectivity (often called an FP etalon). If we assume $L = 1$ cm and $n_r = 1.54$, the free spectral range for near-normal incidence, i.e., for $\theta \cong 0$, is $\Delta\nu_{fsr} = c/2n_rL = 9.7$ GHz. If we now take $R_1 = R_2 = 0.98$, we obtain from Eq. (4.5.14a) a finesse $F \cong 150$, so that $\Delta\nu_c = \Delta\nu_{fsr}/F = 65$ MHz. According to Eq. (4.5.9), for a lossless coating, the peak transmission is $T_{max} = 1$; the minimum transmission, according to Eq. (4.5.10), is $T_{min} \cong 10^{-4}$. Note the very small value of T_{min}.

Example 4.4. *Spectral measurement of an Ar^+-laser output beam.* We consider an Ar-ion laser oscillating on its green line at a wavelength $\lambda = 514.5$-nm. We assume that the laser is oscillating on many longitudinal modes encompassing the full Doppler width of the laser line ($\Delta\nu_0^* = 3.5$ GHz). Thus we have $\Delta\nu_{osc} = \Delta\nu_0^* = 3.5$ GHz. To avoid frequency ambiguity, we must have $\Delta\nu_{fsr} = (c/2L) \geq 3.5$ GHz i.e., $L \leq 4.28$ cm. If we now assume a finesse $F = 150$ and take $L = 4.28$ cm, according to Eq. (4.5.18) we have, for the interferometer resolution, $\Delta\nu_m = \Delta\nu_{osc}/F \cong 23$ MHz. If, for example, the length of the laser cavity is $L_1 = 1.5$ m, consecutive longitudinal modes are separated (see Chap. 5) by $\Delta\nu = c/2L_1 = 100$ MHz. Since $\Delta\nu_m < \Delta\nu$, the FP interferometer is able to resolve these longitudinal modes. Note that, since the frequency of the laser light is $\nu = c/\lambda \cong 5.83 \times 10^{14}$ Hz, the corresponding resolving power of the interferometer is $\nu/\Delta\nu_m = 2.54 \times 10^7$. This is a very high resolving power compared, e.g., to the best that can be obtained with a grating spectrometer ($\nu/\Delta\nu_m < 10^6$).