Optical beams and resonators

Introduction on transverse modes in optical resonators

Laser Physics course SK341X, 2015

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"Lasers", A. E. Siegman, 1986.

Contents

Introduction

<u>Aim of this chapter:</u>

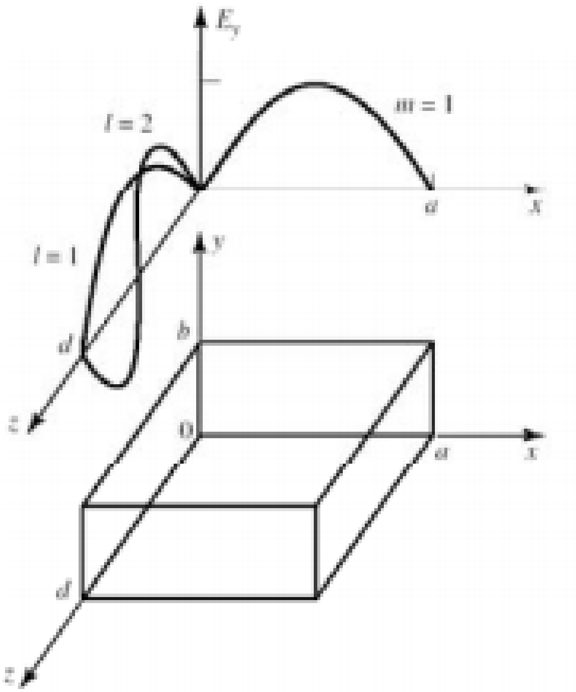
→ Describe the transverse mode properties of laser resonators

Laser resonator

What is a transverse mode ? ⟷ How can we analyzed it ?

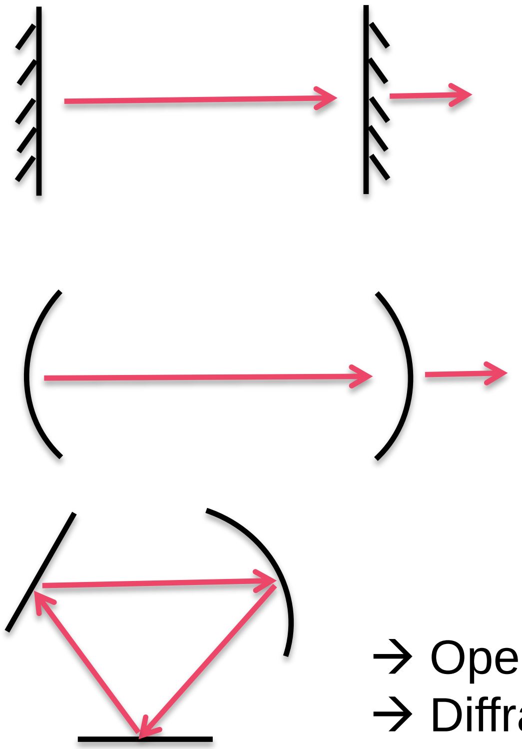
Optical resonator

Microwave cavities



→ Closed cavities

Optical resonator

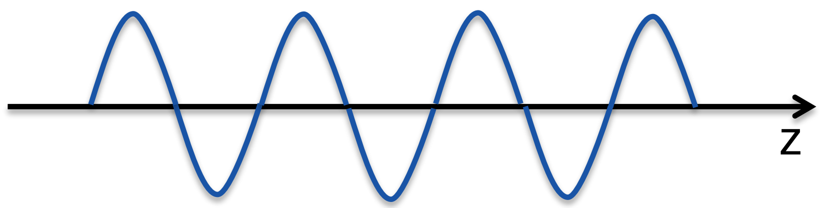


→ Open side cavities
→ Diffraction losses

Prof. K. R. Chu lecture notes, university of Taiwan.

Fundamental concepts – recirculating slab

Earlier chapters: plane-wave approximation, ignoring transverse spatial variation

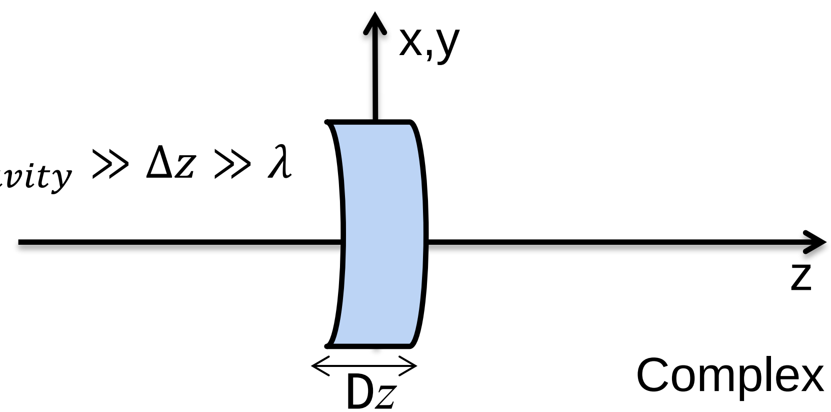


$$E = e^{j(\omega t - kz)}$$

$$k = \frac{\omega}{c}$$

Now: Recirculating "slab" of radiation → this means the optical energy traveling in +z direction contained in a small segment of length Δz in the cavity.

$$L_{cavity} \gg \Delta z \gg \lambda$$



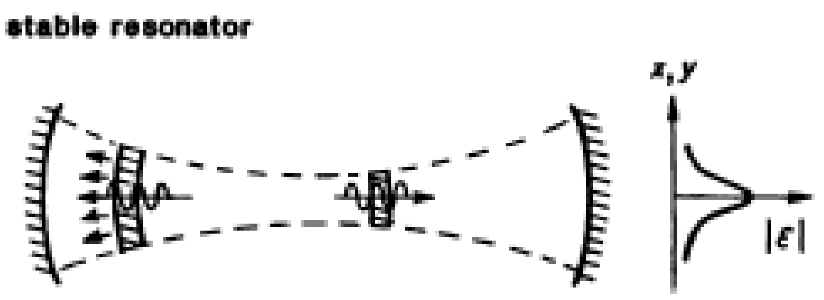
$$\mathrm{E(x,y,z)} = \mathrm{Re}\{\tilde{E}(x,y,z).\, e^{j(\omega t - kz)}\}$$

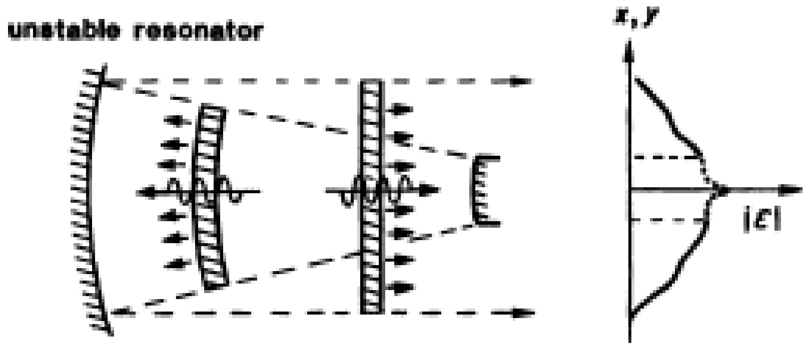
Plane-wave aspect

$$|\tilde{E}(x,y,z)|e^{j\phi(x,y,z)}$$

Complex phasor describing transverse amplitude and phase of the "slab"

Fundamental concepts – Stable/unstable resonators





Geometrically stable or unstable resonator → linked to ray stability inside the cavity

The transverse profile $\tilde{E}(x, y, z)$ is modified each roundtrip due to:

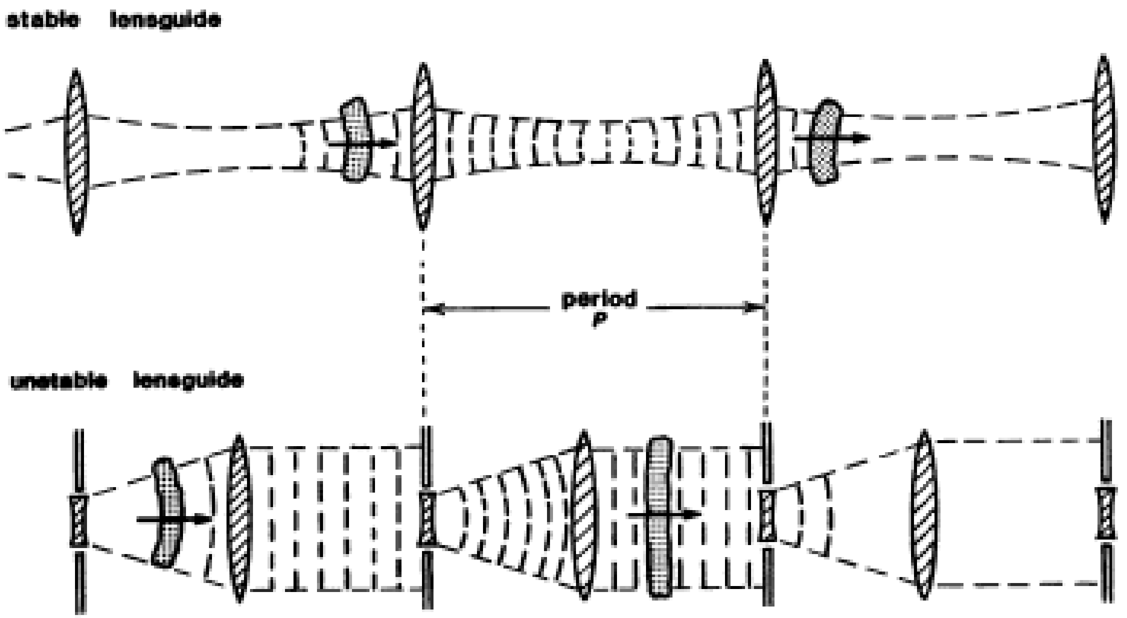
→ Propagation
→ Diffraction
→ Bounces on end mirrors
→ Passes through medium/rods/lenses/apertures…

Determine transverse mode properties of the cavity !

"Lasers", A. E. Siegman, 1986 p561.

Practical concept – Equivalent periodic lensguide

Equivalent periodic lens-guide method:
→ Replace roundtrips in resonator by repeated sections of an iterated periodic optical system



Ex:
Curved mirrors
→ thin lenses

→ Convert resonator geometry into waveguide design !

"Lasers", A. E. Siegman, 1986 p562.

Definition: eigenmodes and eigenvalues

Introducing concept of transverse cavity mode → Eigenmode

QUESTION: $\exists ?\ \widetilde{E_{nm}}(x,y)$; $\widetilde{E_{nm}}^{1}(x,y) = \widetilde{E_{nm}}(x,y)$ after one roundtrip

→ Same transverse form after one roundtrip but reduced amplitude (losses) and arbitrary phase-shift (propagation)

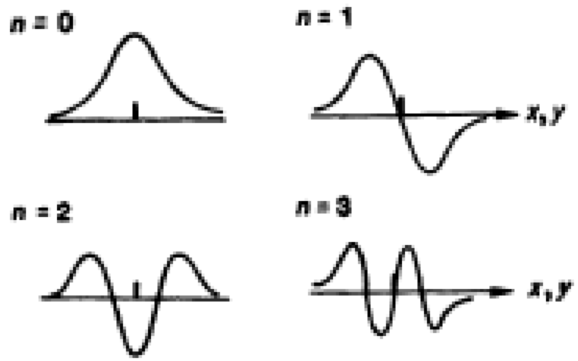
→ Self-reproducing transverse pattern = transverse mode of the resonator

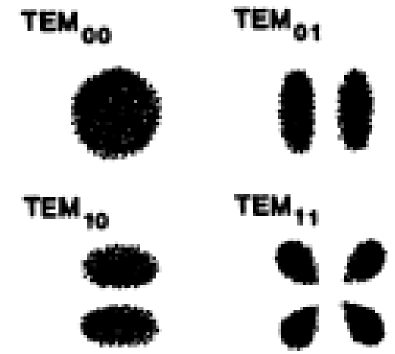
ANSWER: YES!

→ Simplest ones in geometrically stable resonators with curved mirrors
- → Modes are Hermite-Gaussian functions or Laguerre-Gaussian functions (cylindrical coordinates)
- → Plane-waves (or slightly spherical) X $\widetilde{E_{nm}}(x,y)$
- → Polarized ⊥ to direction of propagation → TEM_{nm}

Definition: eigenmodes and eigenvalues

$$E_{nm}(x,y,z) = E_0 \frac{w_0}{w(z)} \cdot H_n\left(\sqrt{2}\frac{x}{w(z)}\right)\exp\left(-\frac{x^2}{w(z)^2}\right) \cdot H_m\left(\sqrt{2}\frac{y}{w(z)}\right)\exp\left(-\frac{y^2}{w(z)^2}\right) \cdot \exp\left(-i\left[kz-(1+n+m)\arctan\frac{z}{z_R}+\frac{k(x^2+y^2)}{2R(z)}\right]\right)$$

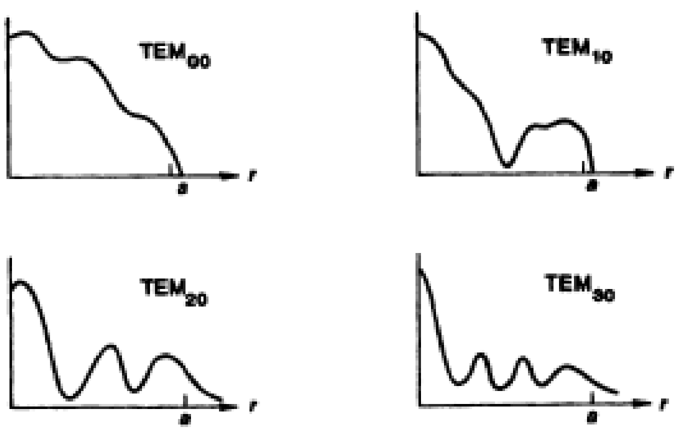




Curved mirrors

Stable cavity transverse modes

Flat mirrors



"Like Bessel functions"

→ Diffraction losses: amount of energy lost past the mirror edges

"Lasers", A. E. Siegman, 1986 p564.

Mathematical definition – Propagation Kernel

Problem: how to quantify propagation effects for the optical "slab" over 1 roundtrip
how to find the self-reproducing transverse modes

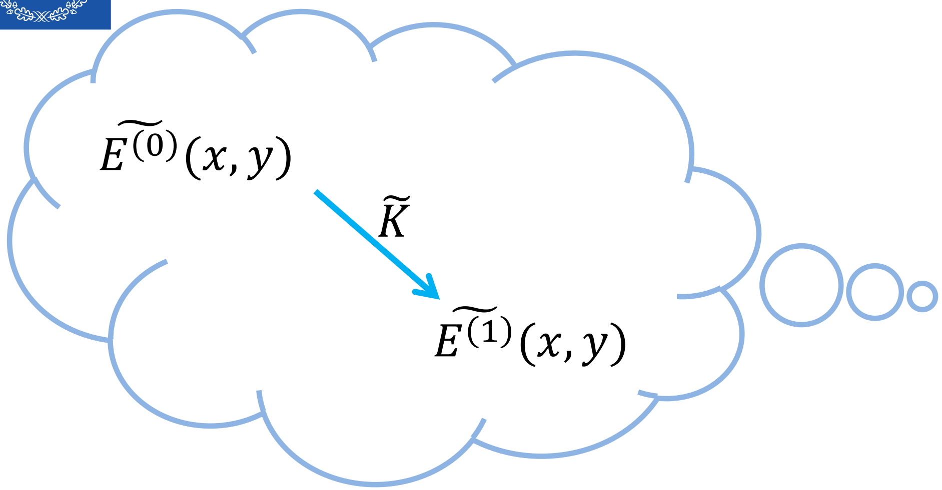
General *linear* transformation between field amplitude after one roundtrip in the plane $\boldsymbol{z = z_0}$ to the field amplitude in **the same plane** one roundtrip earlier
→ Propagation integral

$$\widetilde{E^{(1)}}(x, y, z_0) = e^{-jkp} \iint_{\text{Input plane}} \tilde{K}(x, y, x_0, y_0)\, \widetilde{E^{(0)}}(x_0, y_0, z_0) dx_0 dy_0$$

p: length of one roundtrip
K: propagation Kernel/propagator
→ Depends on the reference plane
→ Contains the details of the cavity (optical elements)

Evaluation of K in the next chapters!

Mathematical definition – Eigen-equation



Operator equation:

∃? $\widetilde{E_{nm}}(x,y)$ and ∃? $\widetilde{\gamma_{nm}}$;

$$\widetilde{\gamma_{nm}}\widetilde{E_{nm}}(x,y) = \iint \widetilde{K}(x,y,x_0,y_0)\widetilde{E_{nm}}(x_0,y_0)dx_0dy_0$$

Solutions of the Eigen-equation

→ determine the transverse modes

$$\widetilde{E_{nm}}^{1}(x,y) = \widetilde{\gamma_{nm}}\widetilde{E_{nm}}^{0}(x,y)e^{-jkp}$$

Mathematical definition – Eigenvalues

Complex eigenvalue $\widetilde{\gamma_{nm}}$:

→ Indices (n,m) for the transverse dimensions of the considered mode
→ $|\widetilde{\gamma_{nm}}| < 1$ in open side resonator, no gain in cavity

Lossless mirrors, power-loss per roundtrip:

$$\Pi_{diff} = 1 - |\widetilde{\gamma_{nm}}|^2$$

From diffraction losses at mirror edges and apertures

No laser gain	Laser gain
$\frac{\widetilde{E_{nm}}^k(x,y)}{\widetilde{E_{nm}}^0(x,y)} = \widetilde{\gamma_{nm}}^k$	$\widetilde{E_{nm}}^1(x,y) = \widetilde{\gamma_{nm}} e^{\alpha_m p_m - jkp} \widetilde{E_{nm}}^0(x,y)$
→ Exponential decay of amplitude	Condition for laser threshold: $\left\lvert \frac{\widetilde{E_{nm}}^1(x,y)}{\widetilde{E_{nm}}^0(x,y)} \right\rvert = \left\lvert \widetilde{\gamma_{nm}} e^{\alpha_m p_m - jkp} \right\rvert = 1$

Mathematical definition – Properties of eigenmodes

Existence: → not automatically guaranteed ($\tilde{K}$ is not always Hermitian)

Orthogonality: → generally not power orthogonal but biorthogonal

$$\iint \widetilde{E_{nm}}(x,y)\widetilde{E_{pq}}^{*}(x,y)dxdy = \delta_{np}\delta_{mq}$$ ✗ **BUT** $$\iint \widetilde{E_{nm}}(x,y)\widetilde{E_{pq}}^{\dagger}(x,y)dxdy = \delta_{np}\delta_{mq}$$

Transverse mode traveling in opposite direction

Completeness: → generally not a complete basis set

$$\tilde{E}(x,y) \overset{?}{\equiv} \sum_{n,m} c_{nm}\widetilde{E_{nm}}(x,y)$$

Attenuation of the modes

Arbitrary initial field $\widetilde{E^{(0)}}$ such that:

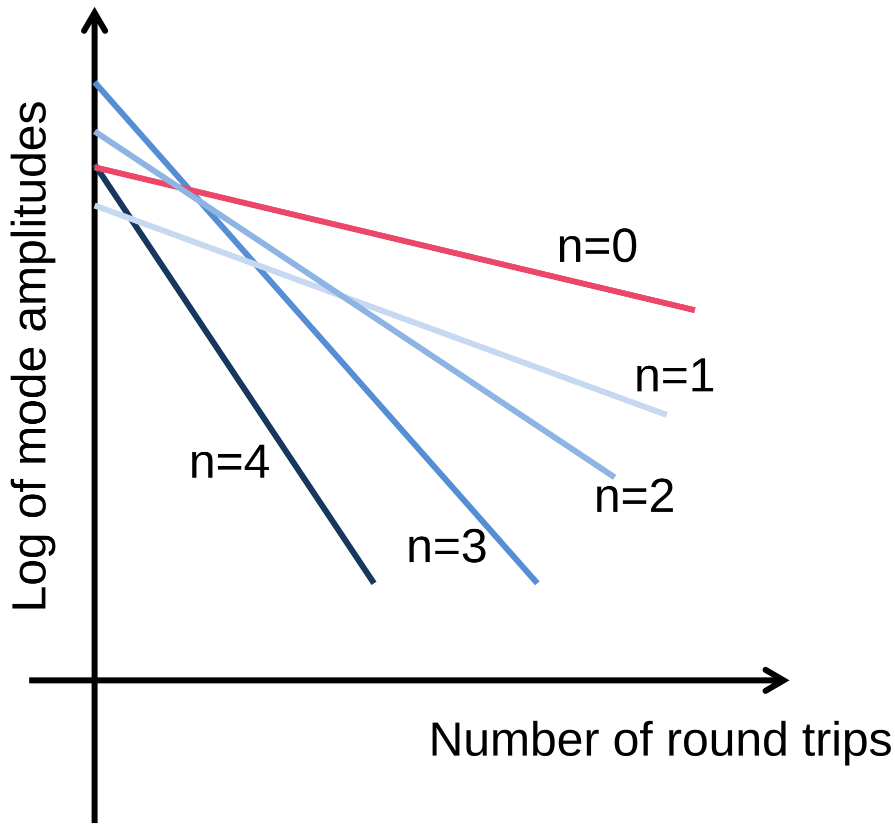
$$\tilde{E}^{(0)}(x,y) = \sum_{nm} c_{nm}\tilde{E}_{nm}(x,y)$$

$$\widetilde{E^{(k)}}(x,y) = \sum_{n,m} c_{nm}\widetilde{\gamma_{nm}}^{k}\,\widetilde{E_{nm}}^{(k)}(x,y)$$

Relative amplitude attenuates as $|\widetilde{\gamma_{nm}}|^k$

nm=00 transverse mode with largest eigenvalue/smallest loss

Become dominant after enough roundtrips

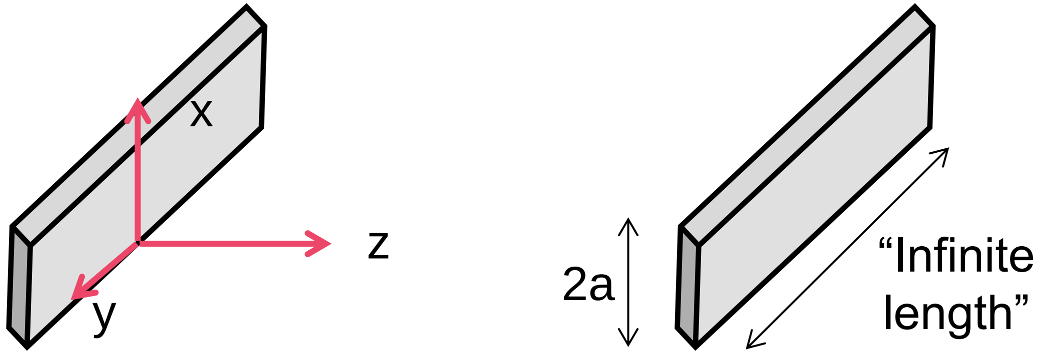


Fox and Li approach

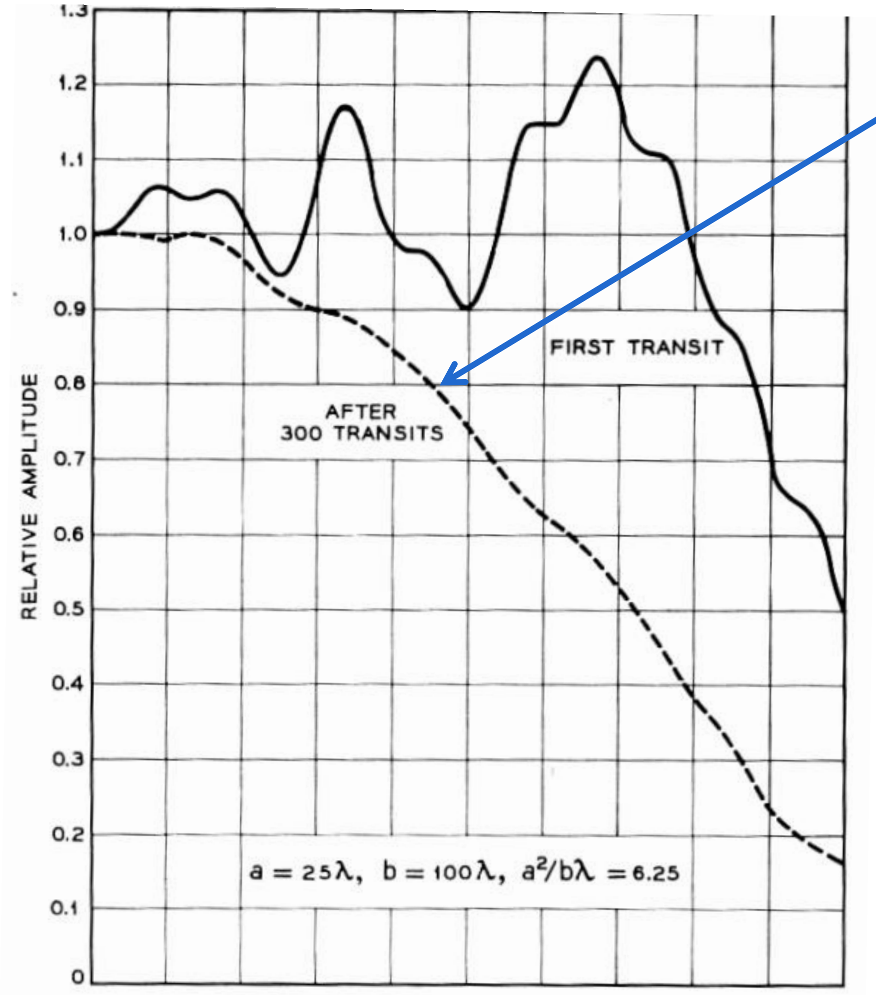
Aim: → Find the lowest-order resonator transverse mode (nm=00)

Numerical computation: → Iterative roundtrips
→ Repeating integration of the propagation equation
→ Huygens integral Kernel for simple cavities

First calculation: → "Strip resonator", variation only in x-direction
→ Uniform field pattern across mirror $\widetilde{E^{(0)}}(x, y) = 1$

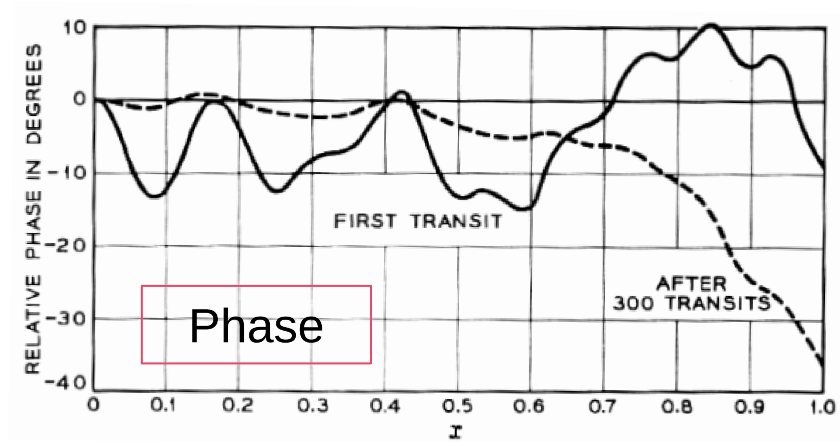


Fox and Li approach - Results



Field amplitude after
k=300 bounces

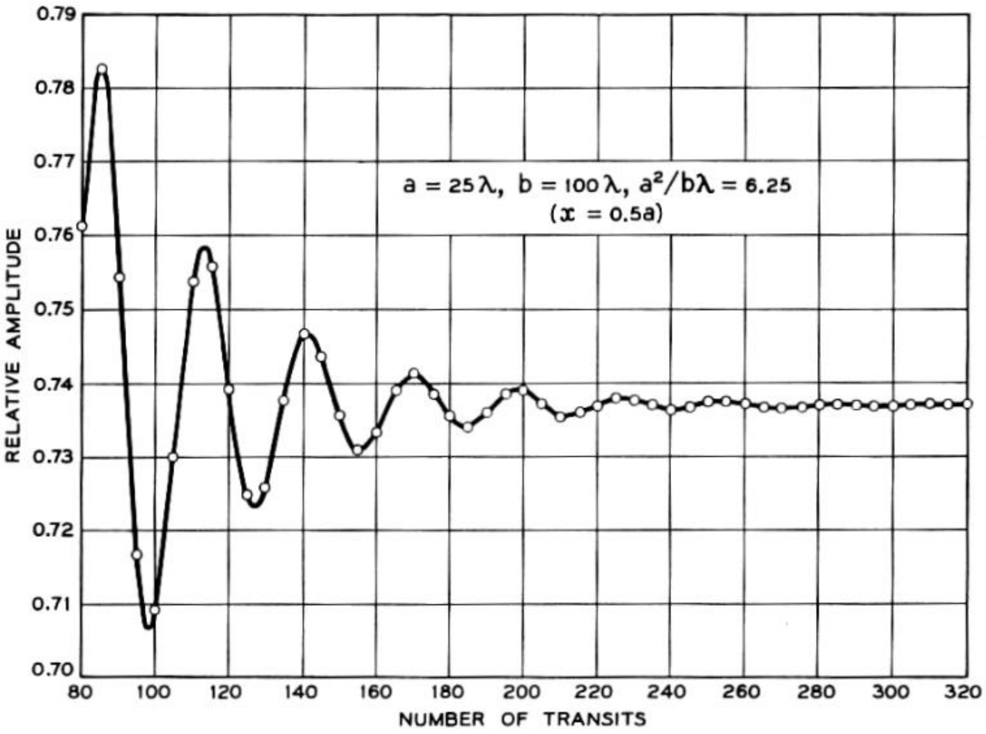
$$\widetilde{\gamma_{00}} = \lim_{k\to\infty} \frac{\widetilde{E^{(k+1)}}(x,y)}{\widetilde{E^{(k)}}(x,y)}$$



Simple resonator with open sides
→ always has lowest-order transverse mode self-reproducing !

"Resonant Modes in a Maser Interferometer", A. G. Fox and T. Li, The Bell System Technical Journal, 1961.

Fox and Li approach – ”Mode Beating”



Plane mirrors

Aim: → Finding higher-order transverse modes

Ex: → 2 dominant fields left
→ Periodic beating/interference

How: → Eigenvalue next higher mode deduce from “dying rate” of the beating

OBS! Have a look at Prony’s method to derive N higher-order transverse modes.

“Resonant Modes in a Maser Interferometer”, A. G. Fox and T. Li, The Bell System Technical Journal, 1961.

Resonator eigenfrequencies

Aim: → Find exact resonance frequencies of transverse modes

How: →Total roundtrip phase-shift of cavity: $q * 2\pi$
→ Roundtrip phase-shift due to laser medium: $\Delta\beta_m p_m$
→ Eigenvalue of transverse mode: $\widetilde{\gamma_{nm}} = |\widetilde{\gamma_{nm}}| e^{j\psi_{nm}}$
→ Propagation: e^{-jkp}

$$e^{-jkp - j\Delta\beta_m p_m + j\psi_{nm}} = e^{-j2\pi q}$$

q is a large integer $\left(q \sim \frac{p}{\lambda}\right)$

$$\omega = \omega_{qnm} = \frac{2\pi c}{p}\left(q + \frac{\psi_{nm}}{2\pi} - \frac{\Delta\beta_m p_m}{2\pi}\right)$$

Where: $k = \frac{\omega}{c}$

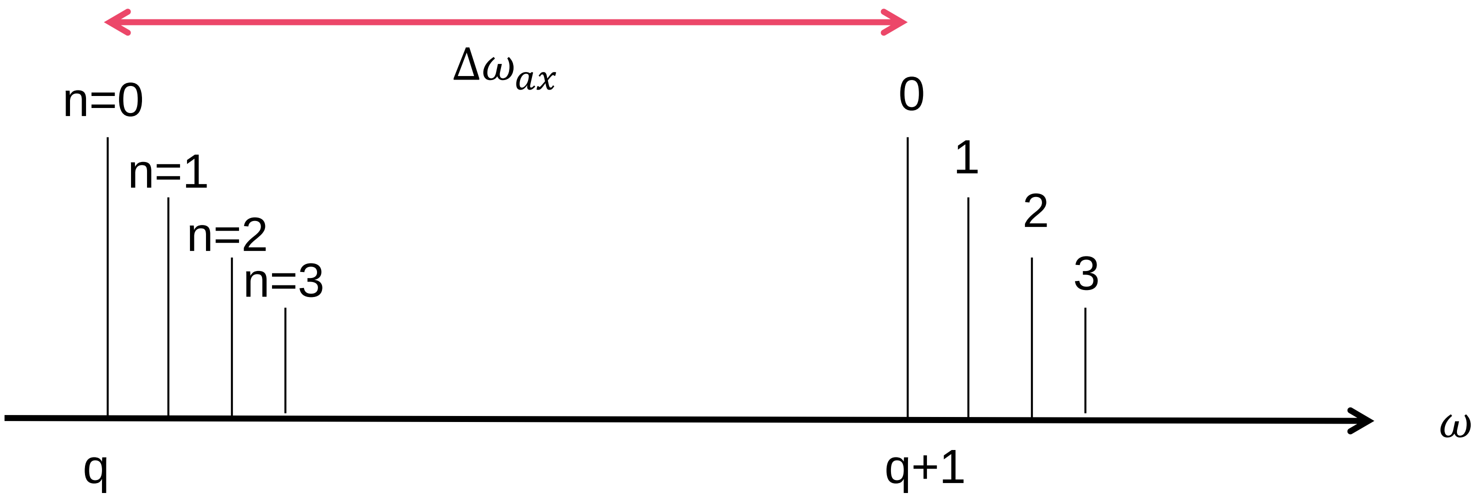
Small correction to plane-wave resonance freq.
→ $\omega_q = q.\frac{2\pi c}{p}$

Small correction slightly ≠ for each transverse mode
→ due to χ' medium

Mode beating 2

Consequences:
→ ≠ transverse modes have slightly ≠ resonant frequencies (because of ψ_{nm})

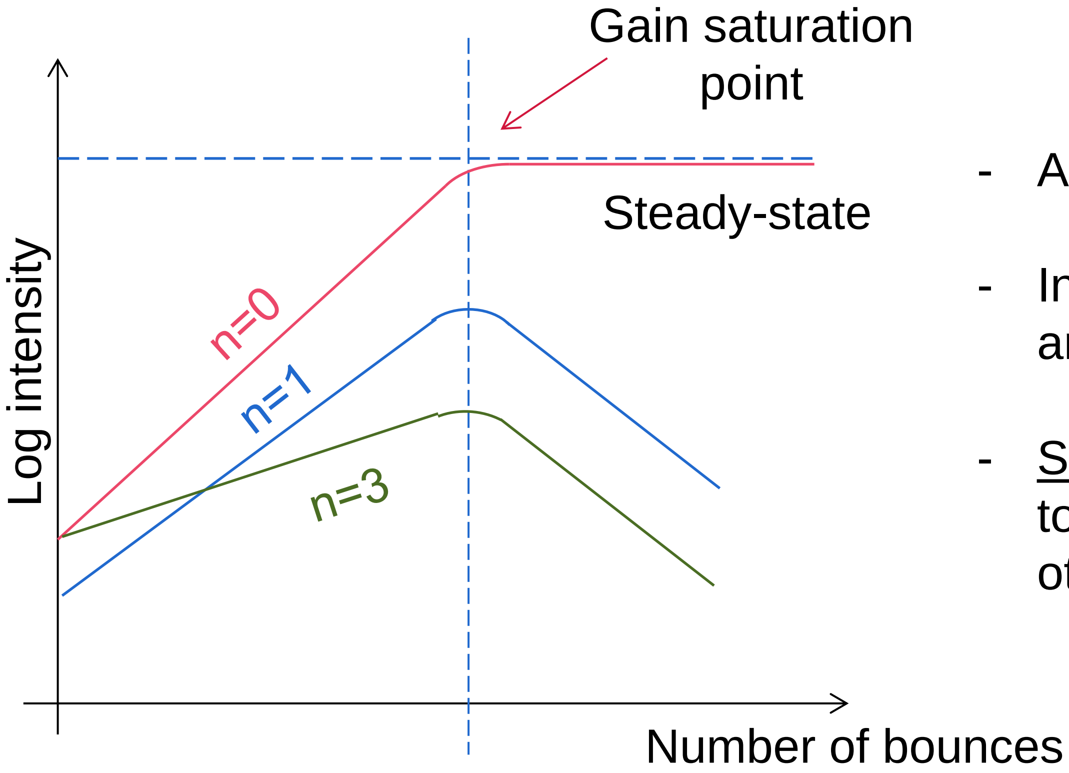
Axial + Transverse mode beating !



Build-up operation

Previous results: optical resonator with initial injected field and **NO GAIN**

OBS! What if we consider gain ?



- Above threshold
- Initial field distribution circulates and grows in amplitude
- Simple situation: 00 mode grows to saturation (steady-state) and other modes die out.

Alternative to Fox and Li approach

Fox and Li approach → **Serious convergence problems !**

Methods based on field tracing: **MPE** → Minimal Polynomial Extrapolation
RRE → Reduced Rank Extrapolation

Computational time ↘ 70%

Dominant resonator Eigen-mode:

$$\vec{V} = (V_1, V_2, V_3, V_4, V_5, V_6)^T = \left(E_x, E_y, E_z, H_x, H_y, H_z\right)^T$$

Calculated thanks to roundtrip operator:

$$\gamma\vec{V}(x, y, z_0) = \hat{\mathcal{R}}\vec{V}(x, y, z_0)$$

Classic Eigen-value problem: $\gamma_l \leftrightarrow V_l$

"Acceleration of dominant transversal laser resonator Eigen-mode calculation by vector extrapolation methods", D. Asoubar and Al., unpublished 2014.

Vector extrapolation

Fox and Li → Relative power loss / roundtrip

$= 1 - |\gamma_l|^2$

→ Iterative power method

Convergence of power method:

$$\left(\frac{|\gamma_{l,2}|}{|\gamma_{l,1}|}\right)^j$$

Problem!

Determination of the following Eigen-mode:

$$\overrightarrow{V^{(j+1)}}(x, y, z_0) = \frac{1}{\alpha(j)} \hat{\mathcal{R}} \overrightarrow{V^{(j)}}(x, y, z_0)$$

Solution: MPE and RRE → can be applied to nonlinear and coupled $\hat{\mathcal{R}}$ types

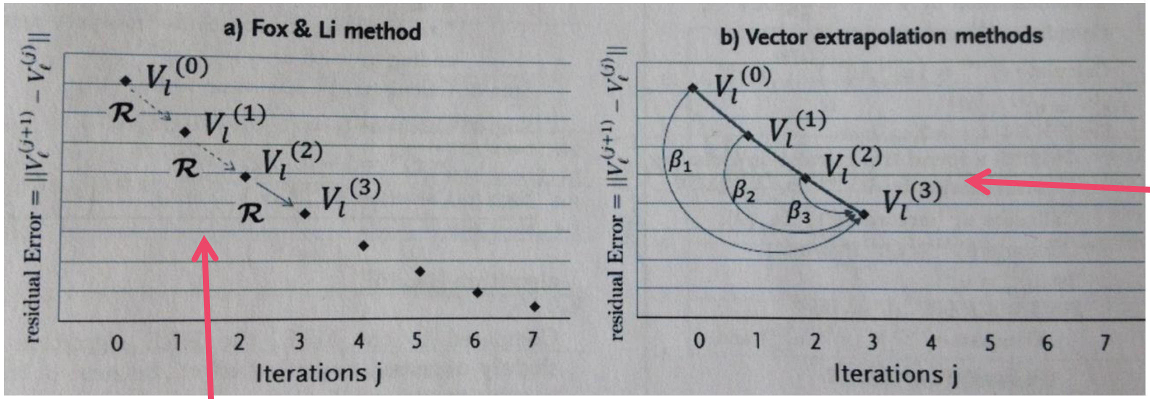
$$\lim_{j \to \infty} V_l{}^j (x, y, z_0) = W_l(x, y, z_0)$$

Weighted sum for k iterations:

$$W_l(x, y, z_0) = \sum_{j=0}^{k} \beta_l{}^j V_l{}^j (x, y, z_0)$$

“Vector extrapolation methods with applications to solution of large systems of equations and to PageRank computations”, A. Sidi, Computers and Mathematics with Applications, vol.56, 2008.

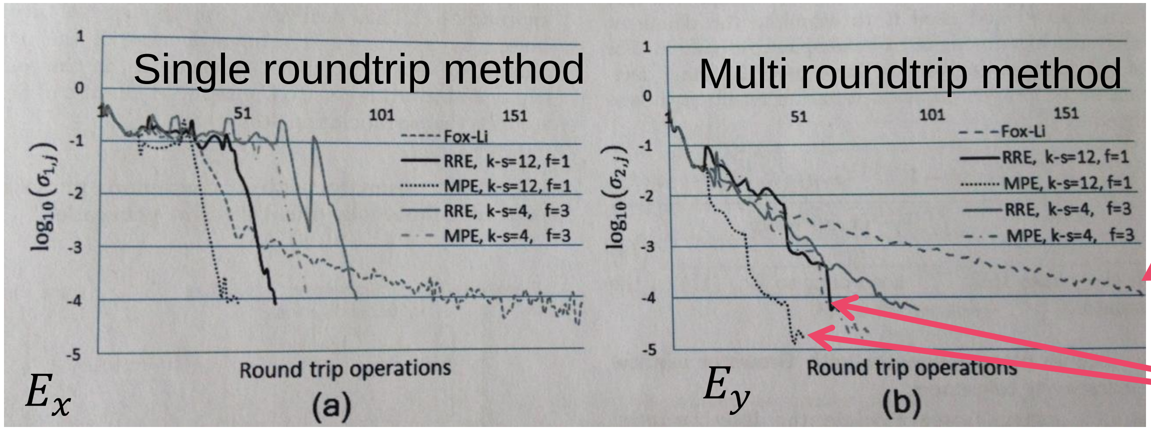
Comparison



MPE, RRE: several iteration results used to construct dominant Eigen-mode

Fox and Li: only the previous iteration result V_l^j is used to construct the next step

Convergence for both methods



Fox & Li

RRE/MPE

"Acceleration of dominant transversal laser resonator Eigen-mode calculation by vector extrapolation methods", D. Asoubar and Al., unpublished 2014.

Conclusion

Real world: → Competition between transverse modes
→ Local saturation of the gain by 00
→ Leaves unsaturated gain at other transverse positions
→ High-gain short pulse laser
→ Insufficient time to grow dominating mode 00
→ ≠ modes can see ≠ gains
→ Narrow atomic linewidth
→ Can favour higher-order modes

Problem: → ≠ transverse modes can oscillate **simultaneously**

Single mode operation: → Minimize losses for 00 mode
→ Allow mode discrimination
→ **Adjustable aperture inside the cavity**

Tools: → For evaluation of roundtrip propagation in resonators
→ Ray matrix (chapter 15)
→ Paraxial wave optics (Chapter 16)

References

Fox and Li numerical computation:

→ "Resonant Modes in a Maser Interferometer", A. G. Fox and T. Li, Bell System Technical Journal, 1961.
→ "Computation of Optical Resonator Modes by the Method of resonance Excitation", A. G. Fox and T. Li, IEEE Journal of Quantum Electronics, Vol.4, No. 7, 1968.
→ "Modes in a Maser Interferometer with Curved and Tilted Mirrors", A. G. Fox and T. Li, proceedings of the IEEE, 1963.
→ "Numerical Simulation of Laser Resonators", J. Yoo, Y. U. Jeong, B. C. Lee and Y. J. Rhee, Journal of the Korean Physical Society, Vol.44, No.2, 2004.

A review of all works on optical resonator with spherical mirrors:

→ "Laser Beams and Resonators", H. Kogelnik and T. Li, Applied Optics vol.5, 1966.

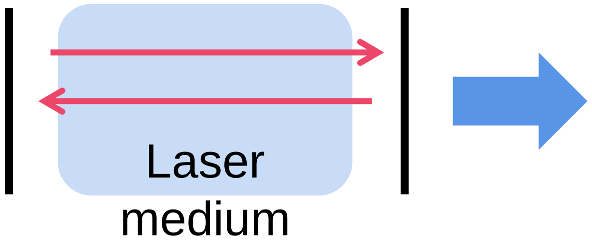
Random lasers and their modes

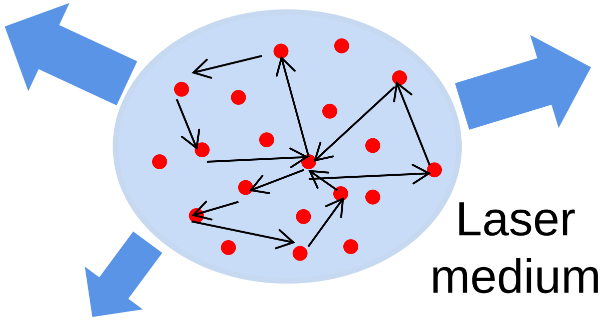
Self-consistency theory

Contents

Random Lasers

Conventional laser	Random laser





Material: provides optical gain
Cavity: traps the light

Total gain in cavity > losses
→ Threshold
→ Lasing

Modes determined by the cavity!

No confinement **BUT**:
- Multiple scattering
- Diffusive, longer paths

Gain path length > losses
→ Lasing effect

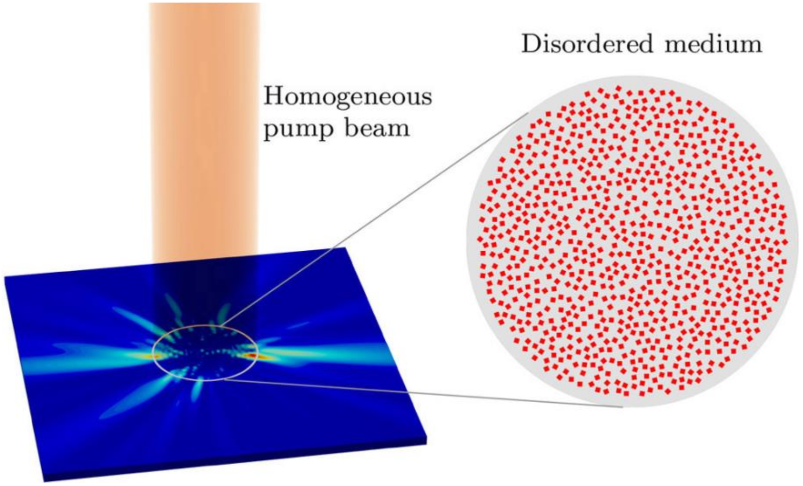
Modes determined by multiple scattering !

Which materials ?

Random laser = disordered amplifying material
multiple scattering

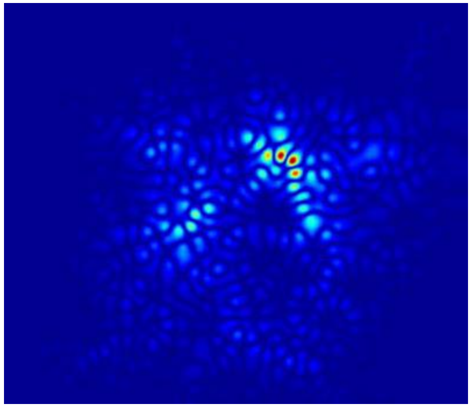
Strong enough scattering to get optically thick material ($\ell \ll L$)

EX: → Reduced laser crystal into powder
→ Suspension of micro-particles in laser dye (Laser paint)
→ Semi-conductor powder
→ Assembling mono-disperse spheres in random fashion (favour resonant scattering and lasing at resonance freq.)



Matthias Liertzer and Stefan Rotter, SPIE 2013.

Random laser process



Parameters:
- Mean free path
- Diffusion constant
- Gain volume governed by the pump spread*

Random laser process:
- Disordered medium
- Light trapped by multiple scattering
- Efficient amplification
- Excited gain material to get population inversion

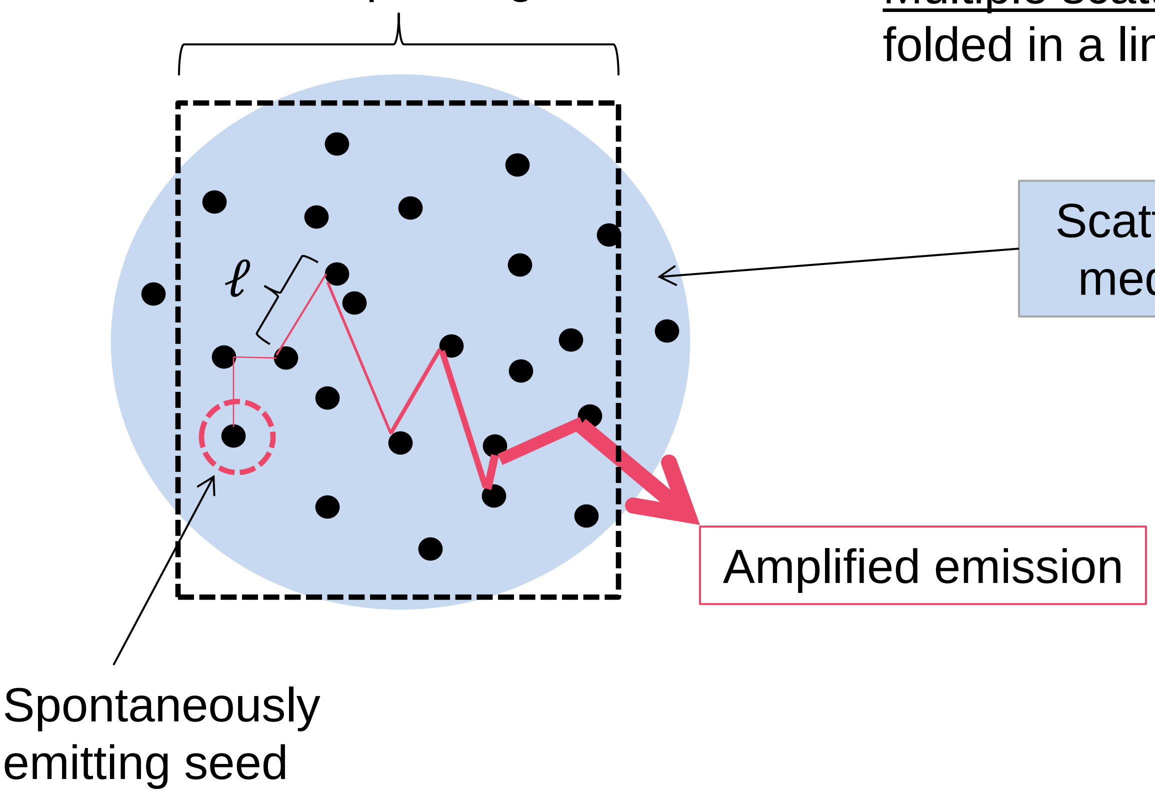
Advantages:
- No confining mirrors
- Coherent

Emission drawbacks:
- At different wavelengths
- In all directions

“Modes of random lasers“, J. Andreasen and al., Advances in Optics and Photonics vol.3, 2011.

Random laser theory

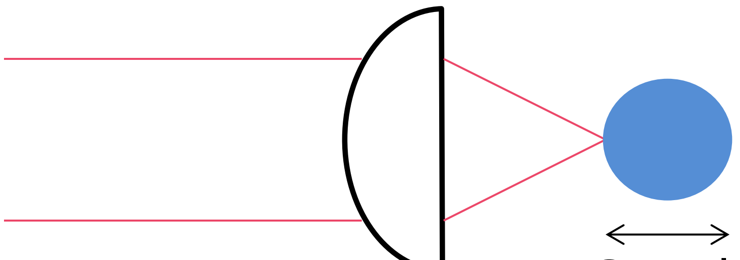
<u>Multiple scattering</u>: light paths are folded in a limited pumped volume

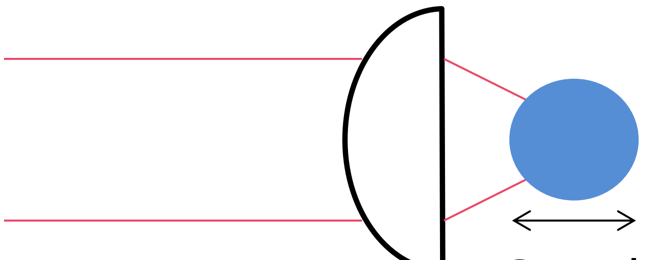


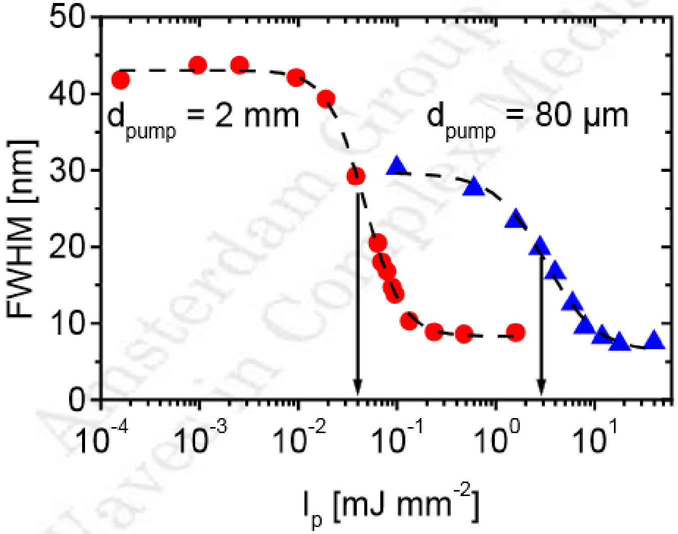
- $\ell \gg L$ → linear propagation
- $\ell \ll L$ → diffusion
- $\ell \rightarrow 0$ → Strong scattering: **Anderson localization** of light

ℓ is the mean free path

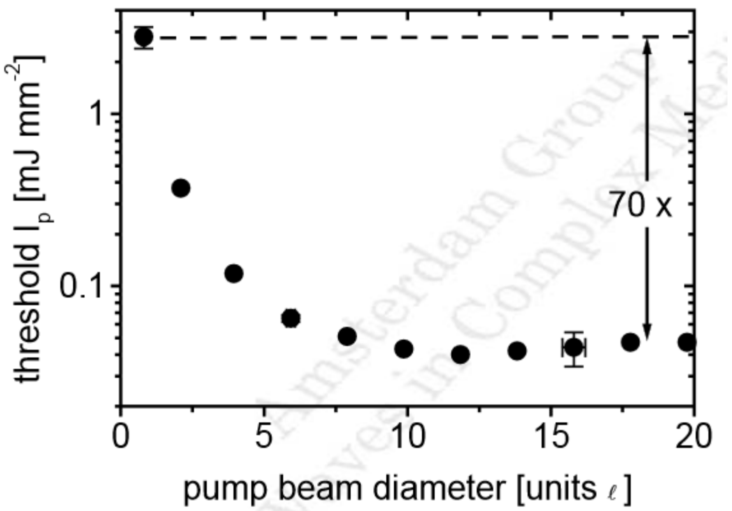
Influence of pump spread on laser threshold







Diameter of the pump is changed



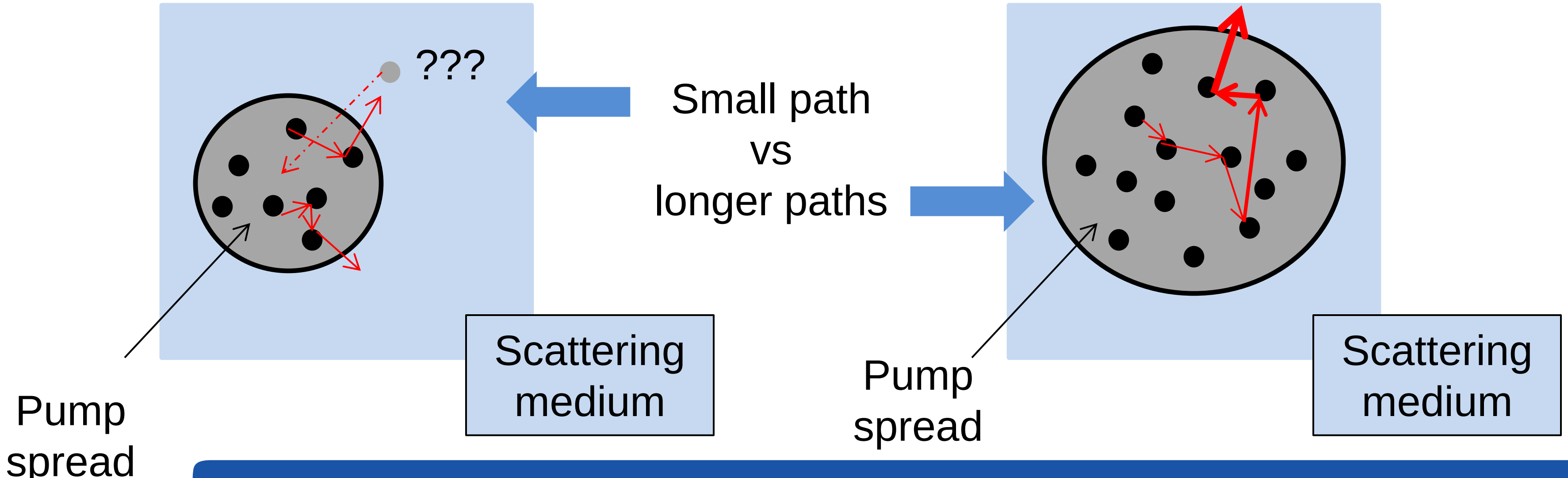
Results obtained by Gijs Van Soest at Amsterdam University, Optics Letters vol. 24, 1999.

Influence of pump spread on laser threshold

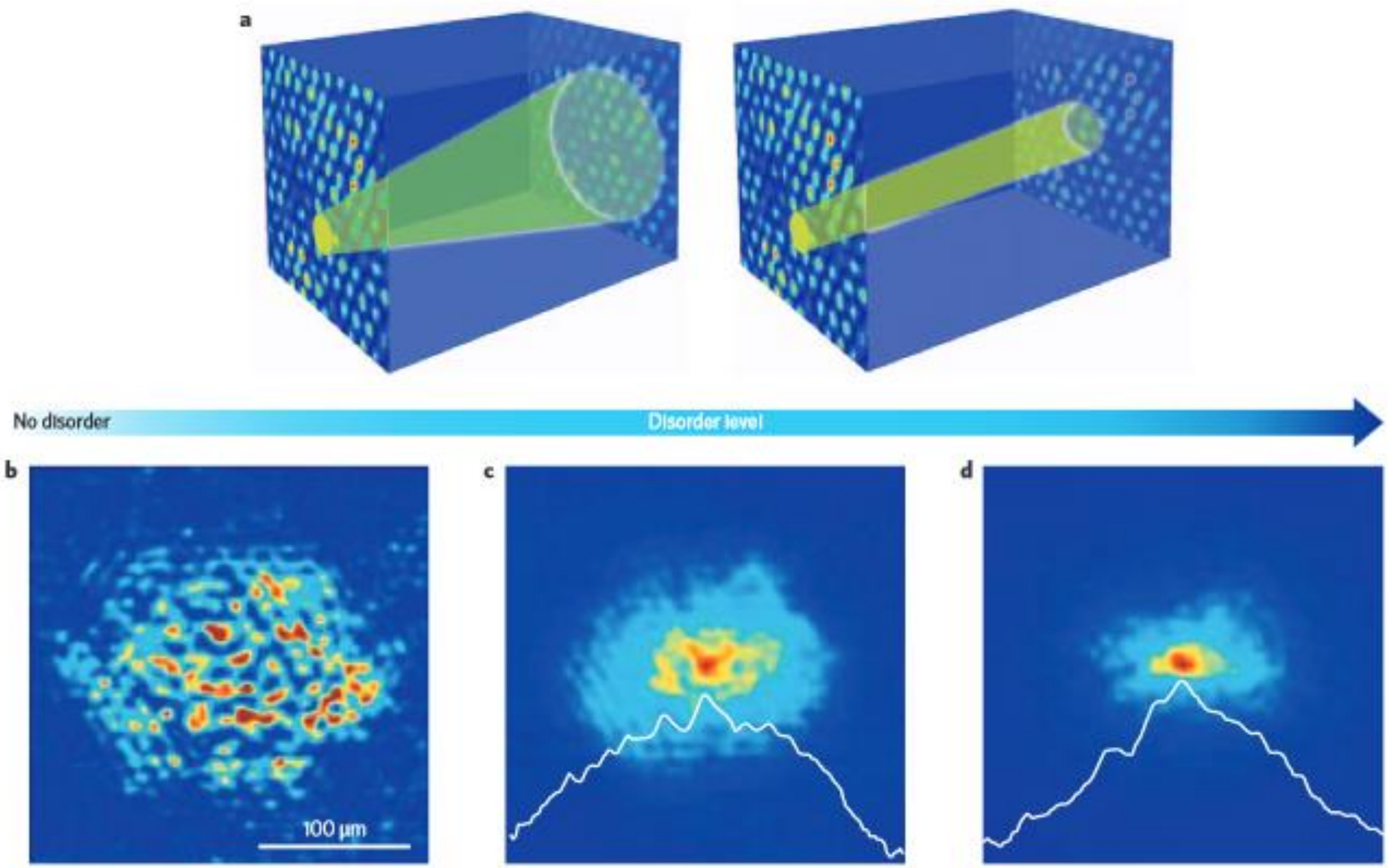
<u>Why ?</u>

Small pump spread → Small gain volume → Light undergoes short paths before leaving the gain volume → Probability to return in gain volume is small (losses)

→ High laser threshold



Anderson localization of light



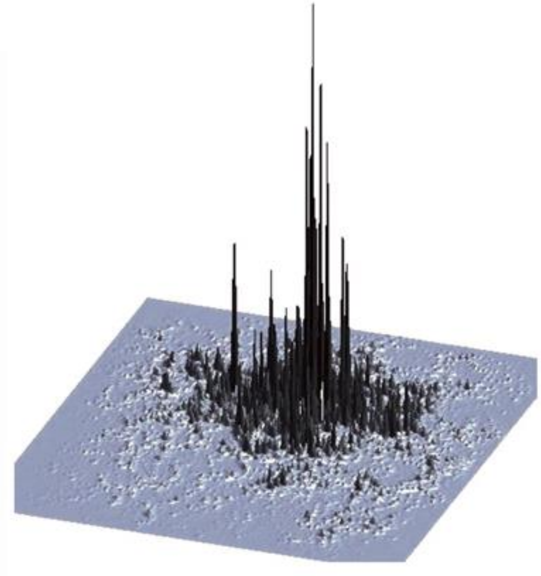
Localization takes place in media where: $k\ell \leq 1$ (Ioffe-Regel criterion)

“Anderson localization of light”, M. Segev and al., Nature Photonics, vol.7, 2013.

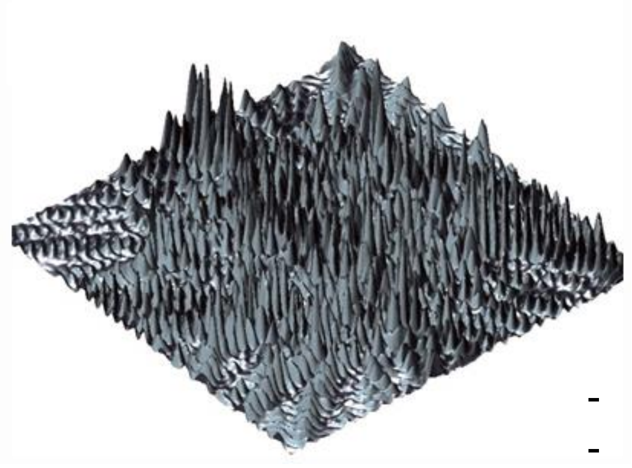
Mode structure of a Random Laser

Interference effects to describe mode structure
→ Multiple scattering → Granular distribution → Speckle

Halt in free propagation of wave
→ Formation of randomly shaped modes with exponentially decaying amplitude



Localized mode



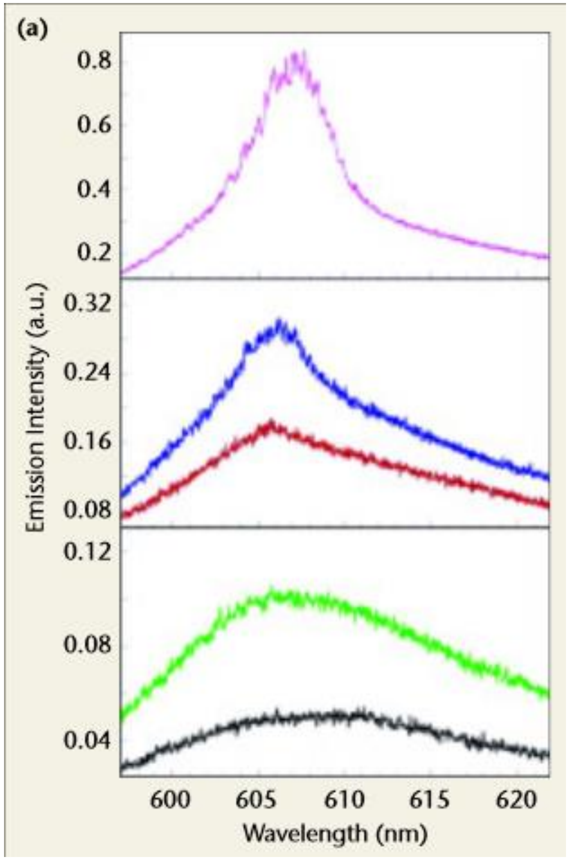
Extended mode

- Easy to get extended modes
- Difficult to design material to get localized modes

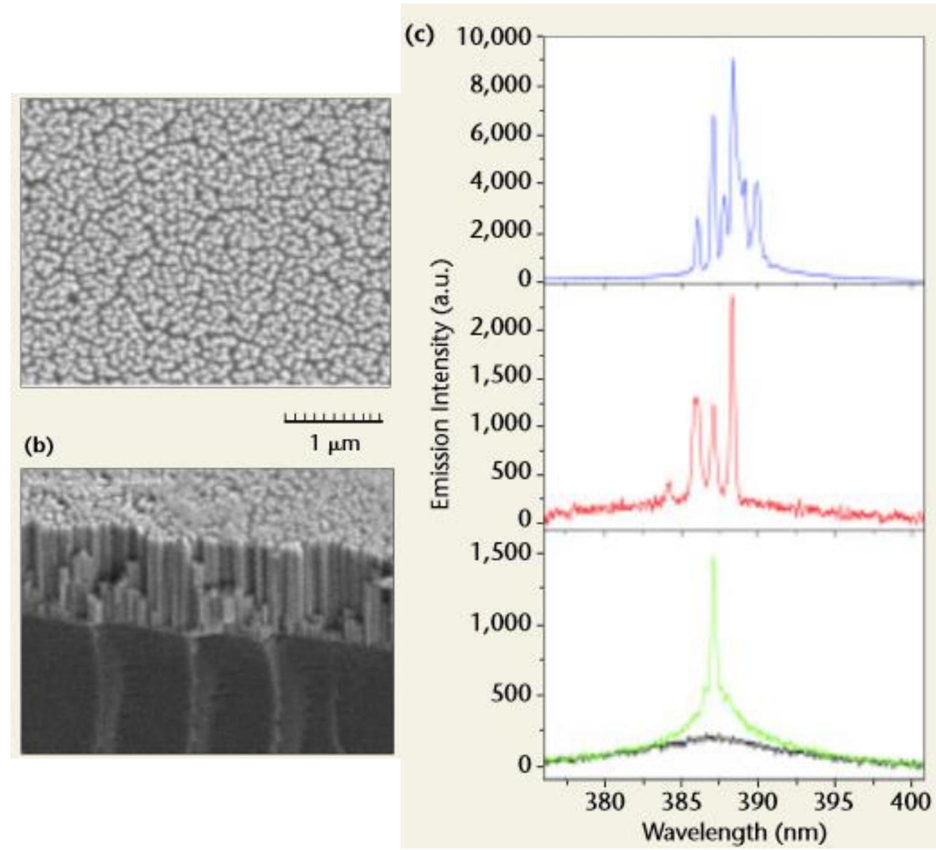
- Strong scattering
- Scattering elements size of λ
- High refractive index (s-c !)

“The physics and applications of random lasers”, D. S. Wiersma, Nature Physics, vol.4, 2008.

Emission spectra of random lasers



Suspension of ZnO microparticles in Rhodamine 640 for different pump powers



ZnO nanorods for different pump powers

“Random lasers: Development, features and applications”, Hui Cao, Optics and Photonics News, 2005.

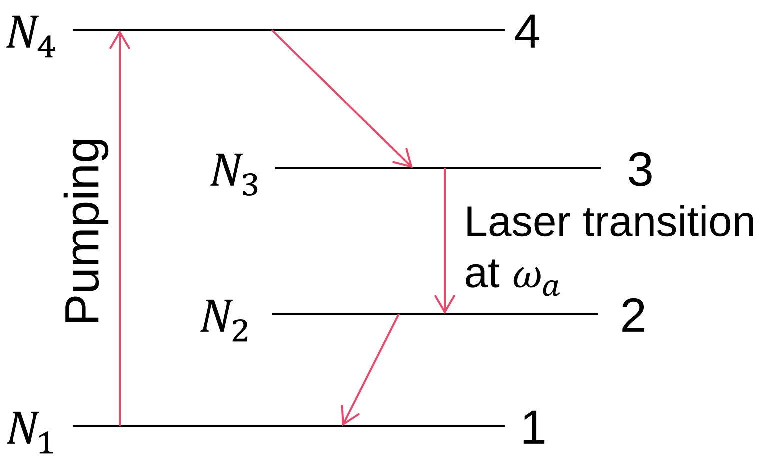
Localized light – Time dependent model

<u>First approximation:</u>

→ Localized modes in scattering system are like modes of standard optical cavities (FP)

→ Quasi-bound states (QB)

Solve time dependent Maxwell equations coupled with population equations 4-level system



$$\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - W_p N_1$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} - \frac{E}{\hbar\omega_a}\left(\frac{dP}{dt}\right)$$

$$\frac{dN_3}{dt} = \frac{N_4}{\tau_{43}} - \frac{N_3}{\tau_{32}} + \frac{E}{\hbar\omega_a}\left(\frac{dP}{dt}\right)$$

$$\frac{dN_4}{dt} = -\frac{N_4}{\tau_{43}} + W_p N_1$$

“Lasers”, A. Siegman, University Science Books, 1986.

Localized light – First draft

The polarization obeys the following equation:

$$\frac{d^2P}{dt^2} + \Delta\omega_a\left(\frac{dP}{dt}\right) + \omega_a^2 P = \kappa\Delta N.E$$

Where:

$$\Delta N = N_2 - N_3$$

$$\Delta\omega_a = \frac{1}{\tau_{32}} + \frac{2}{T_2}$$

$$\kappa = \frac{3c^3}{2\omega_a^2\tau_{32}}$$

+

P being a source term of Maxwell equations:

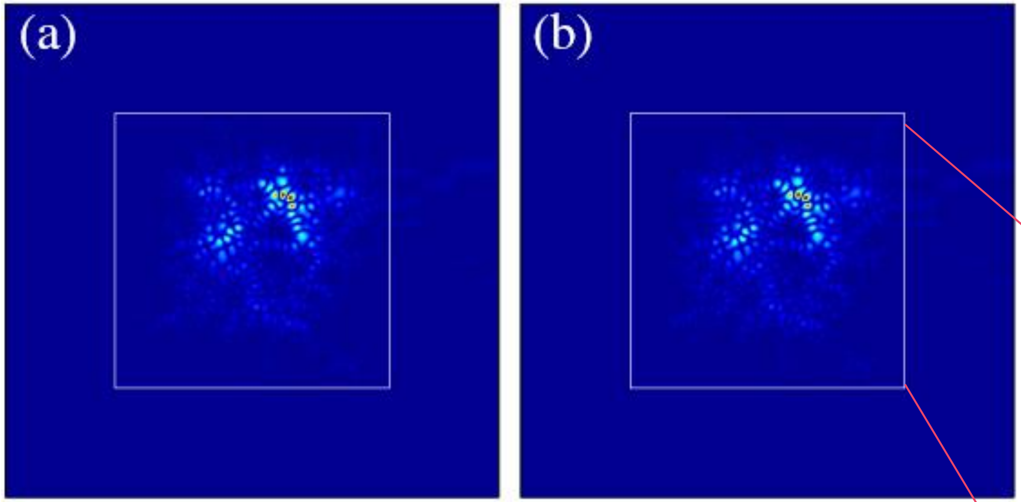
$$\frac{\partial H}{\partial t} = -c\nabla \times E$$

$$\epsilon(r)\left(\frac{\partial E}{\partial t}\right) = c\nabla \times H - 4\pi\left(\frac{\partial P}{\partial t}\right)$$

Describe the randomness of the system

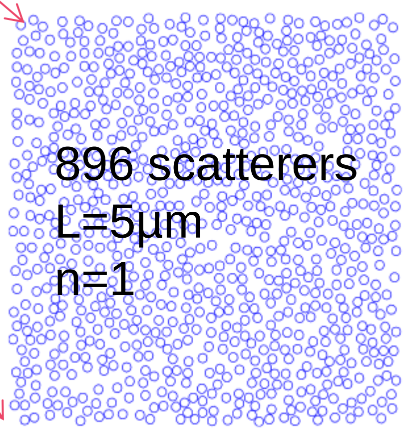
“Lasers”, A. Siegman, University Science Books, 1986.

Localized light – Numerical solutions



Localized case:

a) Lasing mode (TLM)
b) QB state in same random system but without gain



Optical index contrast (between medium and scatterers) $\Delta n = 1$

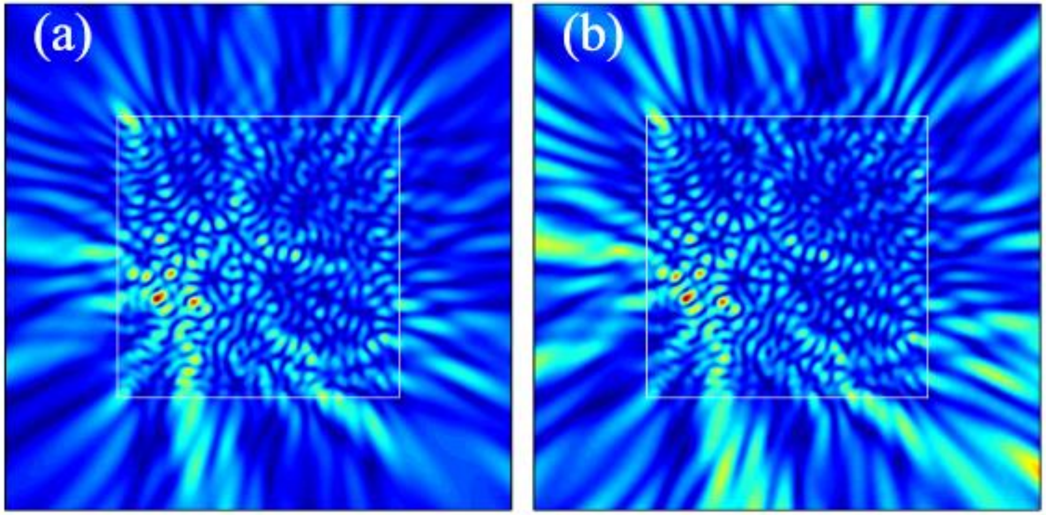
QB states have similar features as Eigen-modes of a conventional cavity

“Modes of random lasers”, J. Andreasen and al., Advances in Optics and Photonics vol.3, 2011.

Diffusive light – First draft

Recently shown:

→ Even diffusive systems with low-Q resonances can exhibit lasing with resonant feedback



Optical index contrast (between medium and scatterers) $\Delta n = 0.25$

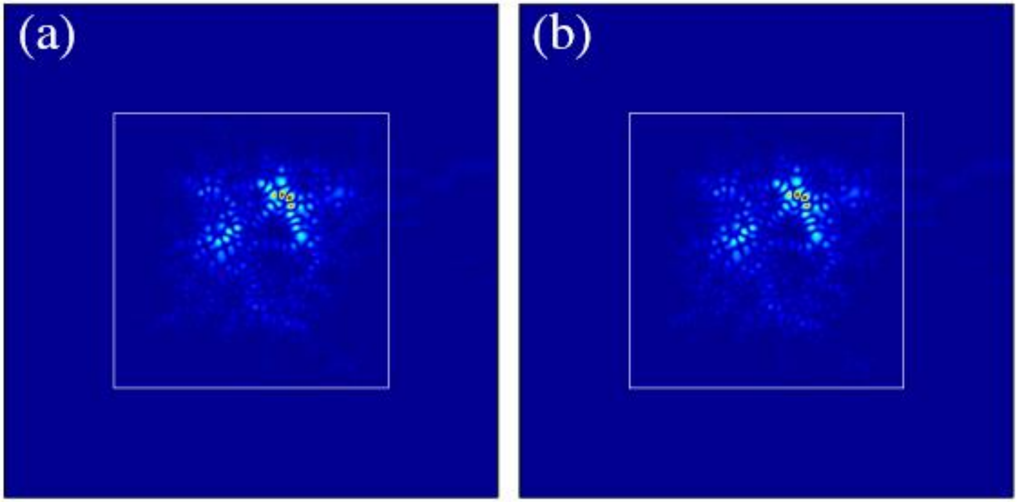
Diffusive case:

a) Lasing mode (TLM)
b) Lasing mode with pump off + P=0 (resonances of passive system)

Modes extend outside the "cavity" and both cases differ outside cavity

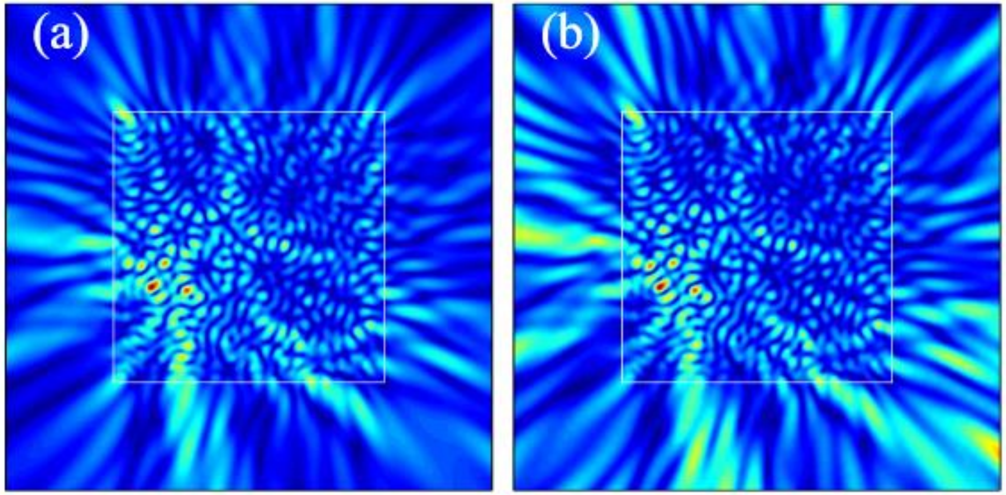
"Modes of random lasers", J. Andreasen and al., Advances in Optics and Photonics vol.3, 2011.

Localized case vs Diffusive case



Localized case:
a) Lasing mode
b) QB state in same random system but without gain

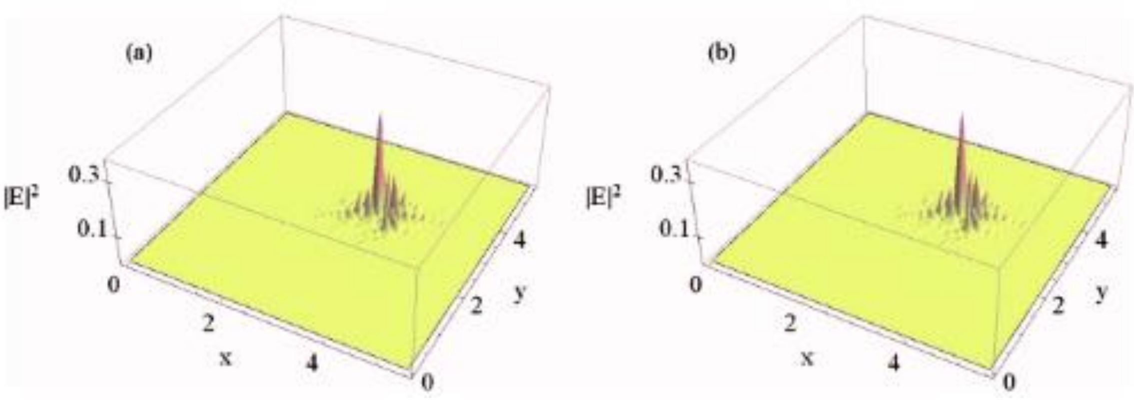
Closeness of lasing modes and passive cavity resonances



Diffusive case:
a) Lasing mode
b) Lasing mode with pump off

Lasing modes rather close to QB states *inside scattering medium*.

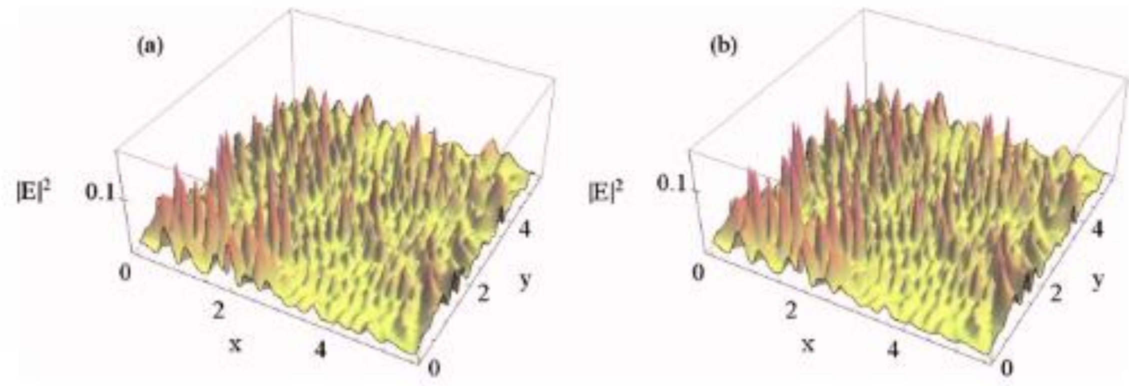
2D random laser – Time independent method



$\sigma_d = 0.05\%$

Localized case:
a) QB state
b) Corresponding lasing mode

$n_s = 2$



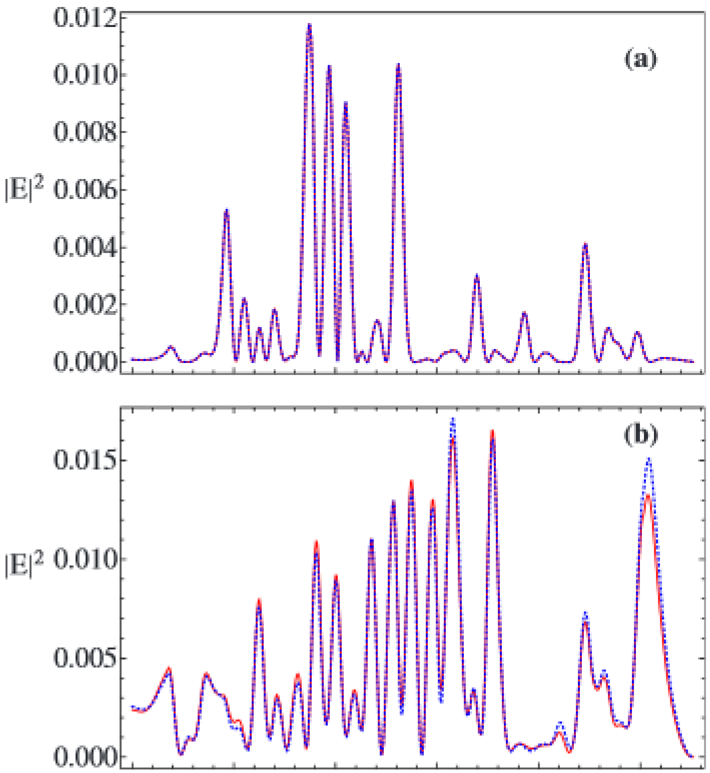
$\sigma_d = 14.5\%$

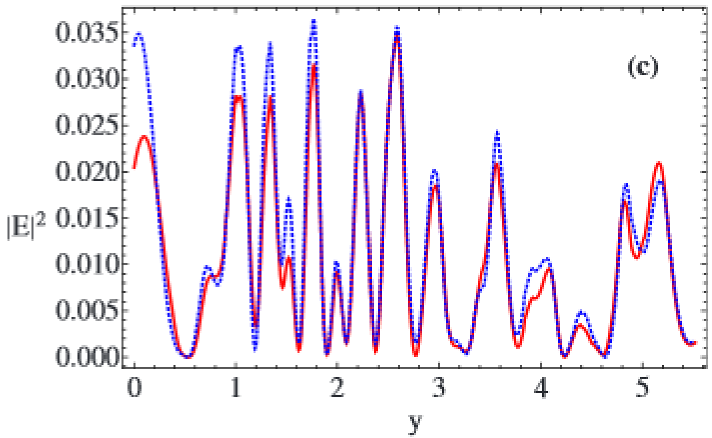
Diffusive case:
a) QB state
b) Lasing mode

$n_s = 1.25$

“Modes of random lasers”, J. Andreasen and al., Advances in Optics and Photonics vol.3, 2011.

Time independent method - Comparison





a) $n_s = 1.75$
b) $n_s = 1.5$
c) $n_s = 1.25$

Localized case

Decrease of scattering

Intensity of QB state (blue) and lasing mode (red).

"Modes of random lasers", J. Andreasen and al., Advances in Optics and Photonics vol.3, 2011.

Summary

Problem with previous descriptions:

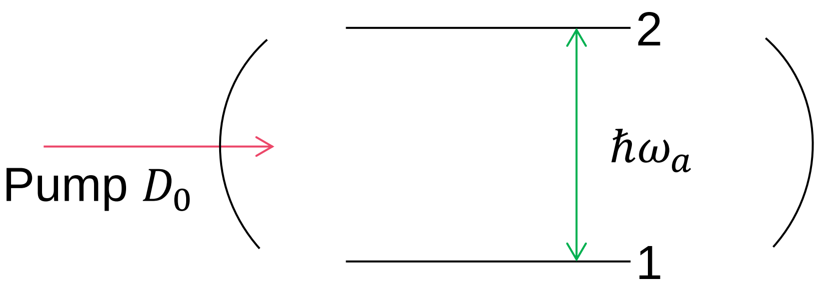
Localized states: → QB and TLM undistinguishable *inside the cavity*
→ TLM defines threshold modes (alteration of the gain medium, spatial hole burning)

New approach:

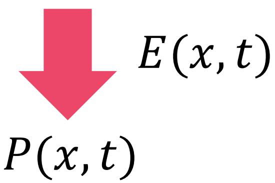
SALT (Steady-state Ab initio Laser Theory)

→ Stationary solutions of Maxwell Bloch lasing equations in multimode regime

Self-consistency theory: Maxwell Bloch



Cavity: $\epsilon(x)$
Population inversion: $D(x,t)$



Induced polarization relaxation rate: $\gamma_\perp$ (width of gain curve)
Inversion relaxation rate: $\gamma_\parallel$
→ **Inversion stationary $\boldsymbol{\gamma_\perp \gg \gamma_\parallel}$**

$$E = E^+ + E^-$$
$$P = P^+ + P^-$$
$$c = 1$$

(Positive and negative frequency components)

Maxwell Bloch equations:

$$\ddot{E}^+ = \frac{1}{\epsilon(x)}\nabla^2 E^+ - \frac{4\pi}{\epsilon(x)}\ddot{P}^+$$
$$\dot{P}^+ = -(i\omega_a + \gamma_\perp)P^+ + \frac{g^2}{i\hbar}E^+ D$$
$$\dot{D} = \gamma_\parallel(D_0 - D) - \frac{2}{i\hbar}\left(E^+(P^+)^\star - P^+(E^+)^\star\right)$$

g is the dipole matrix element and ϵ is the cavity dielectric function

“Ab initio self-consistent laser theory and random lasers”, H. E. Tureci and al., IOP Publishing Nonlinearity 22, 2009.

Self-consistency theory: Lasing equations

Assuming the existence of steady-state multiperiodic solutions of MB equations:

$$E^{+}(x,t) = \sum_{\mu=1}^{N} \psi_{\mu}(x) e^{-ik_{\mu}t}$$

$$P^{+}(x,t) = \sum_{\mu=1}^{N} P_{\mu}(x) e^{-ik_{\mu}t}$$

Unknown lasing frequencies

Unknown lasing mode(s)

As the pump increases, N increases depending on the number of thresholds we hit !

Self consistent equation → how many modes there are at a given pump?

“Ab initio self-consistent laser theory and random lasers”, H. E. Tureci and al., IOP Publishing Nonlinearity 22, 2009.

Self-consistency theory: Lasing equations

Assumptions: → TLM single mode lasing (1 term in the sum)
→ E small at first threshold ($D(x,t) \approx D_0$)

$$P_\mu(x) = -\frac{iD_0 g^2 \psi_\mu(x)}{\hbar(\gamma_\perp - i(k_\mu - k_a))}$$

$k_a = \frac{\omega_a}{c}$ is the frequency of the gain center

We substitute the polarization in Maxwell equation with also $\psi_\mu(x)$ for the electric field:

$$\left[\nabla^2 + \left(\epsilon(x) + \epsilon_g(x)\right) k_\mu^2\right] \psi_\mu(x) = 0$$

Where: $\epsilon_g(x)$ is the dielectric function of the gain medium.

$$\epsilon_g(x) = \frac{D_0}{k_a^2}\left[\frac{\gamma_\perp(k_\mu - k_a)}{\gamma_\perp^2 + (k_\mu - k_a)^2} - \frac{i\gamma_\mu^2}{\gamma_\perp^2 + (k_\mu - k_a)^2}\right]$$

Where:

$$D_0 \rightarrow \frac{D_0}{\frac{\hbar\gamma_\perp}{4\pi k_a^2 g^2}}$$

“Steady-state ab initio laser theory: generalizations and analytic results”, Li Ge and al., Physical Review A 82, 2010.

Self-consistency theory: Lasing equations

Case of very broadband curve: $\gamma_\perp \rightarrow \infty$

(Constant imaginary part)

$$\epsilon_g \rightarrow -\frac{iD_0}{k_a^2} \propto pump\ strength$$

“Steady-state ab initio laser theory: generalizations and analytic results”, Li Ge and al., Physical Review A 82, 2010.

Self-consistency theory: CF states

Define the TLM in the CF basis, solutions of MB equations as an eigenvalue problem.

From previously:

$$\left[\frac{1}{\epsilon(x)}\nabla^2 + k_\mu^2\right]\psi_\mu(x) = -\frac{\epsilon_g k_\mu^2}{\epsilon(x)}\psi_\mu(x)$$

Inversion of the equation with Green function:

$$\psi_\mu(x) = \frac{iD_0\gamma_\perp}{\gamma_\perp - i(k_\mu - k_a)}\frac{k_\mu^2}{k_a^2}\int_D dx' \frac{G(x,x';k_\mu)\psi_\mu(x')}{\epsilon(x')}$$ ★

Spectral representation of the Green function:

$$G(x,x'|k) = \sum_m \frac{\phi_m(x,k)\bar{\phi}_m^\dagger(x',k)}{k^2 - k_m^2}$$

The functions $\phi_m(x,k)$ are the CF states and k_m are the eigenvalues, the CF states are biorthogonal.

Outside cavity: CF states complete basis (real wave vector + constant photon flux)

“Steady-state ab initio laser theory: generalizations and analytic results”, Li Ge and al., Physical Review A 82, 2010.

SALT theory: CF states

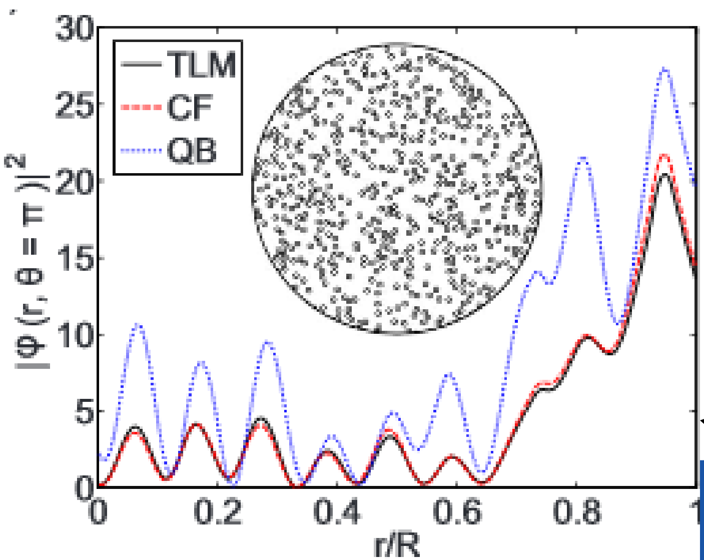
Define TLM modes in CF basis:

$$\psi_\mu(x) = \sum_{m=1}^{\infty} a_m^\mu \phi_m^\mu(x)$$

In previous equation!

$$a_m^\mu = \frac{\frac{D_0 i \gamma_\perp k_\mu^2}{k_a^2}}{\left(\gamma_\perp - i(k_\mu - k_a)\right)\left(k_\mu^2 - k_m^2(k_\mu)\right)} \int_D dx\, \bar{\phi}_m^{\mu\dagger}(x) \sum_p^N \frac{a_p^\mu \phi_p^\mu(x)}{\epsilon(x)} = D_0 \sum_p^N T_{mp}^{(0)} a_p^\mu$$

Threshold matrix with eigenvalues λ_μ



<u>In general:</u> TLM very close to a single CF state at lasing frequency k_μ.

For weak scattering

“Steady-state ab initio laser theory: generalizations and analytic results”, Li Ge and al., Physical Review A 82, 2010.

Nonlinear SALT

Previously: stationary inversion approximation → Uniform inversion ($D(x,t) \approx D_0$)

Nonlinear approach: $D_0 \rightarrow \frac{D_0}{(1+\sum_\nu \Gamma(k_\nu)|\psi_\nu(x)|^2)}$ where ν defines all the above threshold modes

Lorentzian centered at the lasing frequency of mode ν with width $\gamma_\perp$.

In the previous equation:

$$\psi_\mu(x) = \frac{iD_0\gamma_\perp}{\gamma_\perp - i(k_\mu - k_a)} \frac{k_\mu^2}{k_a^2} \int_D dx' \frac{G(x,x';k_\mu)\psi_\mu(x')}{\epsilon(x')(1+\sum_\nu \Gamma_\nu |\psi_\nu(x')|^2)}$$

Each lasing mode interacts with itself and other lasing modes → Mode competition via hole-burning!

Applications of Random Lasers

Display applications:
Electrically tuned directionality, plane emission

Domestication:
Tunability of emission spectrum and directionality
→ "**pump-shaping**" of the modes

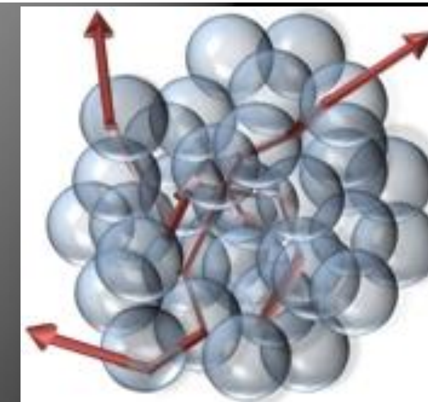
- → Cheap
- → Broad angular distribution (up to 4π)
- → Suspensions of particles
- → Localization and random lasing (emission spectrum)

Can be applied as coatings on arbitrary shaped surfaces
→ **Environment lightning (paint laser)**

Unique emission spectrum:
Specific localized modes
→Coding objects (bank notes…)

Medical application:
Emission spectrum of cancerous human tissues doped with laser dye
→ Tumour diagnostics

Conclusion



Random lasers: Disordered/scattering medium

How to describe random laser mode structure ? → QB states of conventional cavities are not enough, especially in the weak scattering case…

SALT tool:

- Study random lasers with full nonlinear interactions in 2D/3D
- Eliminate time dependence (can study more complex cavities)
- Provides a new description of the lasing modes based on CF states

Further theories to explore: wave chaos theory, random matrix theory, etc.

References on Random Lasers

<u>Basics:</u>

- "The physics and applications of random lasers", D. S. Wiersma, Nature Physics vol.4, 2008.
- "Random lasers: development, features and applications", Hui Cao, Optics and Photonics News, 2005.
- "Anderson localization of light", M. Segev and al., Nature Photonics vol.7, 2013.

<u>Mode theory:</u>

- "Modes of random lasers", J. Andreasen and al., Advances in Optics and Photonics vol.3, 2011.
- "Steady-state ab initio laser theory: generalization and analytic results", Li Ge and al., Physical Review A 82, 2010.
- "Ab initio self-consistent laser theory and random lasers", H. E. Tureci and al., IOP Publishing Nonlinearity 22, 2009.

<u>"Pump-shaping" applications:</u>

- "Pump-controlled directional light emission from random lasers", T. Hisch and al., Phys. Rev. Lett. 111, 2013.