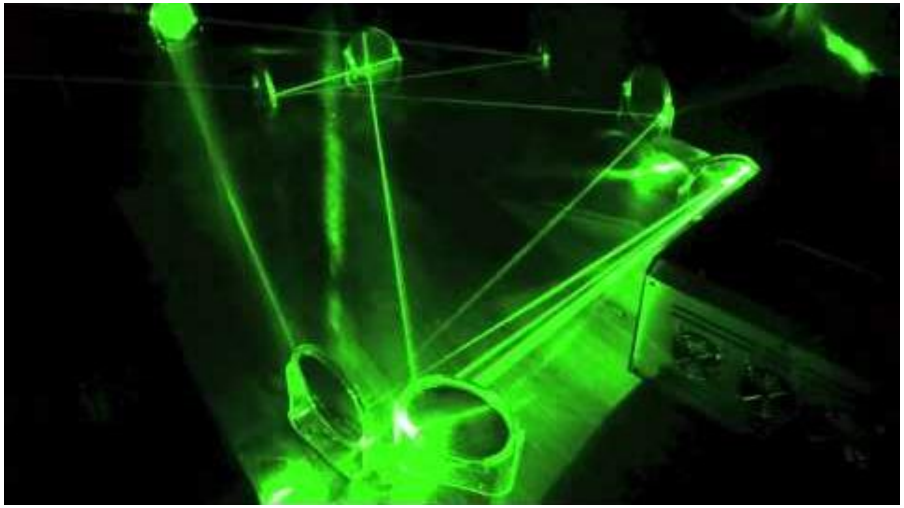
Laser Mirrors and Regenerative Feedback

Chapter 11 – "Lasers" – A.Siegman

9th April 2015

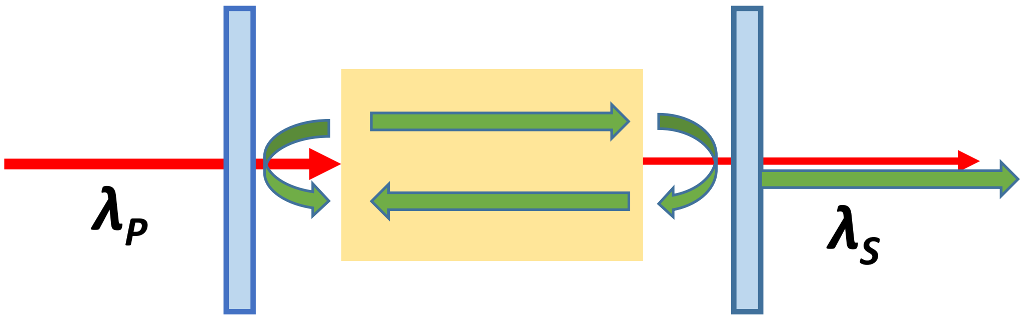
RS Coetzee



Outline

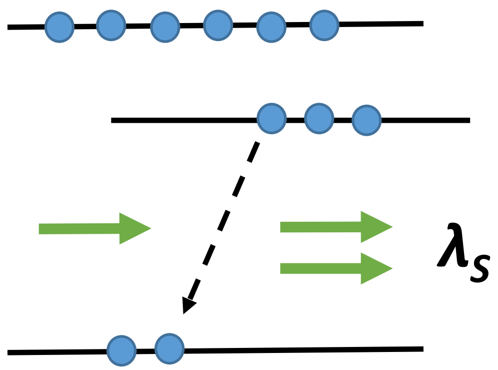
- Introduction.
- (Brief) Review of Laser mirrors & optical elements.
- Fabry-Perot Interferometers & etalons.
- Resonant optical cavities.
- The Delta Notation for Cavity Gains and Losses.
- Cavity mode frequencies.
- Regenerative Laser Amplification.
- The Highly Regenerative Limit → Approaching Threshold

Introduction

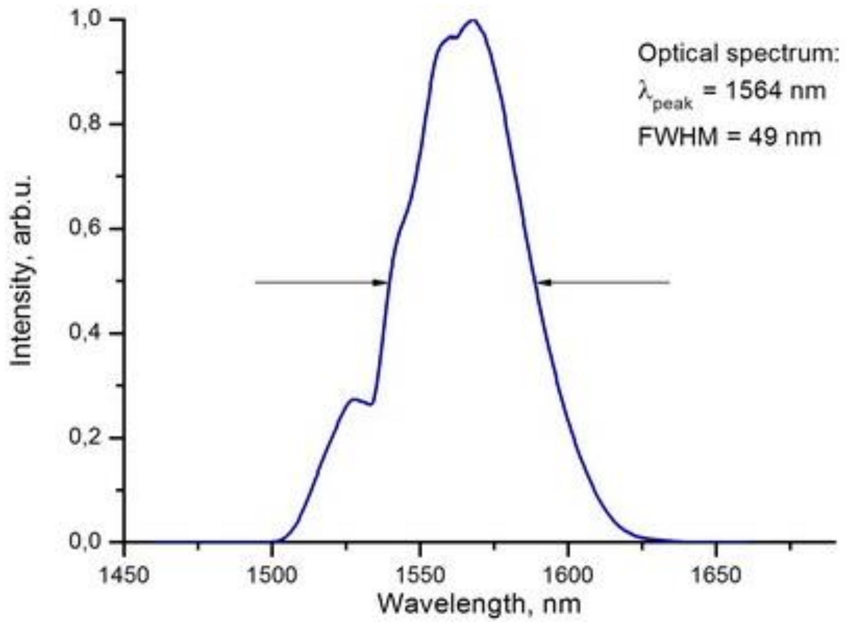


3 Essential Components that constitute a Laser

1. Pump Source
2. Gain Medium
3. Optical Cavity

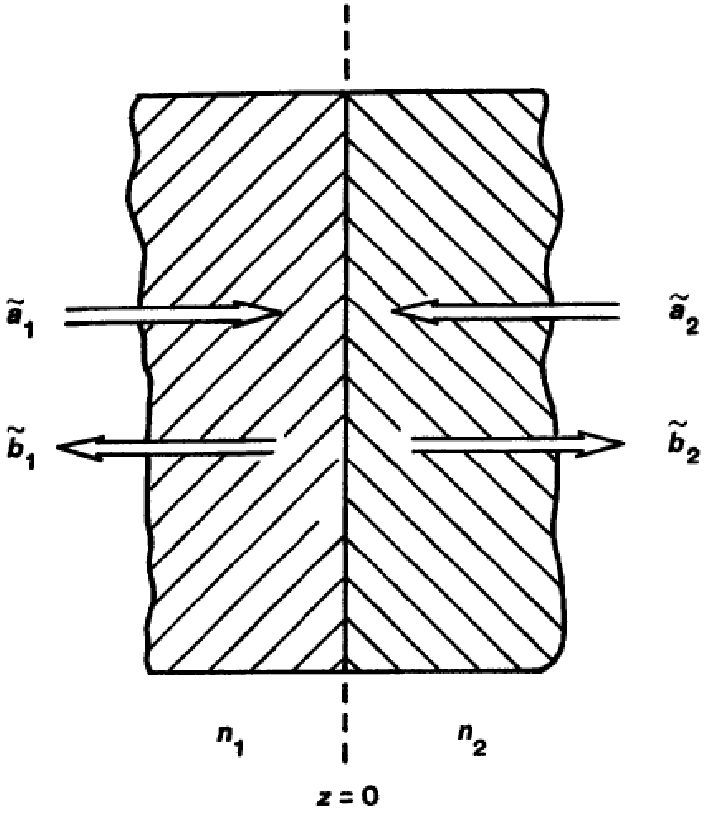


With Regenerative Feedback Via Mirrors



Laser Mirrors – Dielectric Slab

$$\mathcal{E}_i(z,t) = \mathrm{Re}\left\{\tilde{a}_i \exp[j(\omega t \mp \beta_i z)] + \tilde{b}_i \exp[j(\omega t \pm \beta_i z)]\right\}, \qquad i = 1, 2,$$



$$\tilde{b}_1 = r\,\tilde{a}_1 + t\,\tilde{a}_2,$$

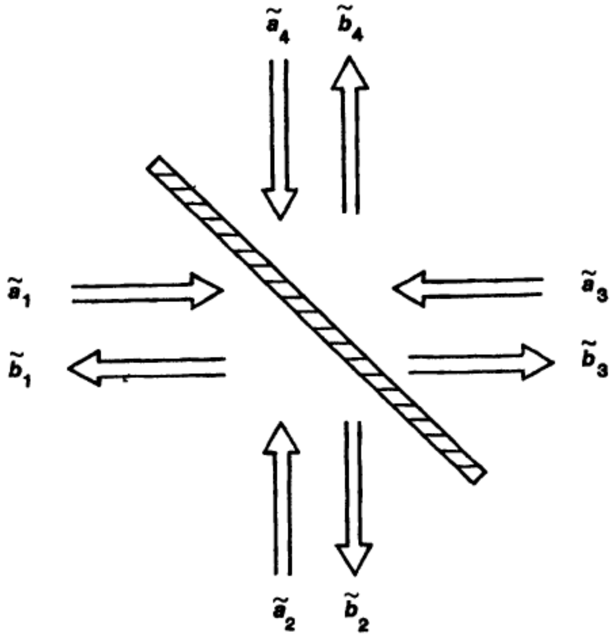
$$\tilde{b}_2 = t\,\tilde{a}_1 - r\,\tilde{a}_2,$$

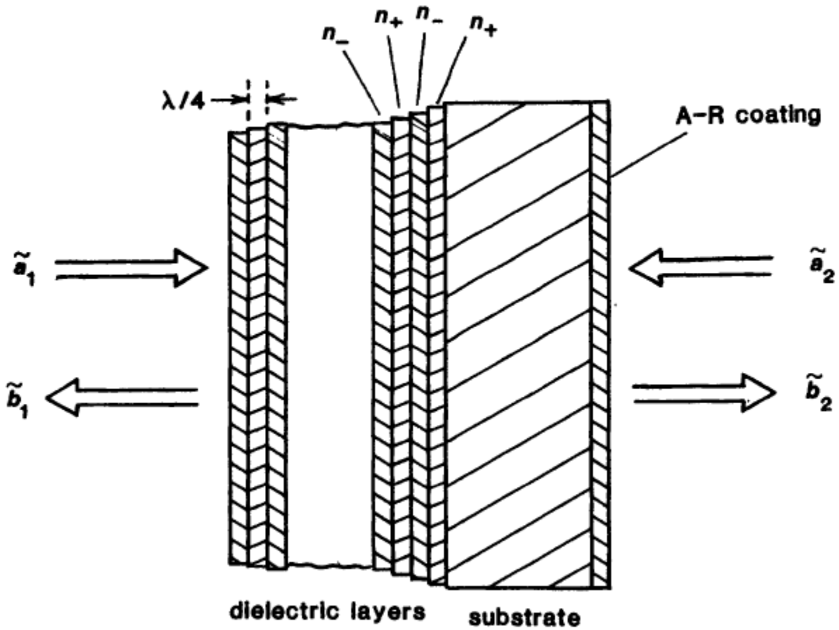
$$\begin{bmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{bmatrix} = \begin{bmatrix} r & t \\ t & -r \end{bmatrix} \times \begin{bmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{bmatrix}$$

$$r = \frac{n_1 - n_2}{n_1 + n_2} \qquad \text{and} \qquad t = \frac{2\sqrt{n_1 n_2}}{n_1 + n_2},$$

Fresnel Equations - 0° incidence

Laser Mirrors – Dielectric Slab





$$\begin{bmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_3 \\ \tilde{b}_4 \end{bmatrix} = \begin{bmatrix} \tilde{r}_{11} & \tilde{t}_{12} & \tilde{t}_{13} & \tilde{t}_{14} \\ \tilde{t}_{21} & \tilde{r}_{22} & \tilde{t}_{23} & \tilde{t}_{24} \\ \tilde{t}_{31} & \tilde{t}_{32} & \tilde{r}_{33} & \tilde{t}_{34} \\ \tilde{t}_{41} & \tilde{t}_{42} & \tilde{t}_{43} & \tilde{r}_{44} \end{bmatrix} \times \begin{bmatrix} \tilde{a}_1 \\ \tilde{a}_2 \\ \tilde{a}_3 \\ \tilde{a}_4 \end{bmatrix}$$

$$\boldsymbol{b} = \boldsymbol{S} \times \boldsymbol{a}$$

Laser Mirrors – Dielectric Slab

$$P_{\text{out}} = \boldsymbol{b}^{\dagger}\boldsymbol{b} = (\boldsymbol{S}\boldsymbol{a})^{\dagger}(\boldsymbol{S}\boldsymbol{a})$$
$$= (\boldsymbol{a}^{\dagger}\boldsymbol{S}^{\dagger})(\boldsymbol{S}\boldsymbol{a}) = \boldsymbol{a}^{\dagger}(\boldsymbol{S}^{\dagger}\boldsymbol{S})\boldsymbol{a}$$

$$P_{\text{out}} = \sum_{j=1}^{N} \tilde{b}_j^* \tilde{b}_j = \left[\tilde{b}_1^*, \tilde{b}_2^*, \tilde{b}_3^*, \ldots\right] \times \begin{bmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_3 \\ \ldots \end{bmatrix} = \boldsymbol{b}^{\dagger} \times \boldsymbol{b},$$

For this to be true, we must have: $\boldsymbol{S}^{\dagger}\boldsymbol{S} = \boldsymbol{I} \quad \text{or} \quad \boldsymbol{S}^{\dagger} \equiv \boldsymbol{S}^{-1}$

i.e. the Scattering matrix S, is unitary for a lossless network.

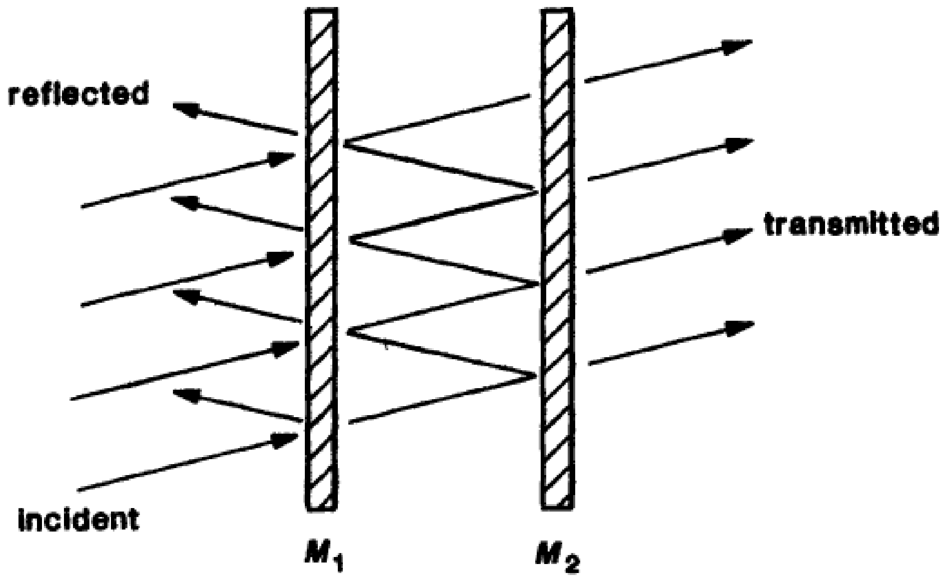
Applying this constraint requires that (e.g. for a two-port network)

$$|\tilde{t}_{12}| = |\tilde{t}_{21}|, \qquad |\tilde{r}_{11}| = |\tilde{r}_{22}|,$$
$$|\tilde{r}_{11}|^2 + |\tilde{t}_{21}|^2 = |\tilde{r}_{22}|^2 + |\tilde{t}_{12}|^2 = 1,$$
$$\tilde{r}_{11}\tilde{t}_{12}^* + \tilde{t}_{12}\tilde{r}_{22}^* = 0$$

$$\boldsymbol{S} = \begin{bmatrix} r & t \\ t & -r \end{bmatrix} \quad \text{and} \quad \boldsymbol{S} = \begin{bmatrix} r & jt \\ jt & r \end{bmatrix}$$

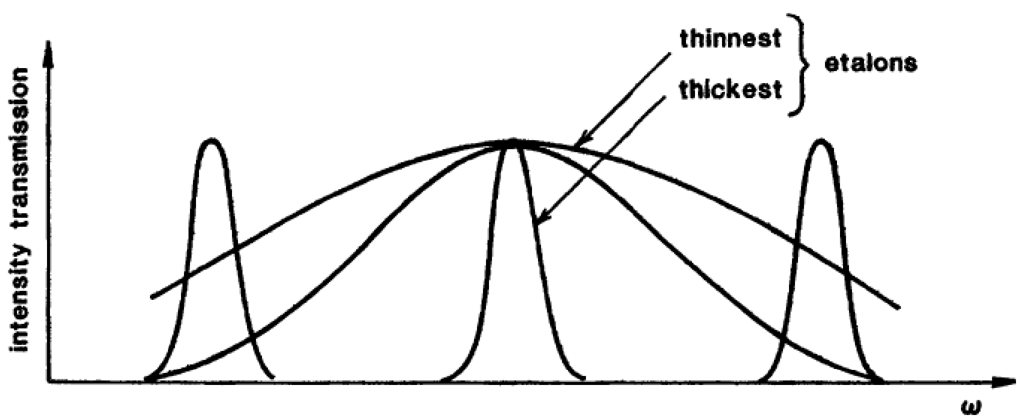
$$\boldsymbol{S} = \begin{bmatrix} r & t \\ t & r \end{bmatrix}$$

The Fabry Perot Interferometer



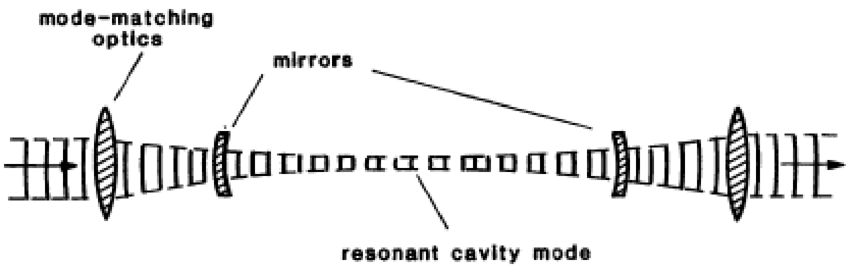
The Fabry-Perot Interferometer

- M_1 & M_2 Highly Reflective.
- Discrete resonances and Transmission windows.
- Used as an optical filter, to measure frequency spectrum.

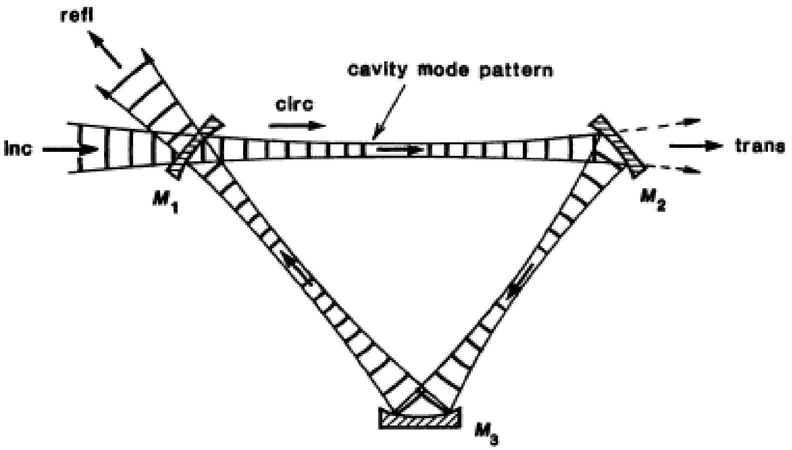


Resonant Optical Cavities

Typical passive cavities:

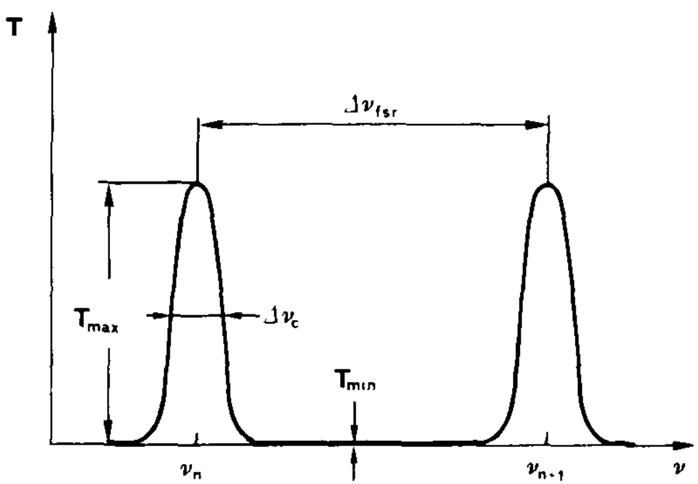


Linear Cavity

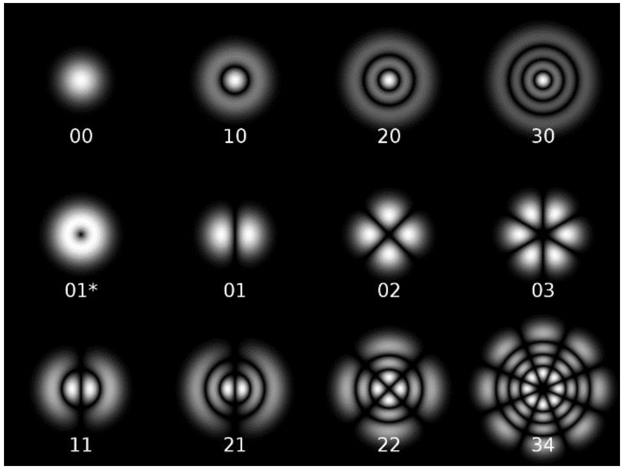


Ring Cavity

Longitudinal/Cavity modes

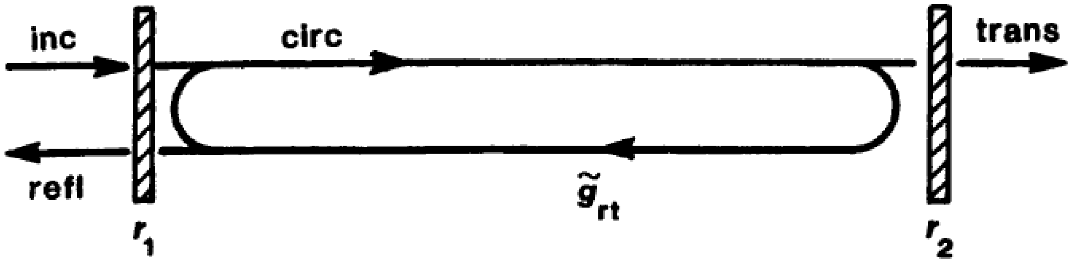


Transversal/Spatial modes



Resonant Optical Cavities

Let us take a closer look at the field inside a general passive cavity…



$$\tilde{E}_{\text{circ}} = jt_1\tilde{E}_{\text{inc}} + \tilde{g}_{\text{rt}}(\omega)\,\tilde{E}_{\text{circ}},$$

$$\tilde{g}_{\text{rt}}(\omega) \equiv r_1 r_2 (r_3 \ldots) \times \exp\left[-\alpha_0 p - j\omega p/c\right]. \qquad where:\ p = 2L$$

↓

“net complex round trip gain for a plane wave”

$$\tilde{E}_{\text{circ}} = jt_1\tilde{E}_{\text{inc}} + r_1 r_2 (r_3 \ldots) \exp[-\alpha_0 p - j\omega p/c]\,\tilde{E}_{\text{circ}}.$$

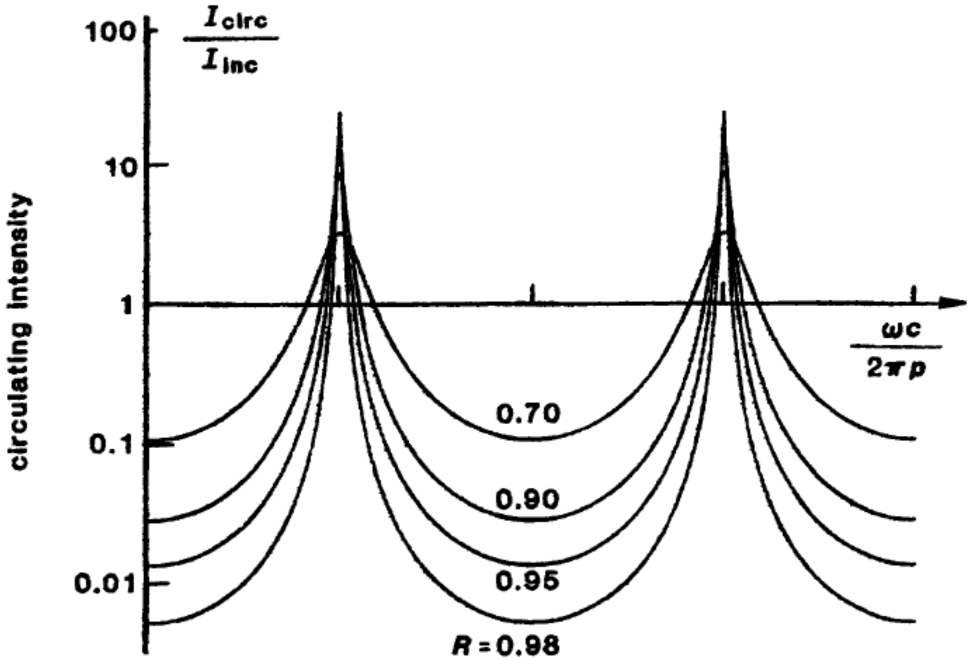
Resonant Optical Cavities

$$\frac{\tilde{E}_{\text{circ}}}{\tilde{E}_{\text{inc}}} = \frac{jt_1}{1 - \tilde{g}_{\text{rt}}(\omega)} = \frac{jt_1}{1 - r_1 r_2 (r_3 \ldots) \exp[-\alpha_0 p - j\omega p/c]}.$$

We note that when the phase $\left(\frac{wp}{c}\right)$ is an integer multiple of 2π; that is:

$$\omega = \omega_q \equiv q \times 2\pi \times (c/p)$$

We observe large, resonant peaks in the circulating intensity.



Resonant Optical Cavities

How large can this circulating intensity become (at resonance) ?

Assume a Symmetric linear cavity, lossless ($\alpha_0 p \approx 0$), $R_1 = R_2 = R$. Then from Eq (2):

$$\left.\frac{\tilde{E}_{\text{circ}}}{\tilde{E}_{\text{inc}}}\right|_{\omega=\omega_q} = \frac{jt}{1 - r_1 r_2 e^{-\alpha_0 p}} \approx \frac{jt}{1 - r^2} = \frac{j}{t},$$

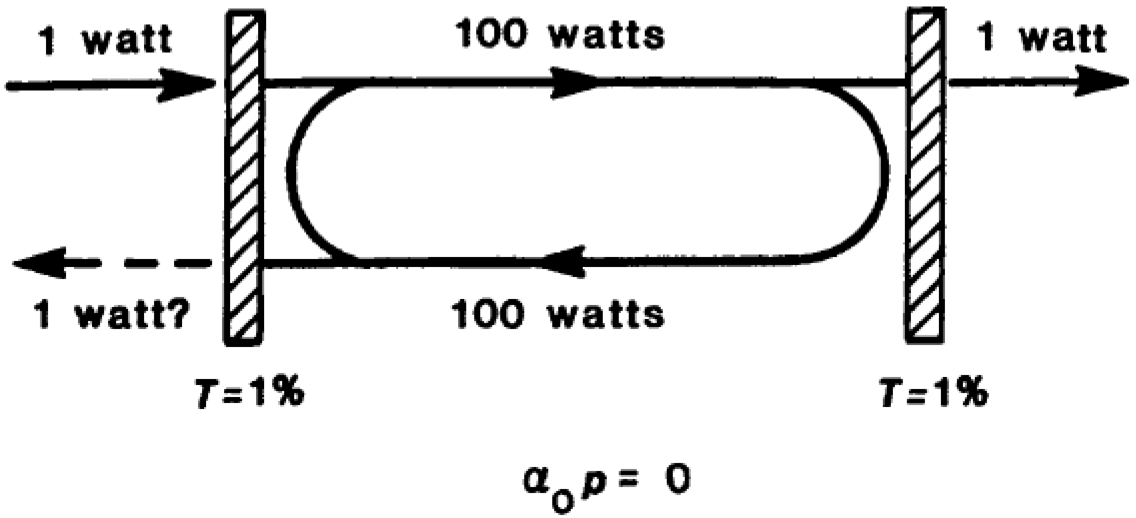
$$\left.\frac{I_{\text{circ}}}{I_{\text{inc}}}\right|_{\omega=\omega_q} \approx \left|\frac{1}{t}\right|^2 = \frac{1}{T},$$

Where T is the power transmission through mirror

Assume T = 1%, $R_1 = R_2 = 99\%$

$$I_{\text{circ}} \approx 100 \times I_{\text{inc}} \qquad \text{for} \qquad \begin{cases} R_1 = R_2 = 0.99, \\ \alpha_0 p \ll 0.01. \end{cases}$$

Resonant Optical Cavities

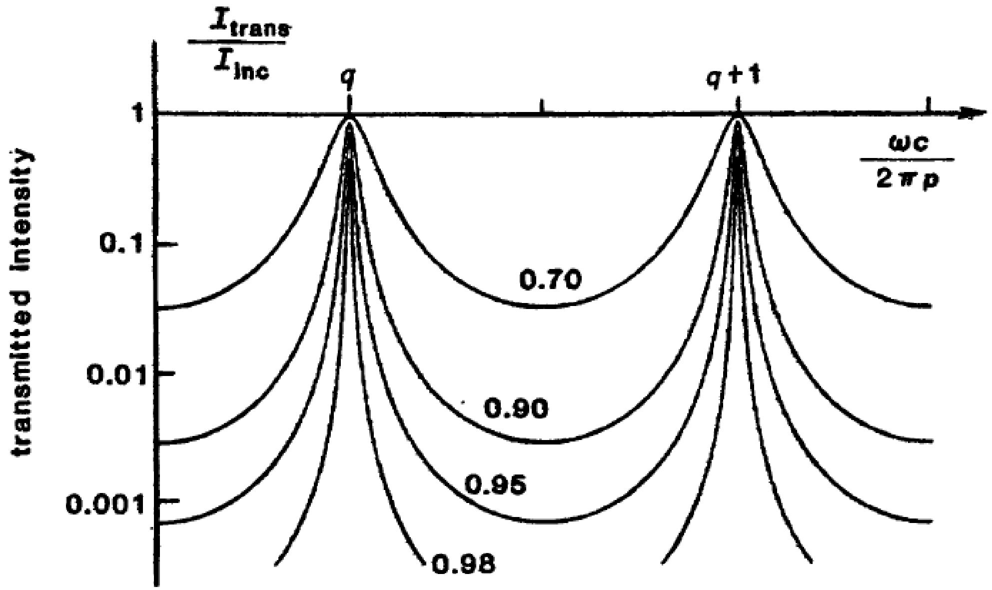


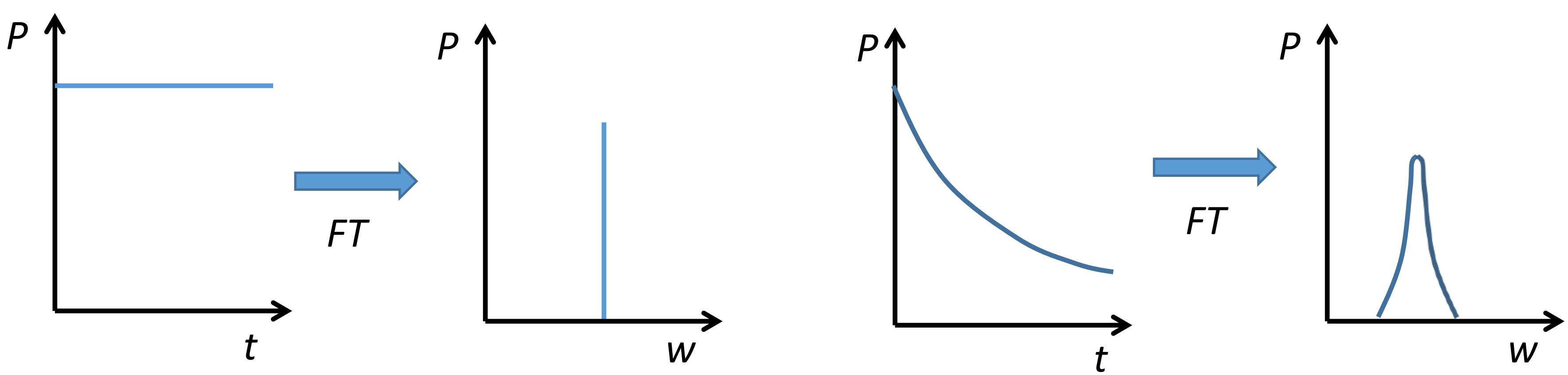
Energy conservation violated? No…The stored energy within the cavity cannot be extracted (continuously).

Can be extracted on a transient basis → Cavity dumping, using some Switch within the cavity.

Resonant Optical Cavities

For Lasers, we are also interested in the Transmitted Intensity:

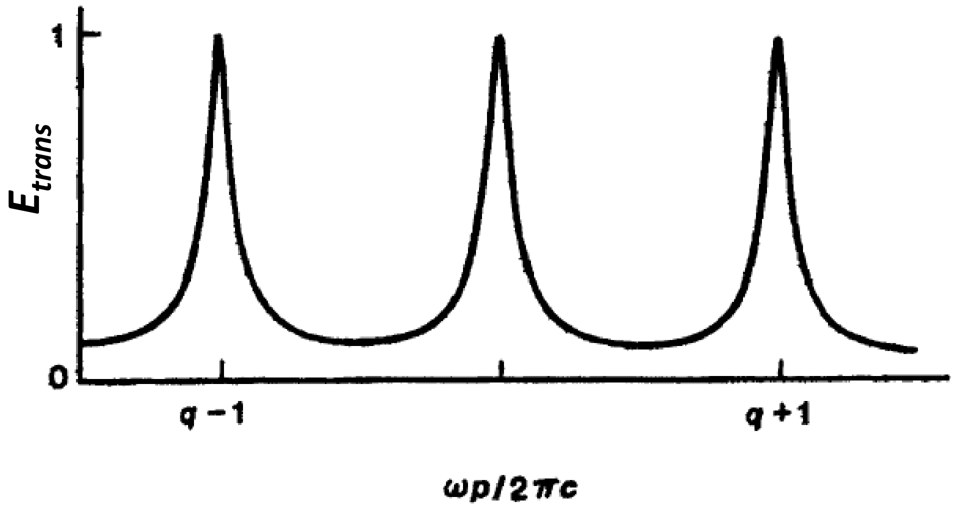




Resonant Optical Cavities

$$\tilde{E}_{\text{trans}} = j t_2 \exp[-\alpha_0 p_1 - j\omega p_1/c] \times \tilde{E}_{\text{circ}}.$$

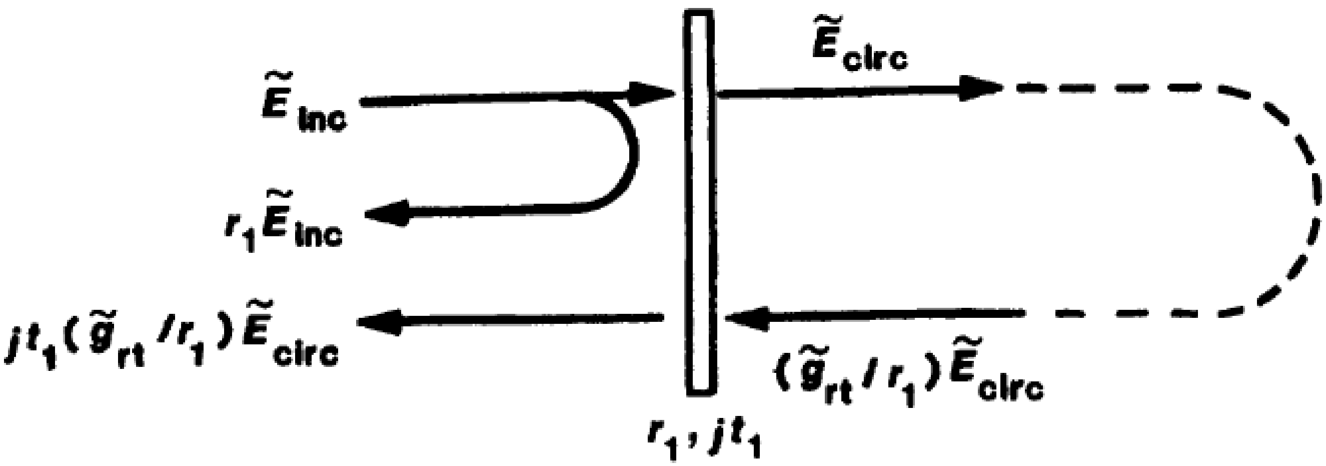
$$\frac{\tilde{E}_{\text{trans}}}{\tilde{E}_{\text{inc}}} = \frac{-t_1 t_2 \exp[-\alpha_0 L - j\omega L/c]}{1 - r_1 r_2 \exp[-2\alpha_0 L - 2j\omega L/c]} = -\frac{t_1 t_2}{\sqrt{r_1 r_2}} \frac{\sqrt{\tilde{g}_{\text{rt}}(\omega)}}{1 - \tilde{g}_{\text{rt}}(\omega)}$$



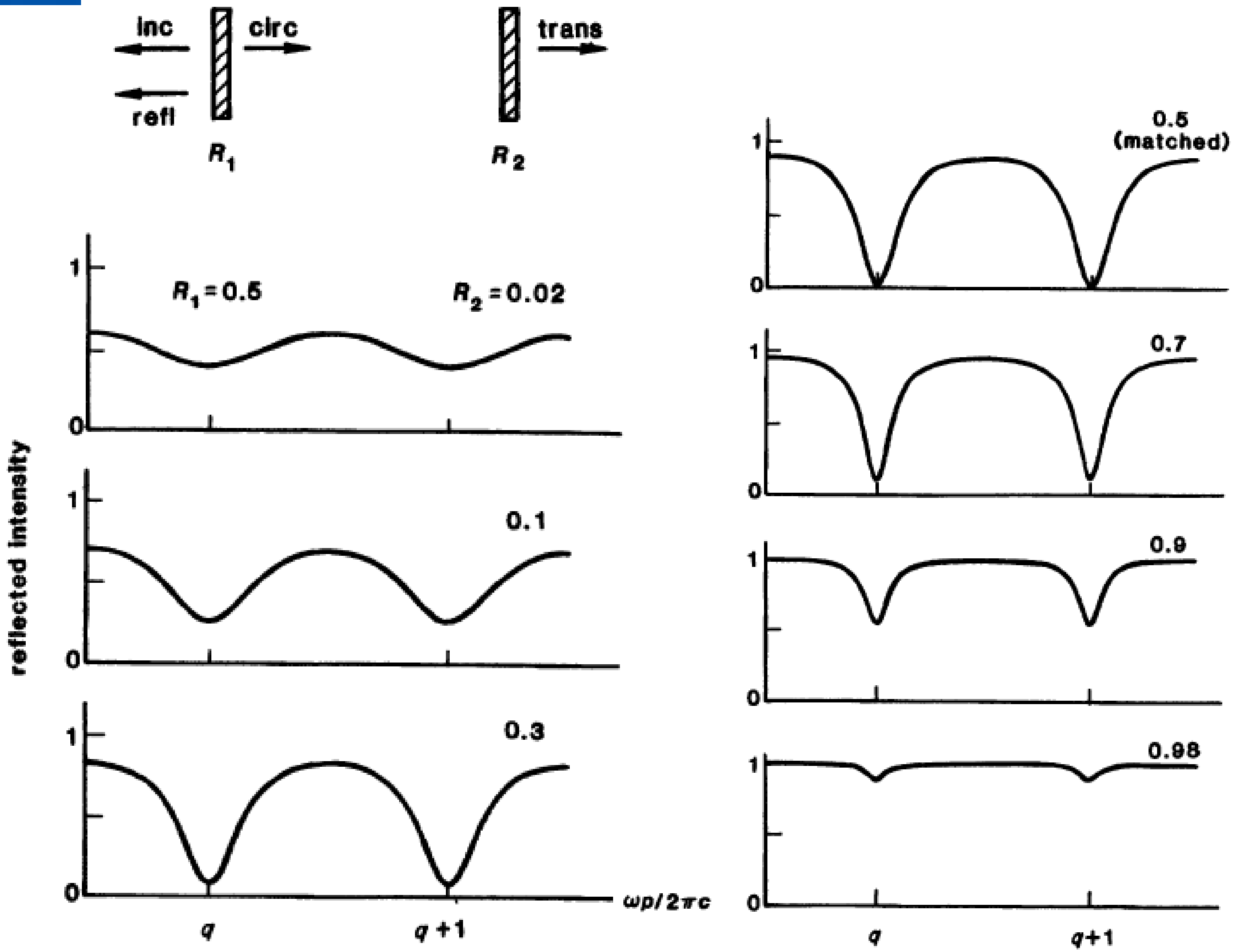
Resonant Optical Cavities

$$\tilde{E}_{\text{refl}} = r_1 \tilde{E}_{\text{inc}} + j t_1 \left(\tilde{g}_{\text{rt}} / r_1 \right) \tilde{E}_{\text{circ}}.$$

$$\frac{\tilde{E}_{\text{refl}}}{\tilde{E}_{\text{inc}}} = \frac{r_1 - r_2 e^{-\alpha_0 p - j\omega p/c}}{1 - r_1 r_2 e^{-\alpha_0 p - j\omega p/c}} = \frac{1}{r_1} \times \frac{r_1^2 - \tilde{g}_{\text{rt}}(\omega)}{1 - \tilde{g}_{\text{rt}}(\omega)}.$$



Resonant Optical Cavities



The Delta Notation for Cavity Gains and Losses

Typically, R is defined as a simple number, i.e. R = 95% → R =0.95

Introduce a new definition:

$$R_1 \equiv e^{-\delta_1} \qquad \text{(exact definition, arbitrary } \delta_1\text{)},$$
$$\approx 1 - \delta_1 \qquad \text{(approximate definition, } \delta_1 \ll 1\text{)}.$$

$$\delta_i \equiv \ln\left(\frac{1}{R_i}\right) = 2\ln\left(\frac{1}{r_i}\right)$$

"Mirror coupling coefficient"

$$R_1 \equiv r_1^2 \equiv e^{-\delta_1}$$

Now re-write round trip gain for a cavity:

$$|\tilde{g}_{\mathrm{rt}}|^2 = R_1 R_2 e^{2\alpha_m p_m - 2\alpha_0 p} = e^{\delta_m - \delta_0 - \delta_1 - \delta_2}$$

With: $\delta_0 \equiv 2\alpha_0 p$ and $\delta_m \equiv 2\alpha_m p_m$.

The idea is to express any roundtrip gain or loss in the form:

$$\delta_x \equiv \ln[\text{power gain, or power loss, ratio per round trip}].$$

The Delta Notation for Cavity Gains and Losses

$$\delta_c \equiv \delta_0 + \delta_1 + \delta_2 = 2\alpha_0 p + \ln\left(\frac{1}{R_1 R_2}\right)$$

Now, If we had to insert a gain medium into the cavity:

$$|\tilde{g}_{\mathrm{rt}}|^2 = e^{\delta_m - \delta_c} \approx 1 + \delta_m - \delta_c \quad \text{if} \quad |\delta_m - \delta_c| \ll 1.$$

It is also useful to express the "Q" factor of the (passive) cavity in this delta notation:

$$I_{\mathrm{circ}}(t) = I_{\mathrm{circ}}(t_0) \times \exp[-N\delta_c] = I_{\mathrm{circ}}(t_0) \times \exp\left[-\frac{\delta_c}{T_{\mathrm{rt}}}(t - t_0)\right]$$

OR

$$I_{\mathrm{circ}}(t) = I_{\mathrm{circ}}(t_0) \times \exp\left[-\frac{\omega_a}{Q_c}(t - t_0)\right]$$

Where:

$$Q_c = \frac{\omega_a T_{\mathrm{rt}}}{\delta_c} = \frac{2\pi p}{\lambda}\frac{1}{\delta_c}$$

The Delta Notation for Cavity Gains and Losses

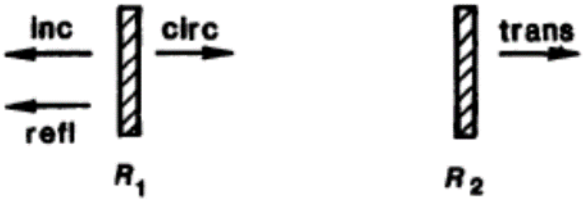
Similarly, for the circulating and transmitted intensities (at resonance):

$$\left.\frac{I_{\mathrm{circ}}}{I_{\mathrm{inc}}}\right|_{\omega=\omega_q} \approx \frac{4\delta_1}{(\delta_1+\delta_2+\delta_0)^2}$$

$$\left.\frac{I_{\mathrm{trans}}}{I_{\mathrm{inc}}}\right|_{\omega=\omega_q} \approx \frac{4\delta_1\delta_2}{(\delta_1+\delta_2+\delta_0)^2} = \frac{4\delta_1\delta_2}{\delta_c^2}.$$

Reflected intensity:

$$\left.\frac{\tilde{E}_{\mathrm{refl}}}{\tilde{E}_{\mathrm{inc}}}\right|_{\omega=\omega_q} \approx \frac{\delta_2+\delta_0-\delta_1}{\delta_2+\delta_0+\delta_1}$$



$$\left.\frac{\tilde{E}_{\mathrm{refl}}}{\tilde{E}_{\mathrm{inc}}}\right|_{\omega=\omega_q} \approx \begin{cases} +1 & \text{if } \delta_2+\delta_0 \gg \delta_1, \\ 0 & \text{if } \delta_2+\delta_0 = \delta_1, \\ -1 & \text{if } \delta_2+\delta_0 \ll \delta_1. \end{cases}$$

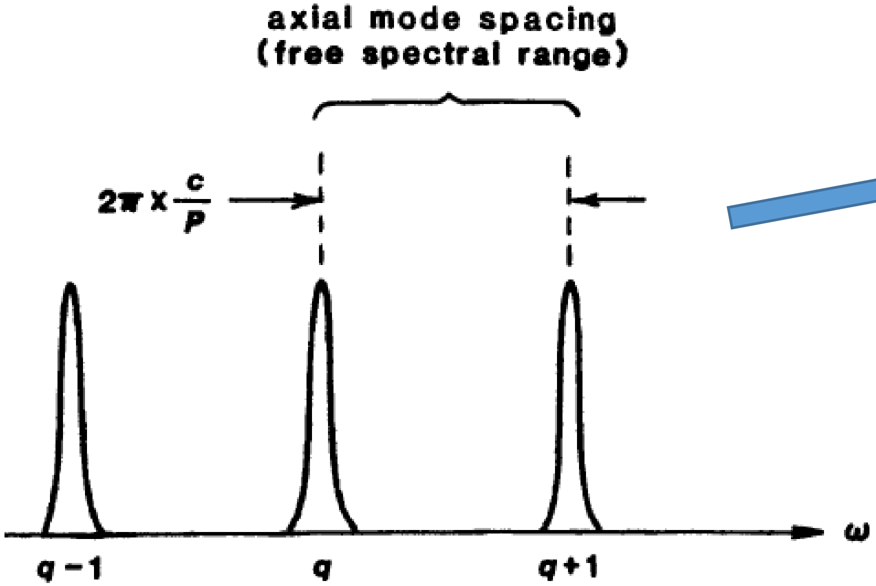
→ *Under-coupled case*

→ *Matched case*

→ *Over-coupled case*

Cavity Mode Frequencies

So far we have seen that a cavity gives rise to periodically spaced resonant frequencies, so called longitudinal or axial modes. A better understanding of these is crucial in understanding laser operation.



Must obey the "self-consistency" condition

$$\omega = \omega_q = q \times 2\pi \times \frac{c}{p} = q \times 2\pi \times \frac{c}{2L},$$

$$\Delta\omega_{ax} \equiv \omega_{q+1} - \omega_q = 2\pi \times \frac{c}{p} = 2\pi \times \frac{c}{2L}.$$

For a cavity in air ($n \approx 1$)

$$\Delta\omega_{\mathrm{ax}} \approx \frac{2\pi c_0}{(n + n'\omega)p} = 2\pi \times \frac{1}{1 + (\omega/n)(dn/d\omega)} \times \frac{c}{p},$$

For a cavity in other media

$$\Delta f_{\mathrm{ax}} = \frac{c}{2L} \approx \begin{cases} 150\ \mathrm{MHz} & \text{if } L = 1\ \mathrm{m}, \\ 500\ \mathrm{MHz} & \text{if } L = 30\ \mathrm{cm}, \\ 2{,}000\ \mathrm{MHz} & \text{if } L = 5\ \mathrm{cm} \text{ and } n = 1.5. \end{cases}$$

Cavity Mode Frequencies

For a given laser spectrum, there are a large amount of modes present.

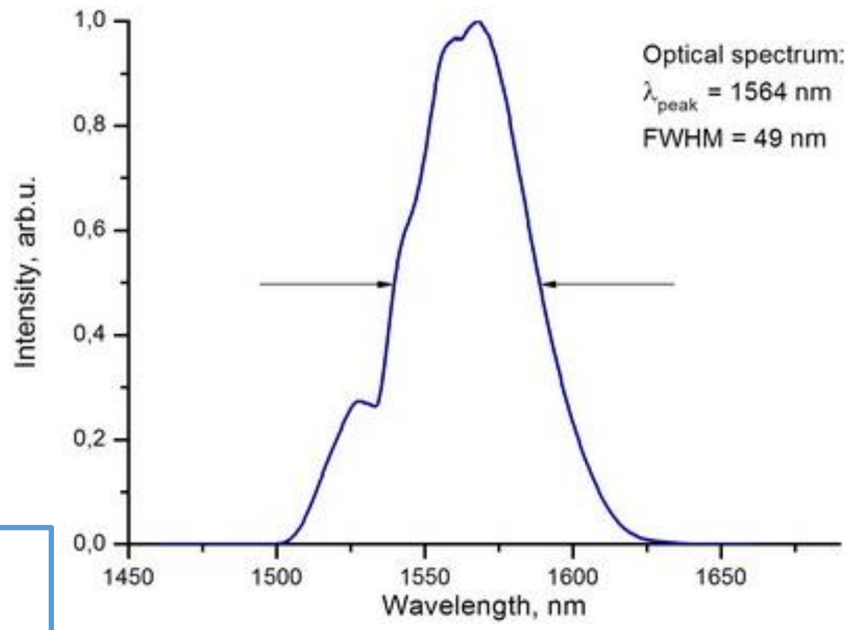
$$q = \frac{\omega_q}{\Delta\omega_{\mathrm{ax}}} = \frac{p}{\lambda_q} = \frac{L}{\lambda_q/2}.$$

For typical laser cavities: $q \sim 10^7$

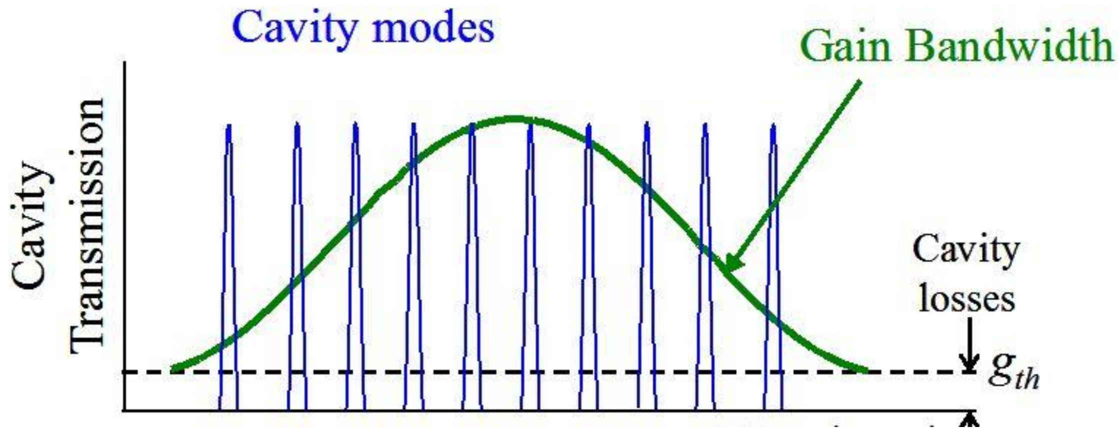
For thin etalons, etc: $q \sim 10^3 - 10^5$

Many modes → mode competition

This can have serious implications on the stability of the laser. Certain applications require only a single frequency, the rest is essentially noise.



Cavity Mode Frequencies



Each mode has an associated gain and loss value. Means to reduce the number of modes/narrow the laser linewidth (a “Single frequency laser”):

- Injection Seeding.
- Narrowing gain bandwidth → Bragg and Diffraction gratings, intra-cavity etalons.
- Short cavity Length → Large free spectral range & smaller mode number.
- High Finesse → High mirror reflectivity's.

$$\text{finesse}, \mathcal{F} \equiv \frac{\pi\sqrt{g_{\mathrm{rt}}}}{1 - g_{\mathrm{rt}}} \approx \frac{\Delta\omega_{\mathrm{ax}}}{\Delta\omega_{\mathrm{cav}}}.$$

Cavity Mode Frequencies

Other issues arise which affect the stability of the axial modes

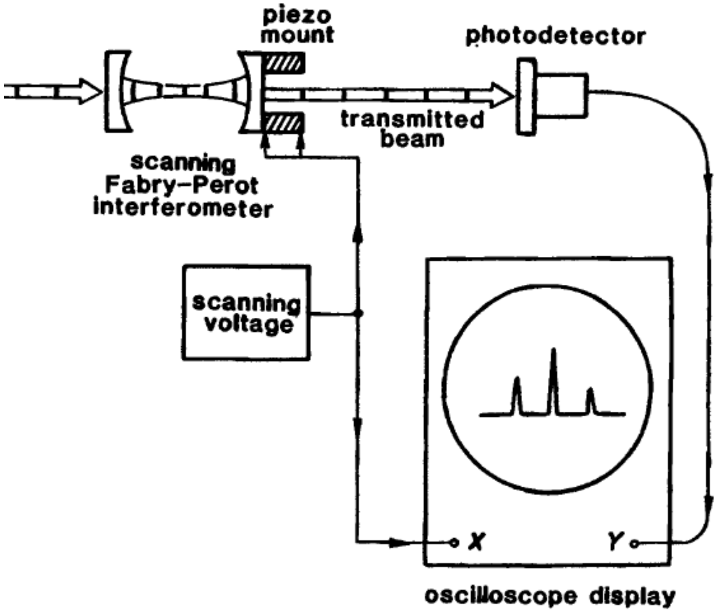
$$\delta\omega_q \approx -\frac{\delta p}{\lambda} \times \Delta\omega_{\mathrm{ax}} \approx -\frac{\delta L}{\lambda/2} \times \Delta\omega_{\mathrm{ax}}.$$

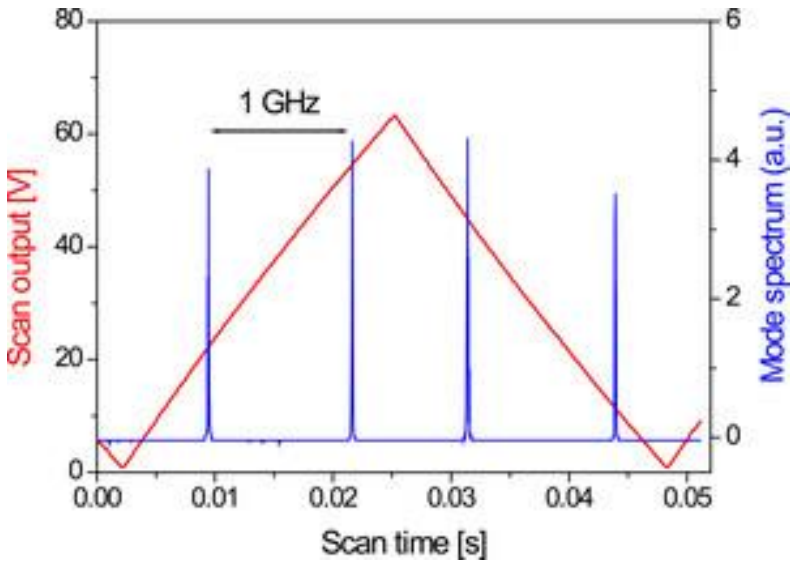
Thermal drift → refractive index is temperature dependent → optical path length will vary with time, affecting the cavity parameters.

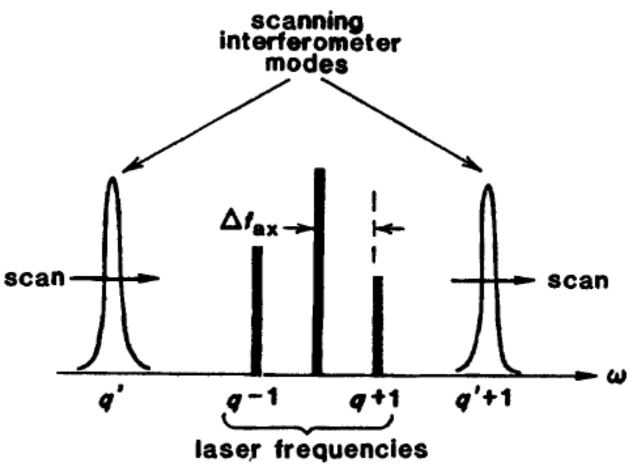
Mechanical drift → compensated for with piezoelectric transducer.

Example: Mode-locked lasers are particularly susceptible to these effects and require active frequency stabilization to maintain mode-locking.

The Scanning Fabry Perot Interferometer







Regenerative Laser Amplification

So far we have studied passive optical cavities. Naturally, the next step is to now study such a cavity which contains a gain medium.

Passive cavity →

$$\tilde{g}_{\mathrm{rt}}(\omega) \equiv r_1 r_2 (r_3 \ldots) \times \exp\left[-\alpha_0 p - j\omega p/c\right].$$

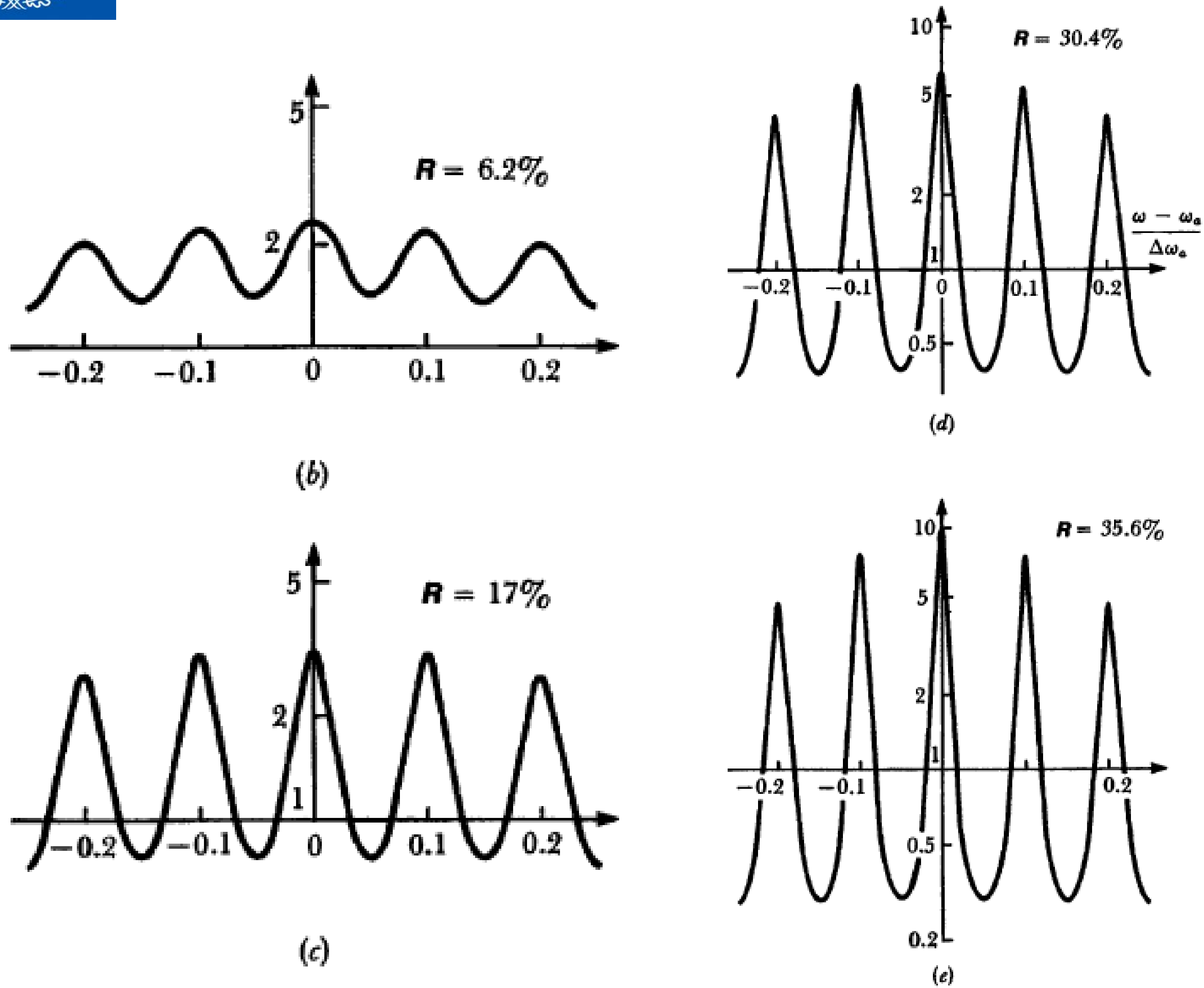
Now add a gain medium with gain coefficient $\alpha_m(\omega) p_m$ and additional phase shift $-j\Delta\beta_m(\omega) p_m$.

Active cavity →

$$\tilde{g}_{\mathrm{rt}}(\omega) = r_1 r_2 (r_3 \ldots) \times \exp\left[\alpha_m p_m - \alpha_0 p - j\omega p/c - j\Delta\beta_m(\omega) p_m\right]$$

$$\frac{\tilde{E}_{\mathrm{trans}}}{\tilde{E}_{\mathrm{inc}}} = \frac{-t_1 t_2 \exp[-\alpha_0 L - j\omega L/c]}{1 - r_1 r_2 \exp[-2\alpha_0 L - 2j\omega L/c]} = -\frac{t_1 t_2}{\sqrt{r_1 r_2}} \frac{\sqrt{\tilde{g}_{\mathrm{rt}}(\omega)}}{1 - \tilde{g}_{\mathrm{rt}}(\omega)}$$

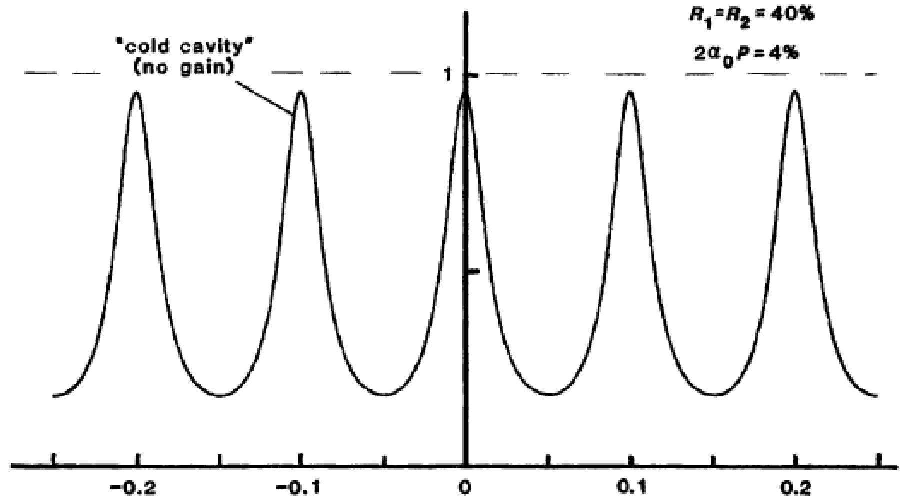
Regenerative Laser Amplification



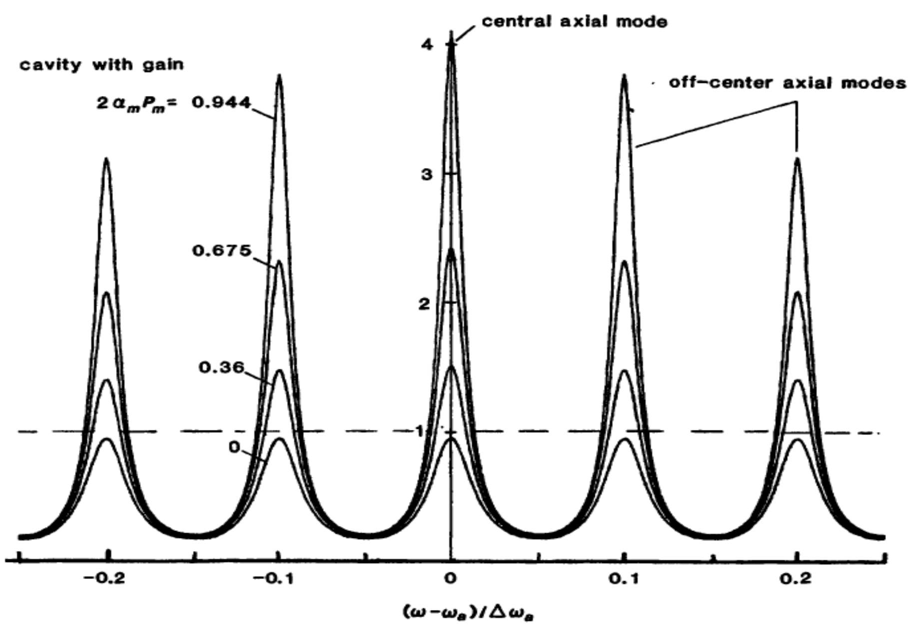
Regenerative Laser Amplification

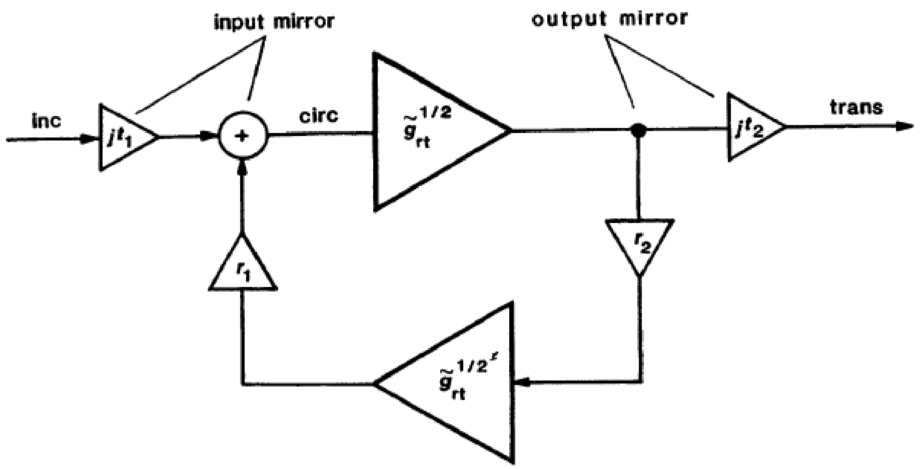
Keeping mirror reflectivity's constant:

Passive cavity

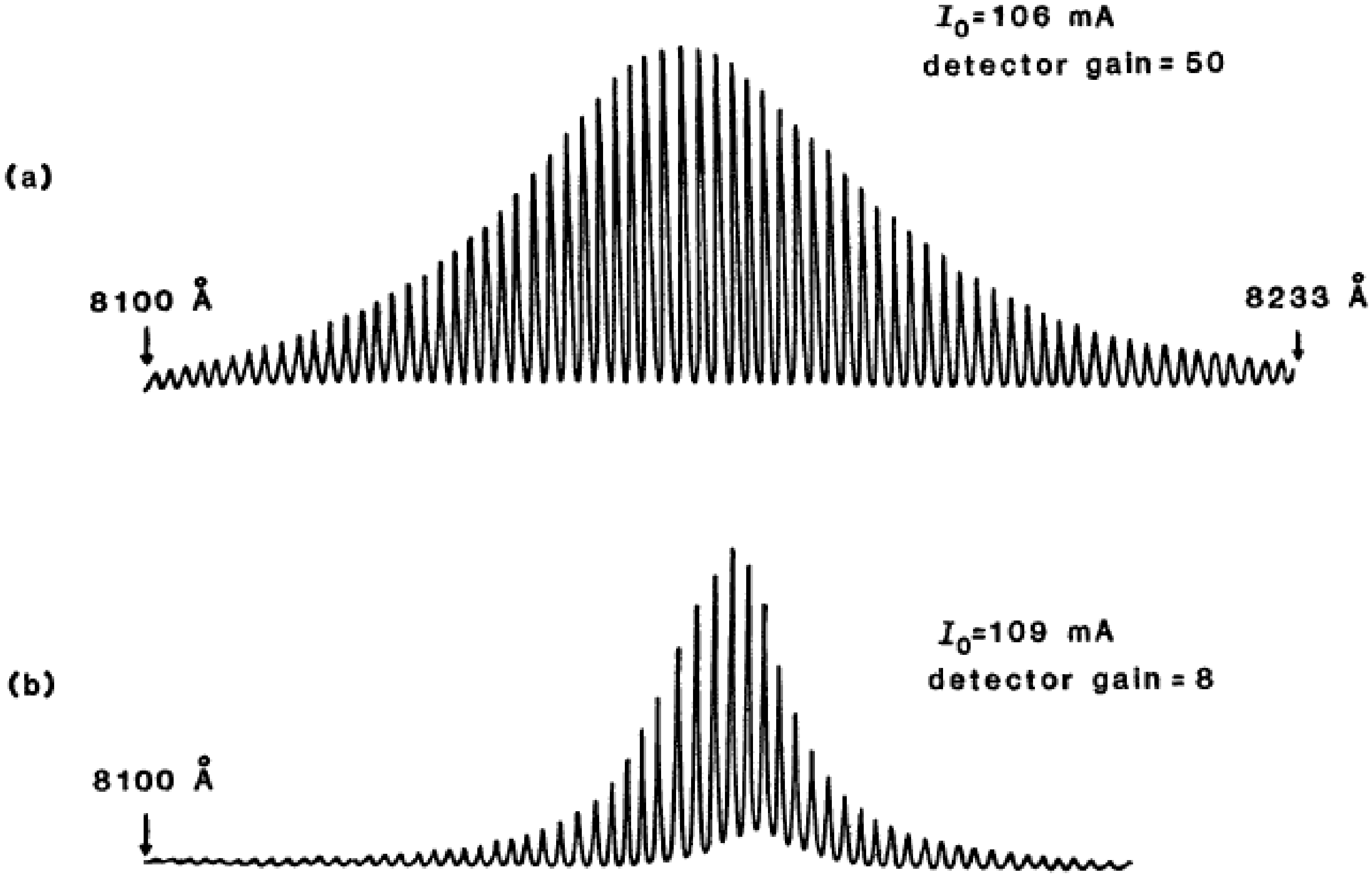


Active cavity

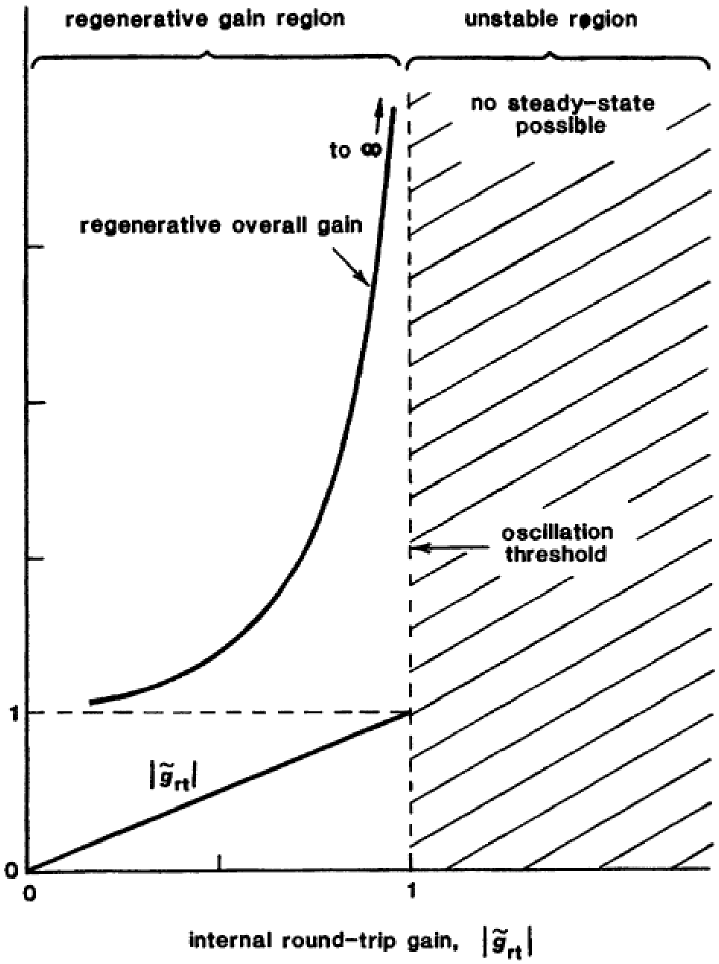




Regenerative Laser Amplification



The Highly Regenerative Limit



As we turn up the gain (or lower the cavity losses) we notice that:

- The gain peaks increase substantiality.
- The gain peaks become narrower.
- Each peak approaches a fixed gain-bandwidth product.

$$\frac{\tilde{E}_{\text{trans}}}{\tilde{E}_{\text{inc}}} = -\frac{t_1 t_2}{\sqrt{r_1 r_2}} \frac{\sqrt{\tilde{g}_{\text{rt}}(\omega)}}{1 - \tilde{g}_{\text{rt}}(\omega)}$$

When $g_{rt} \rightarrow 1, E_{trans} \rightarrow \infty$

Let us study what happens when the gain approaches unity from below…

The Highly Regenerative Limit

Assume: $$\tilde{g}_{\rm rt}(\omega) \equiv g_{\rm rt}(\omega) e^{-j\phi(\omega)}$$

$$\phi(\omega) \approx \frac{\omega p}{c} = \frac{\omega_q p}{c} + \frac{(\omega - \omega_q)p}{c} = q \times 2\pi + \delta\phi(\omega)$$

Where: $$\delta\phi(\omega) \equiv \frac{\omega - \omega_q}{c} p \approx 2\pi \times \frac{\omega - \omega_q}{\Delta\omega_{\rm ax}}$$

We consider a narrow axial mode, few frequencies around the peak frequency: $\omega \approx \omega_q$ and $|\omega - \omega_q| \ll \Delta\omega_{\rm ax}$

$$e^{-j\phi(\omega)} = e^{-j\delta\phi(\omega)} \approx 1 - j\delta\phi(\omega) = 1 - j2\pi \frac{\omega - \omega_q}{\Delta\omega_{\rm ax}}.$$

The Highly Regenerative Limit

$$\left.\frac{\tilde{E}_{\rm trans}}{\tilde{E}_{\rm inc}}\right|_{\omega\approx\omega_q} = -\frac{t_1 t_2}{\sqrt{r_1 r_2}}\frac{g_{\rm rt}^{1/2}(\omega)e^{-j\phi(\omega)/2}}{1-g_{\rm rt}(\omega)e^{-j\phi(\omega)}}$$

$$\approx -\frac{t_1 t_2}{\sqrt{r_1 r_2}}\frac{g_{\rm rt,q}^{1/2}e^{-j\phi(\omega)/2}}{1-g_{\rm rt,q}+j\left(2\pi g_{\rm rt,q}/\Delta\omega_{\rm ax}\right)\times(\omega-\omega_q)}$$

→ Lorentzian

Re-write:

$$\left.\frac{\tilde{E}_{\rm trans}}{\tilde{E}_{\rm inc}}\right|_{\omega\approx\omega_q} = -e^{-j\phi(\omega)/2}\frac{g_{0,q}}{1+2j(\omega-\omega_q)/\Delta\omega_{\rm 3dB,q}}$$

Where:

$$g_{0,q} \equiv \frac{t_1 t_2}{\sqrt{r_1 r_2}}\frac{g_{\rm rt,q}^{1/2}}{1-g_{\rm rt,q}}$$

$$\Delta\omega_{\rm 3dB,q} \approx \frac{1-g_{\rm rt,q}}{g_{\rm rt,q}}\times\frac{\Delta\omega_{\rm ax}}{\pi}$$

Thus we see that for high values of $g_{rt} \Rightarrow g_{0,q} \longrightarrow \infty$; $\Delta\omega_{3dB,q} \rightarrow 0$.

The Highly Regenerative Limit

We observe that that product of $g_{0,q}$ and $\Delta\omega_{3dB,q}$ yields:

$$[g_0 \Delta\omega_{\mathrm{3dB}}]_q \approx g_{\mathrm{rt},q}^{-1/2} \times \frac{t_1 t_2}{\sqrt{r_1 r_2}} \times \frac{\Delta\omega_{\mathrm{ax}}}{\pi}$$

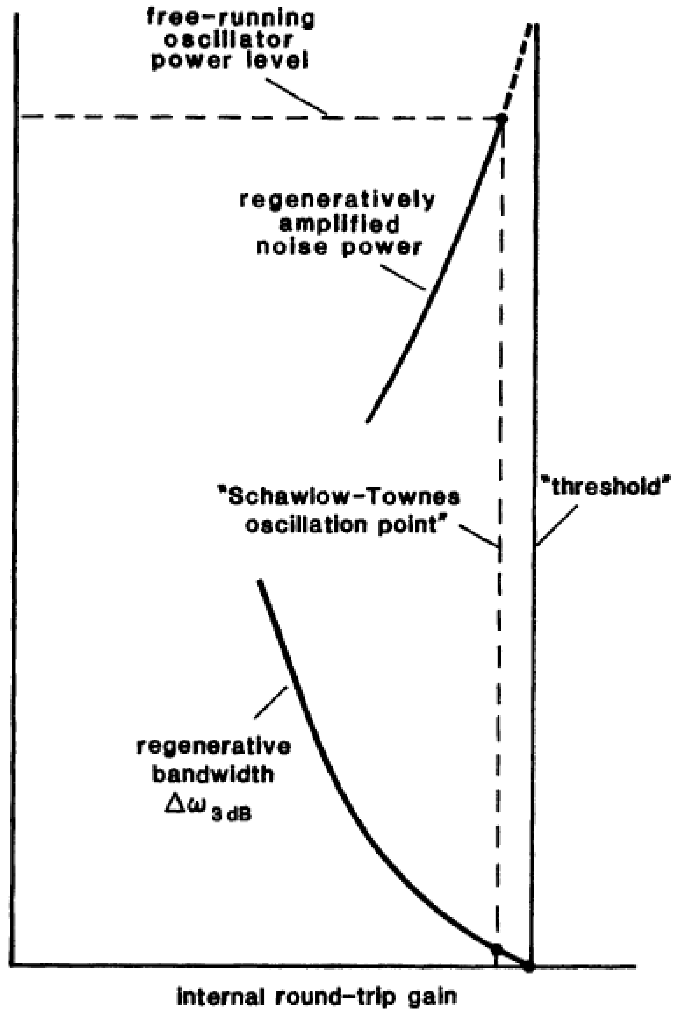
But $g_{rt} \longrightarrow 1$ (High gain limit) ⟹

$$g_0 \Delta\omega_{\mathrm{3dB}} \approx \frac{t_1 t_2}{\sqrt{r_1 r_2}} \times \frac{\Delta\omega_{\mathrm{ax}}}{\pi}$$

Gain-bandwidth product, applicable for all cavity modes,

and only dependent on coupling/cavity parameters.

The Highly Regenerative Limit



Schawlow-Townes model

Laser oscillator is regarded as a very high gain, very narrowband, regenerative noise amplifier just below threshold.

Incoherent, Gaussian noise source. → *Coherent, sinusoidal oscillator.*

$$\Delta\omega_{osc} = \Delta\omega_{3dB} \approx (2) \times \frac{N_2}{N_2 - N_1} \times \frac{\pi\hbar\omega\Delta\omega_c^2}{P_{osc}},$$

"Schawlow-Townes formula"

Regarded as a limit on how narrow the laser linewidth may be.

Thank you!