Chapter 10:
Nonlinear optical pulse propagation

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Outline

- Pulse amplification in homogenous media
- Nonlinear dispersive systems
- Nonlinear Schrödinger Equation
- Solitons in optical fibers

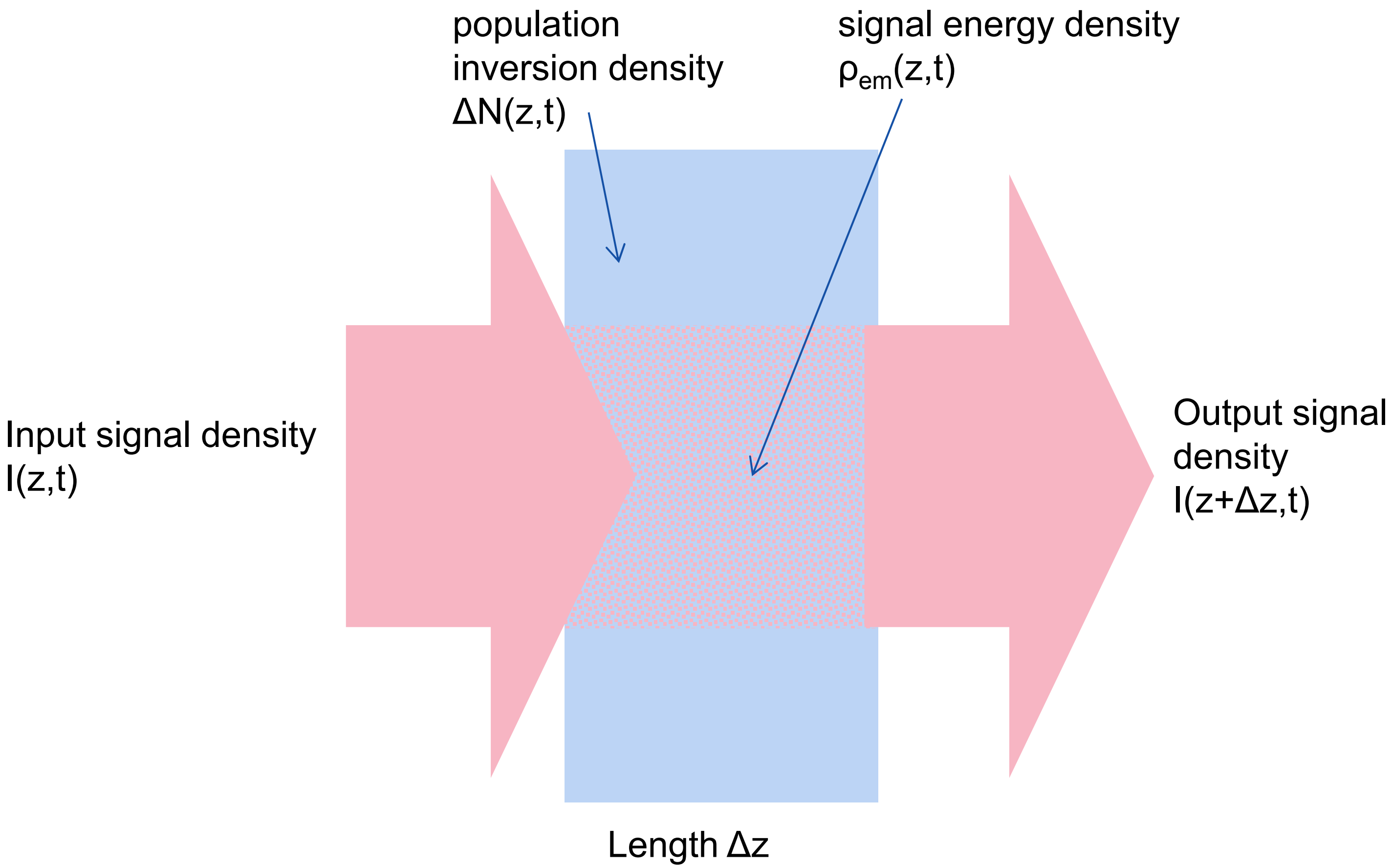
Pulse amplification in homogenous gain media

Approximations

1. Rate equation approximations are still valid, even though pulse amplification often involves short pulses with fast time variation and high intensities

2. The amplified pulse durations are short enough to neglect any pumping effects and any upper-level relaxation effects during the transit time of the pulse

Pulse amplification in homogenous gain media



Pulse amplification in homogenous gain media

The rate of change of the stored signal in the length Δz of the medium is given by

$$\frac{\partial}{\partial \hat{t}} \hat{\rho}_{em}(\hat{z},\hat{t})\Delta\hat{z} = \hat{I}(\hat{z},\hat{t}) - \hat{I}(\hat{z}+\Delta\hat{z},\hat{t}) + \sigma\Delta\widehat{N}(\hat{z},\hat{t})\hat{I}(\hat{z},\hat{t})\Delta\hat{z}$$

Using $\hat{I}(\hat{z},\hat{t})$=$c\hat{\rho}_{em}(\hat{z},\hat{t})$ and combining with the rate equation for the inverted population

$$\frac{\partial \hat{I}(\hat{z},\hat{t})}{\partial \hat{t}} + c\frac{\partial \hat{I}(\hat{z},\hat{t})}{\partial \hat{z}} = c\sigma\Delta\widehat{N}(\hat{z},\hat{t})\hat{I}(\hat{z},\hat{t})$$

$$\frac{\partial \Delta\widehat{N}(\hat{z},\hat{t})}{\partial \hat{t}} = -\left(\frac{2^*\sigma}{\hbar\omega}\right)\Delta\widehat{N}(\hat{z},\hat{t})\hat{I}(\hat{z},\hat{t})$$

Pulse amplification in homogenous gain media

Transformation from laboratory coordinates to a coordinate system traveling along the pulse

$$z \equiv \hat{z} \qquad t \equiv \hat{t} - \hat{z}/c$$

$$I(z,t) \equiv \hat{I}(\hat{z},\hat{t}) \quad \mathrm{N}(z,t) \equiv \Delta\widehat{N}(\hat{z},\hat{t})$$

Previous equations then transform to

$$\frac{\partial I(z,t)}{\partial z} = \sigma N(z,t) I(z,t) \qquad (1)$$

$$\frac{\partial N}{\partial t} = -\left(\frac{2^{*}\sigma}{\hbar\omega}\right) N(z,t) I(z,t) \qquad (2)$$

Pulse amplification in homogenous gain media

Integration of (1) over the entire amplifier and pulse

$$\int_{I=I_{in}(t)}^{I=I_{out}(t)} \frac{dI}{I} = \sigma \int_{z=0}^{z=L} N(z,t)dz$$

Defining the ”total number of atoms”

$$N_{tot}(t) = \int_{z=0}^{z=L} N(z,t)dz$$

Giving the solution

$$I_{out}(t) = I_{in}(t)e^{\sigma N_{tot}(t)} = G(t)I_{in}(t) \qquad (3)$$

Where $G(t) \equiv e^{\sigma N_{tot}(t)}$ is the time varying gain at any instant within the pulse

Pulse amplification in homogenous gain media

Integration of (2) over the entire amplifier length

$$\frac{\partial}{\partial t}\int_{z=0}^{z=L} N(z,t)dz \equiv \frac{\partial N_{tot}(t)}{\partial t} = -\left(\frac{2^*}{\hbar\omega}\right)\int_{z=0}^{z=L} \frac{\partial I(z,t)}{\partial t}$$

Which simplifies to

$$\frac{\partial N_{tot}(t)}{\partial t} = -\frac{2^*}{\hbar\omega}[I_{out}(t) - I_{in}(t)]$$

And substituting (3) into this solution gives

$$\frac{\partial N_{tot}(t)}{\partial t} = -\frac{2^*}{\hbar\omega}\left[\left(e^{\sigma N_{tot}(t)} - 1\right)\cdot I_{in}(t)\right] = \qquad (4)$$

$$= -\frac{2^*}{\hbar\omega}\left[\left(1 - e^{-\sigma N_{tot}(t)}\right)\cdot I_{in}(t)\right]$$

Pulse amplification in homogenous gain media

Suppose the total inversion before the pulse enters the media is

$$N_0 \equiv \int_{z=0}^{z=L} N(z,t)dz$$

The single pass power gain of the amplifier is then

$$G_0 = e^{\sigma N_0}$$

Also, the pulse energies per area are defined as

$$U_{in}(t) \equiv \int_{t_0}^{t} I_{in}(t)dt \quad U_{out}(t) \equiv \int_{t_0}^{t} I_{out}(t)dt$$

The saturation energy for the atomic medium is

$$U_{sat} \equiv \frac{\hbar\omega}{2^* \sigma}$$

Pulse amplification in homogenous gain media

Integration of (4) provides the useful relations

$$U_{in}(t) = U_{sat} \cdot \ln\left(\frac{1 - e^{-\sigma N_0}}{1 - e^{-\sigma N_{tot}(t)}}\right) = U_{sat} \cdot \ln\left(\frac{1 - 1/G_0}{1 - 1/G(t)}\right)$$

$$U_{out}(t) = U_{sat} \cdot \ln\left(\frac{e^{\sigma N_0} - 1}{e^{\sigma N_{tot}(t)} - 1}\right) = U_{sat} \cdot \ln\left(\frac{G_0 - 1}{G(t) - 1}\right)$$

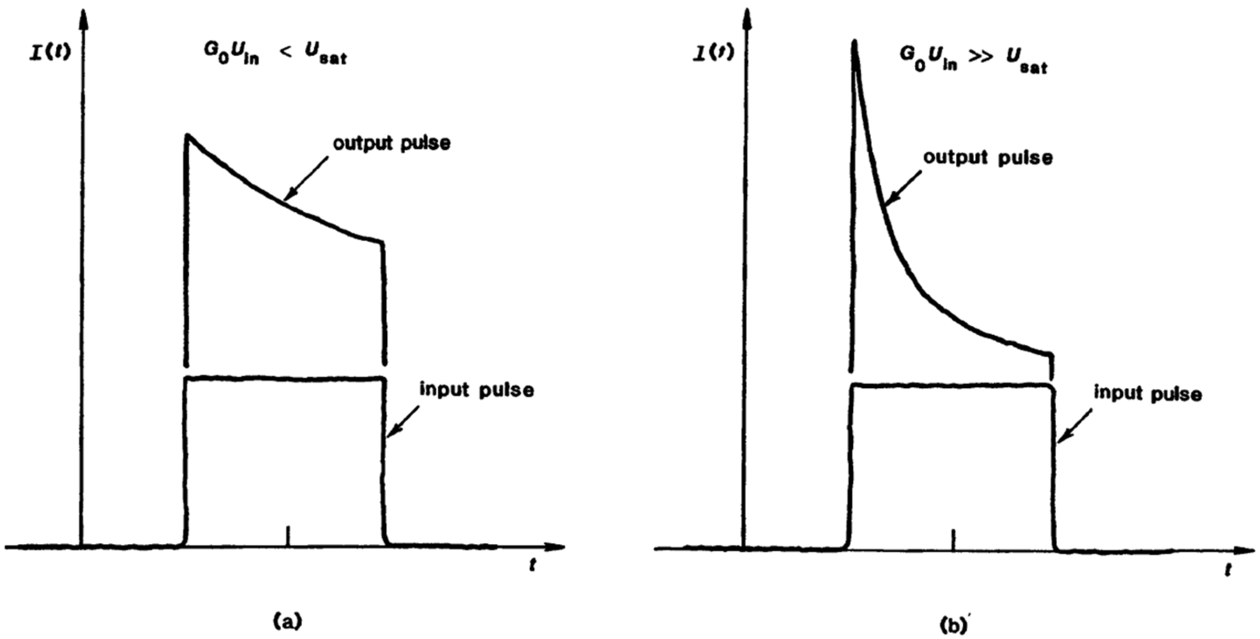
Using these relations one obtain

$$G(t) = \frac{G_0}{G_0 - (G_0 - 1)e^{-U_{in}(t)/U_{sat}}} \qquad (5)$$

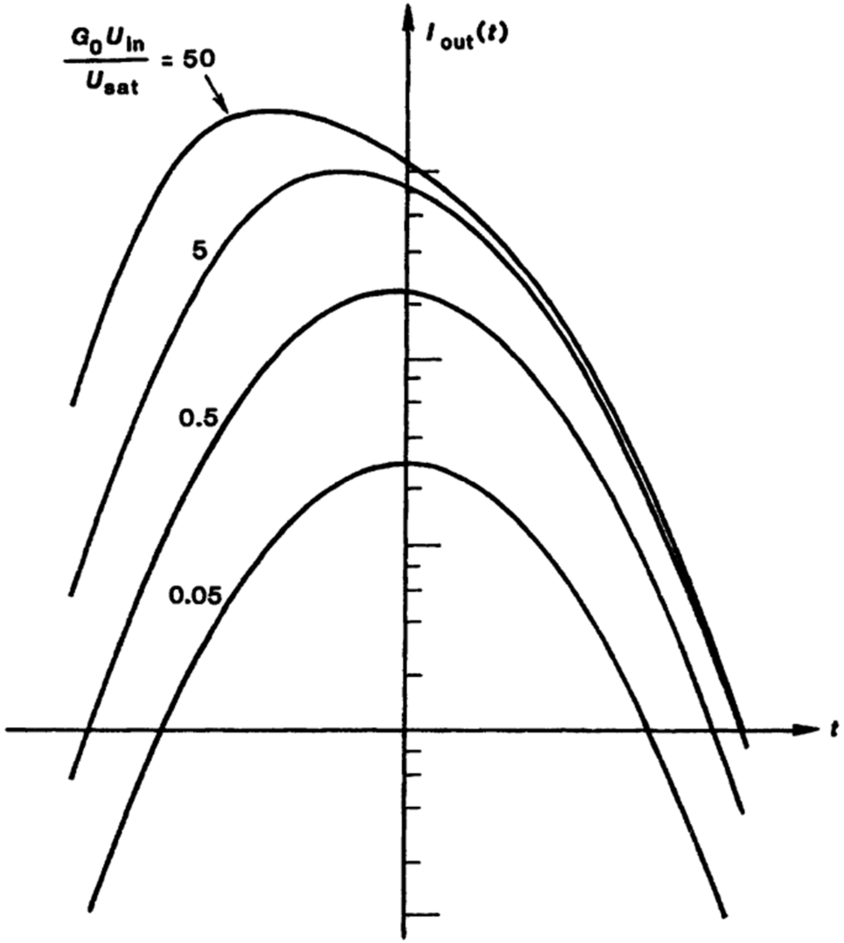
$$G(t) = 1 + (\mathrm{G}_0 - 1)\mathrm{e}^{-U_{out}(t)/U_{sat}}$$

Pulse amplification in homogenous gain media

The relations (5) can be used to detemine the pulseshape after passing though the amplifier



Pulse amplification in homogenous gain media



Pulse amplification in homogenous gain media

The energy extracted from the amplifier is

$$U_{extr} = U_{out} - U_{in} = U_{sat} \cdot \ln(G_0/G_f)$$

$$G_f = \lim_{t \to \infty} G(t)$$

The available energy in the amplifier is defines as

$$U_{avail} = U_{sat} \cdot \ln(G_0) = \frac{\mathrm{N}_0 \hbar\omega}{2^*}$$

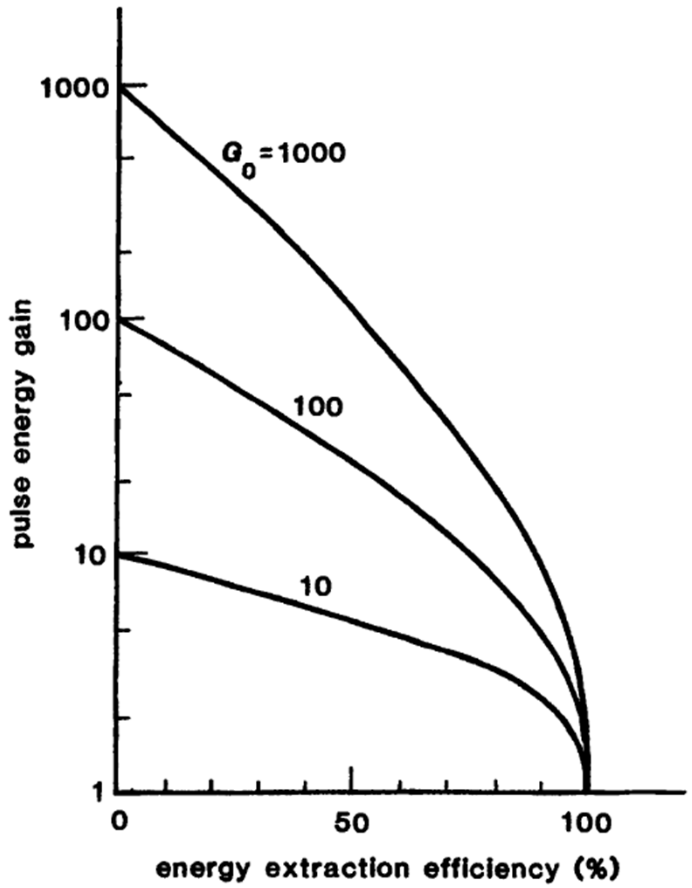
Pulse amplification in homogenous gain media

The pulse energy gain can be defined as

$$G_{pe} \equiv \frac{U_{out}}{U_{in}} = \frac{\ln\left((G_0 - 1)/(G_f - 1)\right)}{\ln\left((G_0 - 1)/(G_f - 1)\right) - \ln(G_0/G_f)}$$

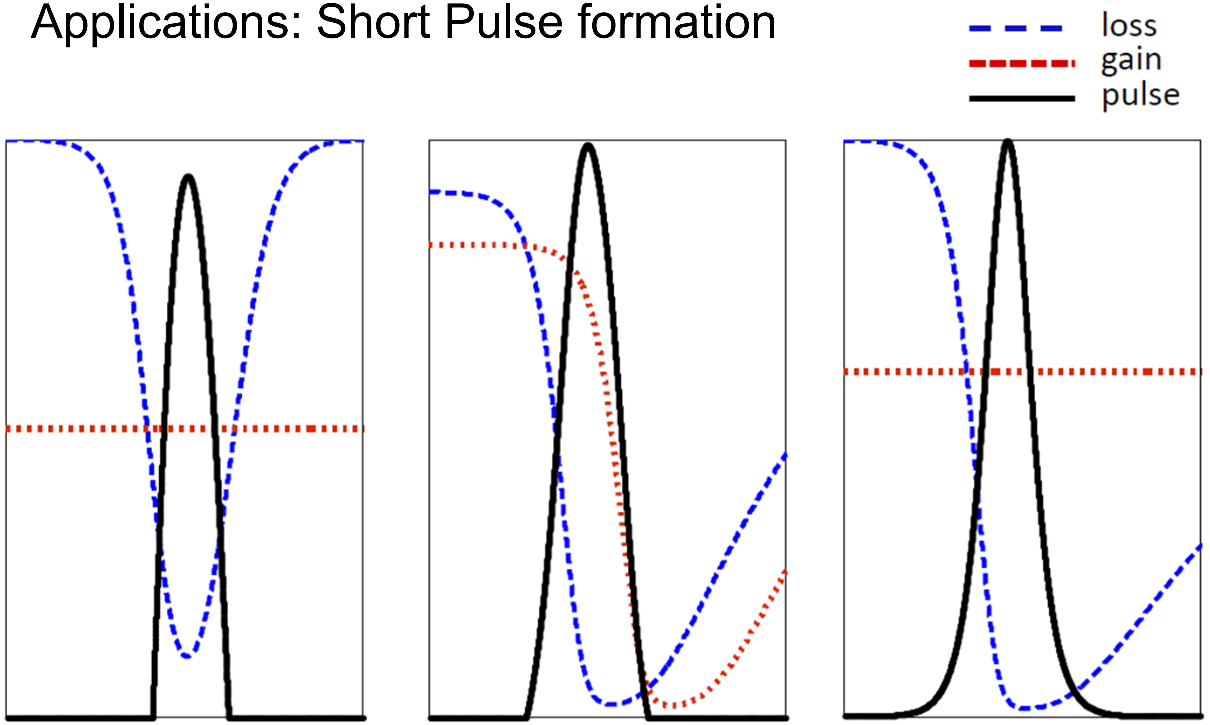
The pulse extraction efficiency

$$\eta \equiv \frac{U_{out} - U_{in}}{U_{avail}} = \frac{\ln(G_0) - \ln(G_f)}{\ln(G_0)}$$



Pulse amplification in homogenous gain media

Applications: Short Pulse formation



A fast saturable absorber
e.g. KLM

Slow gain, slow absorber
e.g. Dye lasers

Soliton formation with
a slow saturable absorer

Pulse propagation in nonlinear dispersive systems

When an electric field is applied to a transparent dielectric medium it gives rise to a macroscopic polarization in the material

$$P(\bar{r},t) = \varepsilon_0\left(\chi_{(1)}E + \chi_{(2)}E^2 + \chi_{(3)}E^3 + \ldots\right)$$

$\chi_{(1)}$ - linear response. Refractive index & Absorption

$\chi_{(2)}$ - SHG, OPO, OPA etc…
Only in noncentrosymmetric materials e.g. KTP, $LiNbO_3$

$\chi_{(3)}$ - THG, FWM, Kerr Effect etc..

Pulse propagation in nonlinear dispersive systems

$\chi_{(3)}$ is present in all optical materials.
For a strong electric field the total displacement d is

$$d = \varepsilon_0(1 + \chi_{(1)})E + \chi_{(3)}E^3$$

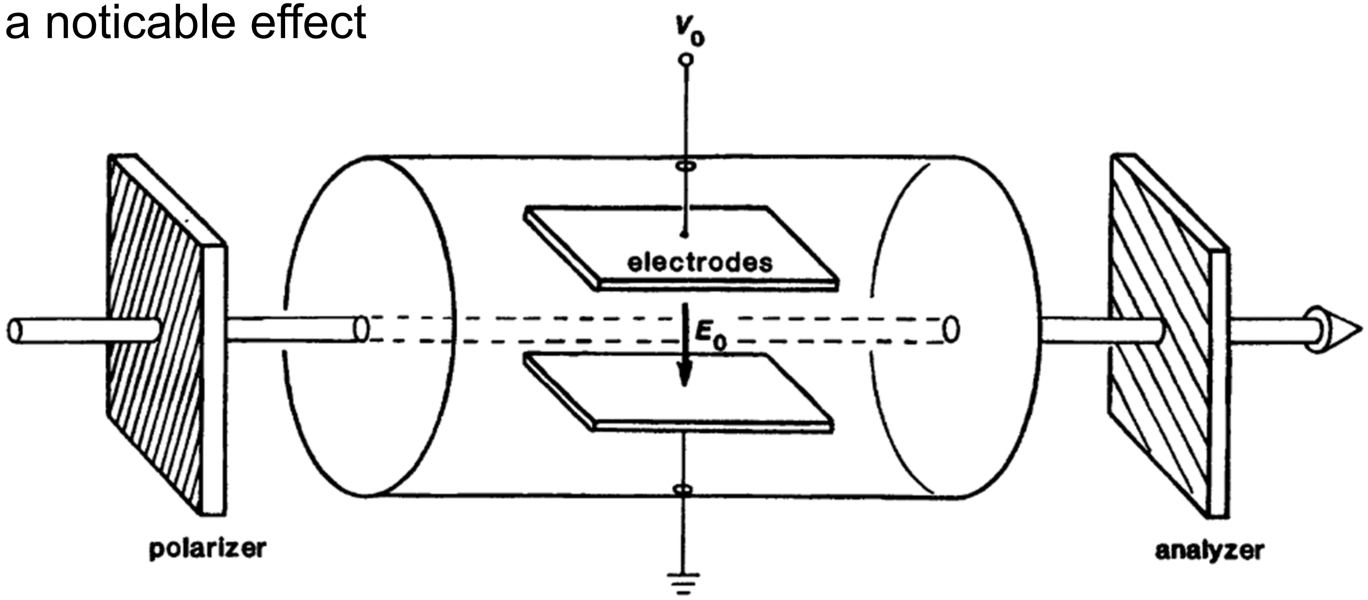
Resulting in a nonlinear respons of the refractive index

$$n = \mathrm{n}_0 + \mathrm{n}_{2E}E^2$$

Can be induced by applying an
electric field and induce birefringence,
e.g in Kerr Light Modulators and
Pockels cells

Pulse propagation in nonlinear dispersive systems

For a Kerr Light modulator voltages in the rage of 25000 V are required for a noticable effect



Kerr Constant $K \ (mV^{-2})$

Nitrobenzen:	4.4×10^{-12}
Water:	5.2×10^{-14}
Oxide glass:	$\sim 1\times10^{-14}$

Pulse propagation in nonlinear dispersive systems

Optical Kerr effect: The optical signal is strong enough to induce the $\chi_{(3)}E^3$ term

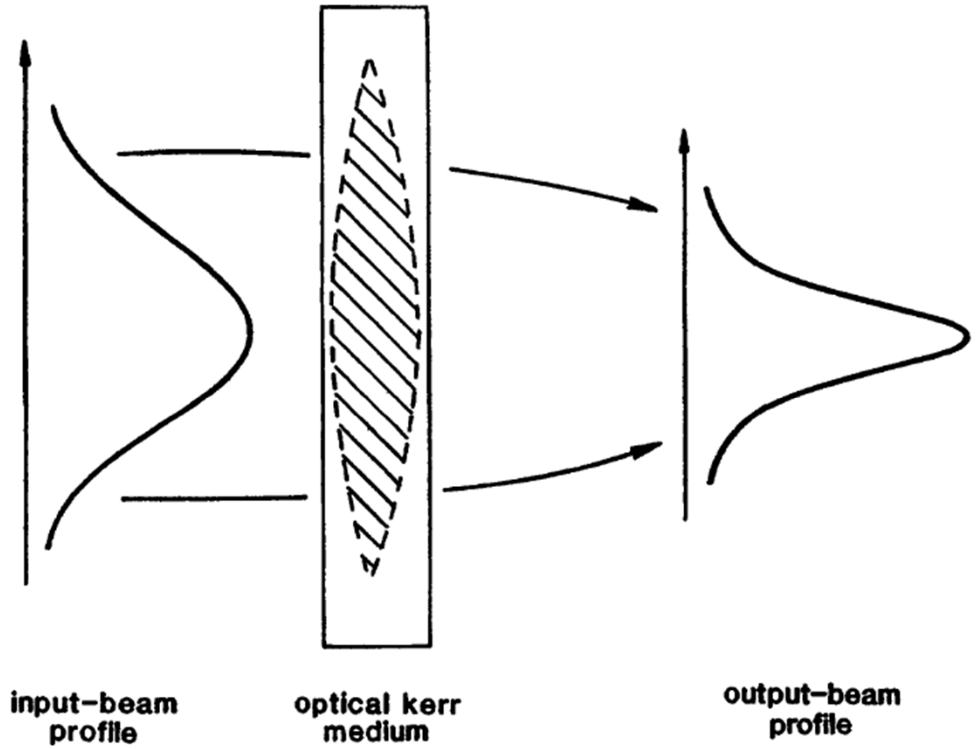
$\chi_{(3)}E^3$
- 3ω generation (generally weak)
- refractive index change $n = n_0 + n_{2I}I$

Present in all optical materials

- Self focusing
- Self phase modulation

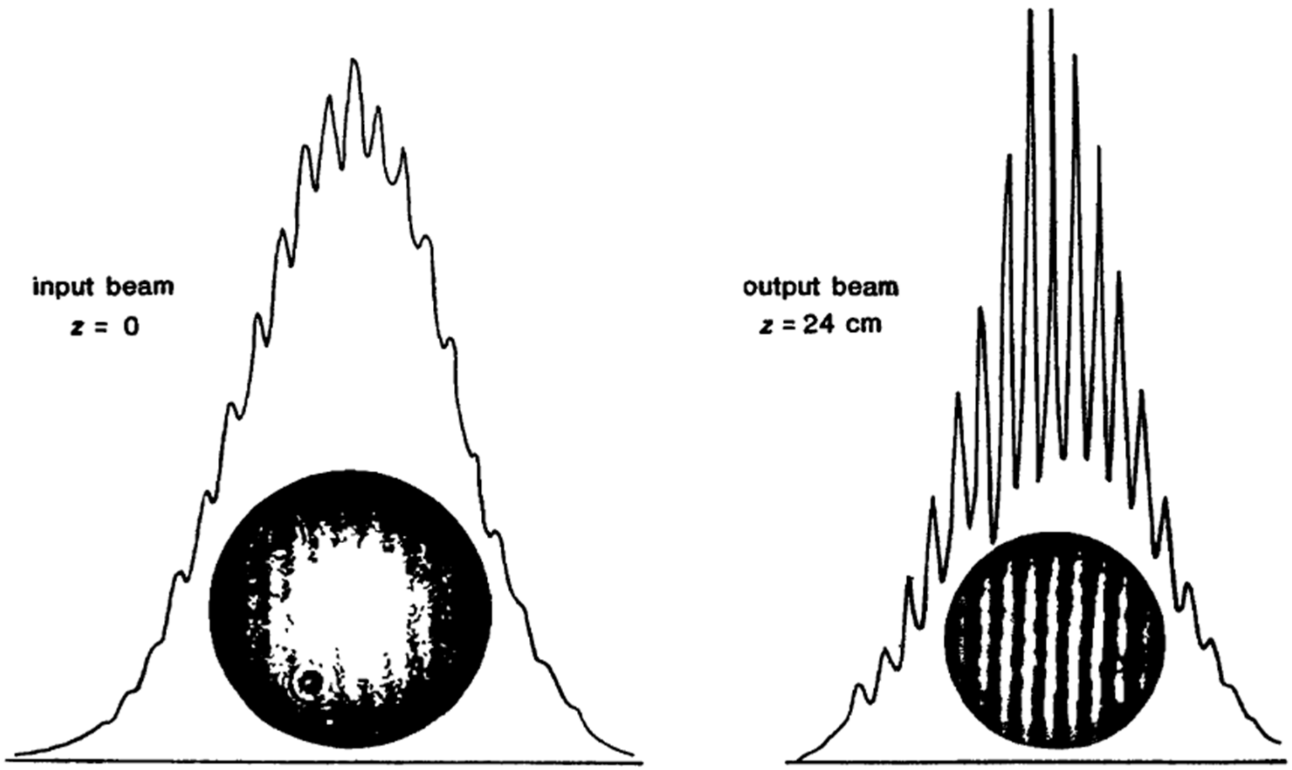
Pulse propagation in nonlinear dispersive systems

Self focusing



Pulse propagation in nonlinear dispersive systems

Self focusing



Pulse propagation in nonlinear dispersive systems

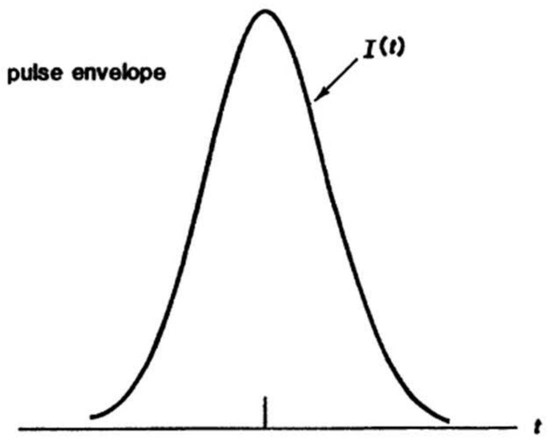
Self phase modulation: the temporal analoge to self focusing

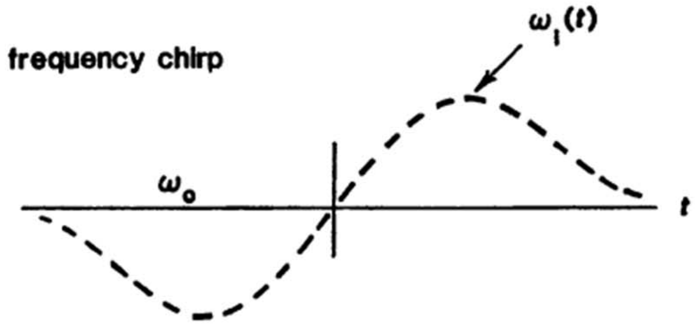
$$\Delta n = n(t) - n_0 = n_{2I} I(t)$$

An optical pulse traveling through a nonlinear material will then experience a phase shift of

$$e^{j\Delta\phi(t)} = e^{-j2\Delta n(t)L/\lambda} = e^{-j2n_{2I}I(t)L/\lambda}$$

Normally $n_{2I} > 0$ so $\frac{\partial}{\partial t}\Delta\phi(t) = \Delta\omega_i < 0$





Pulse propagation in nonlinear dispersive systems

Assume Gaussian unchirped input pulse

$$E(t) = E_0 e^{-at^2} \qquad I(t) = I_0 e^{-2at^2}$$

Passing through a nonlinear medium of length L it acquires a phase shift

$$\phi(t) = -\frac{2\pi(n_0 + n_{2I} I)L}{\lambda}$$

With the phase shift derivative being

$$\frac{\partial \phi}{\partial t} = -\frac{2\pi n_{2I} L}{\lambda}\frac{\partial I}{\partial t} \approx -\frac{4\pi a n_{2I} I_0 L}{\lambda} t e^{-2at^2}$$

That is, the pulse will acquire significant amount of self phase modulation if

$$I_0 = \frac{\lambda}{2\pi n_{2I} L} = \left\{ \begin{matrix} L = 10\ m \\ n_{2I} = 3 \cdot 10^{-16}\ cm^2/W \\ \lambda = 0.5\ \mu m \end{matrix} \right\} \approx 30\ MW/cm^2$$

Pulse propagation in nonlinear dispersive systems

Linear disperion + Nonlinear effect

Negative GVD

$$\beta'' = \frac{\partial v_g}{\partial \omega} < 0$$

v_g increases as ω increases

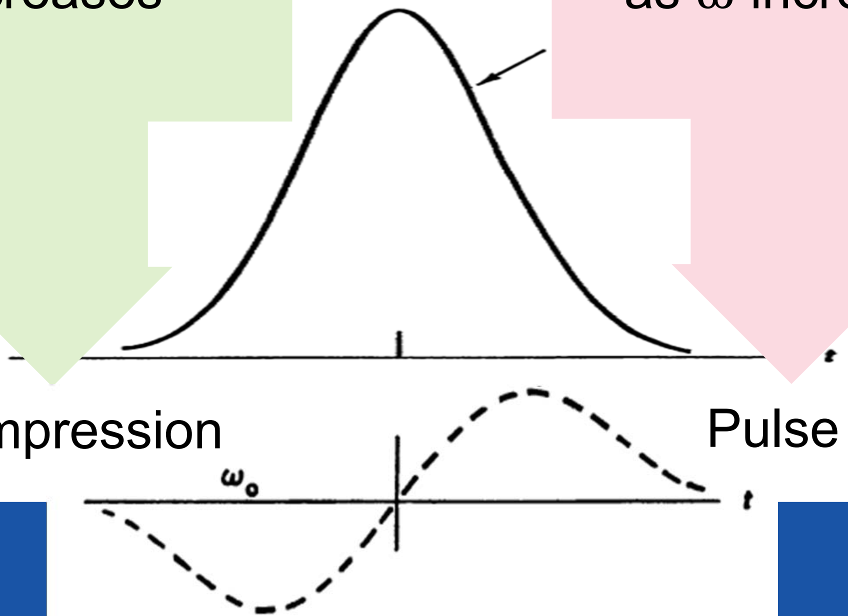
Pulse compression

Positive GVD

$$\beta'' = \frac{\partial v_g}{\partial \omega} > 0$$

v_g decreases as ω increases

Pulse broadening



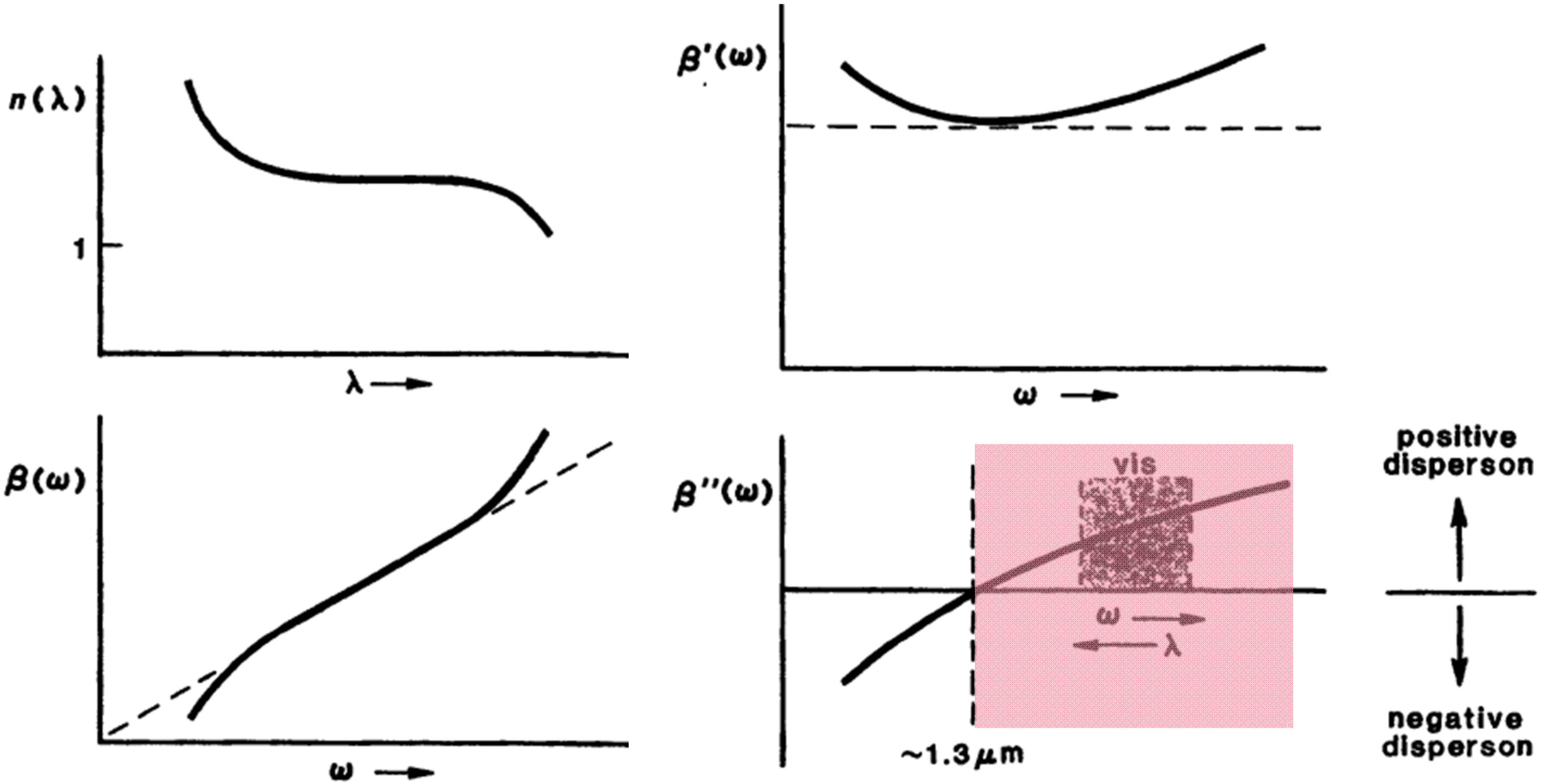
Nonlinear Schrödinger Equation

A pulse propagating in a nonlinear media or along a fiber can be described with the nonlinear Schrödinger equation

$$\left[\frac{\partial}{\partial z} + \beta' \frac{\partial}{\partial t} - j\frac{\beta''}{2}\frac{\partial^2}{\partial t^2} + \frac{j\beta_2 \left|\tilde{E}\right|^2}{2}\right]\tilde{E}(z,t) = 0$$

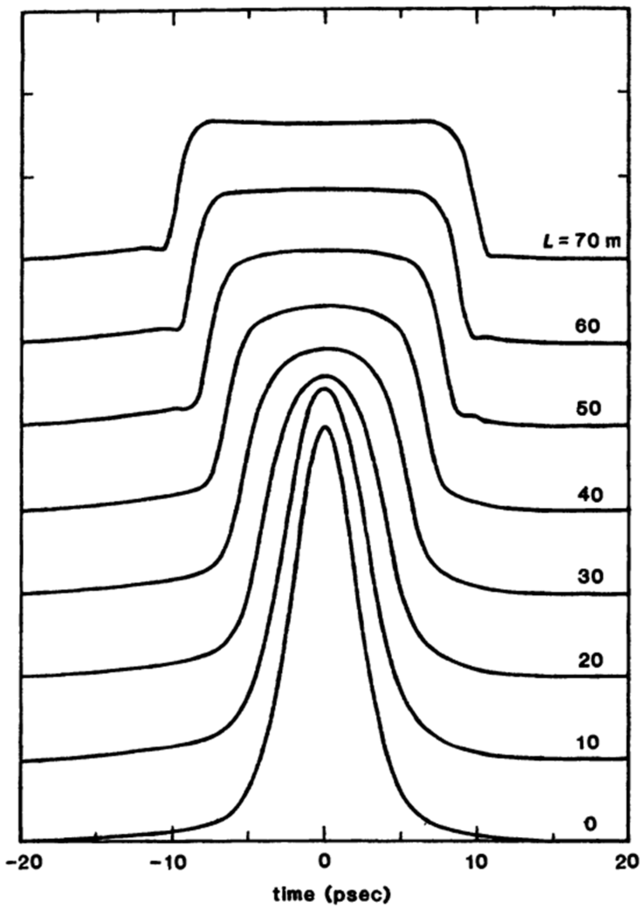
Can be expanded to account for various nonlinear effects such as Raman scattering, SHG etc. Here only the Kerr effect is considered.

Pulse propagation in an optical fiber



Pulse propagation in an optical fiber

FIGURE 10.14
Pulse broadening produced by self-phase modulation plus positive dispersion for a 5.5 ps, 10 W input pulse at $\lambda_0 = 590$ nm traveling through increasing lengths of single-mode fiber.



Pulse propagation in an optical fiber

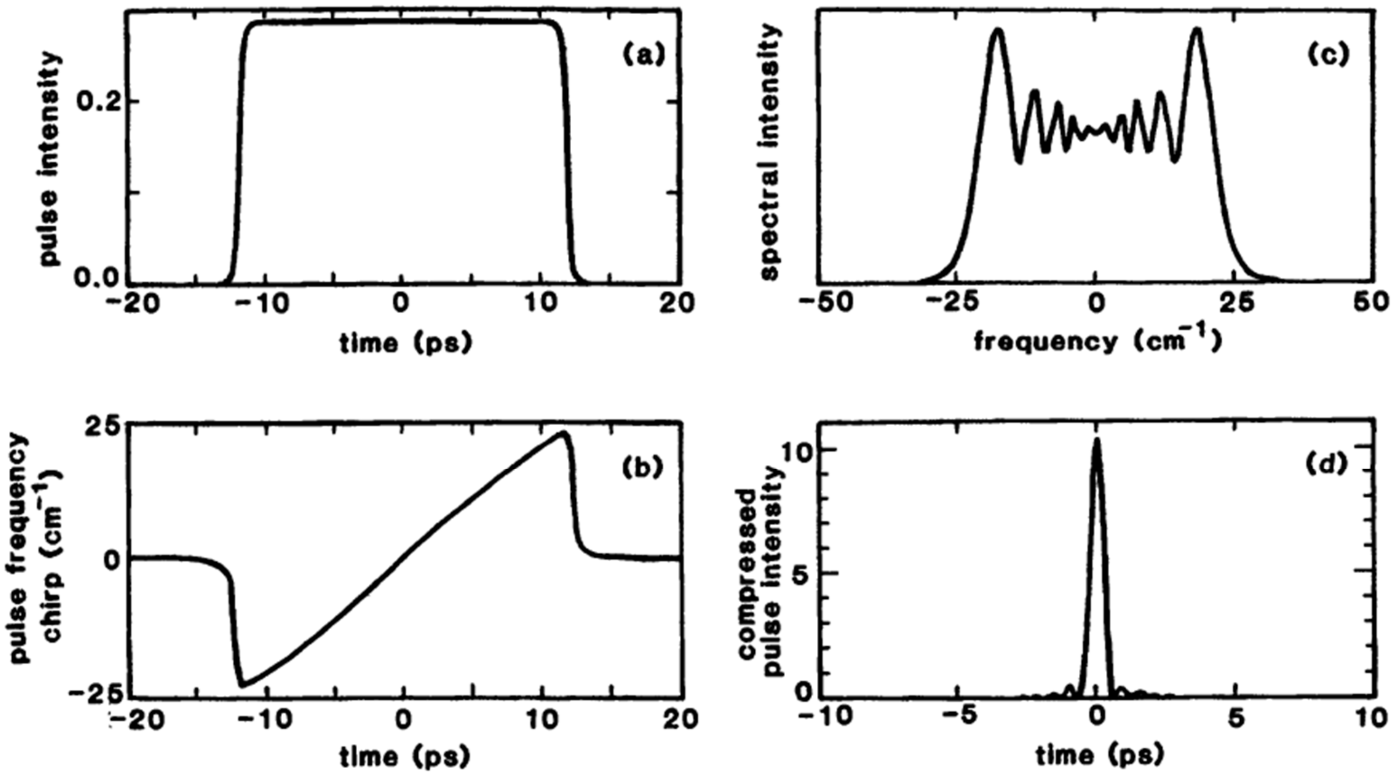
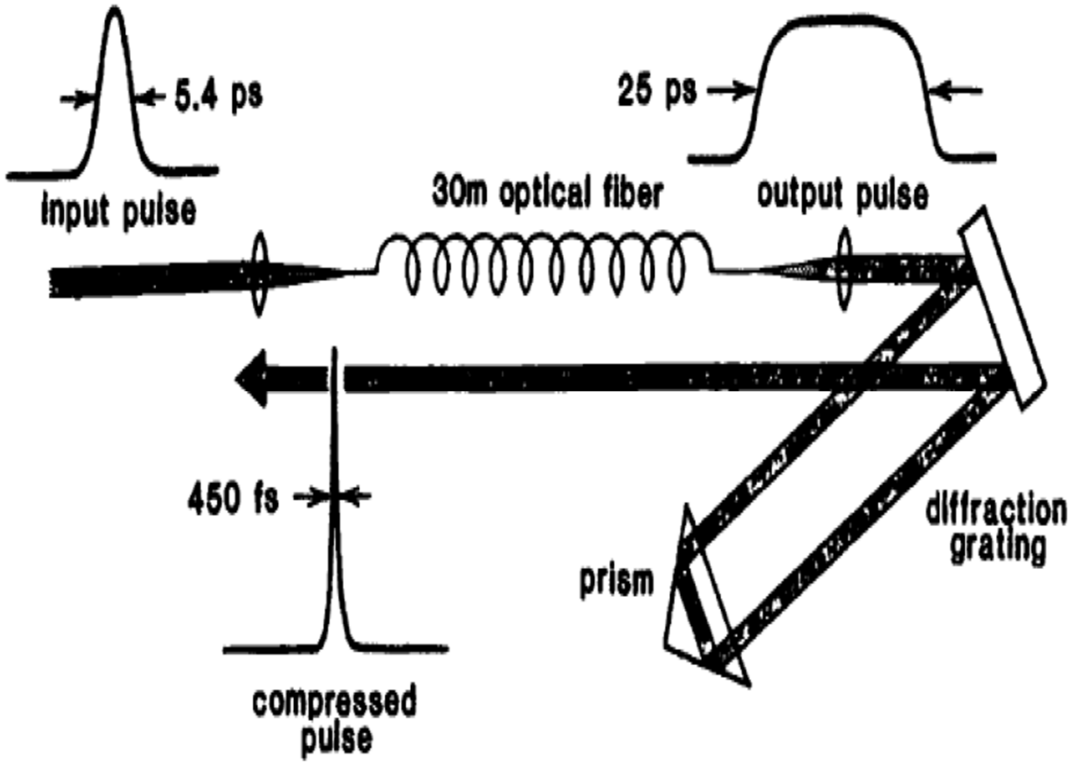


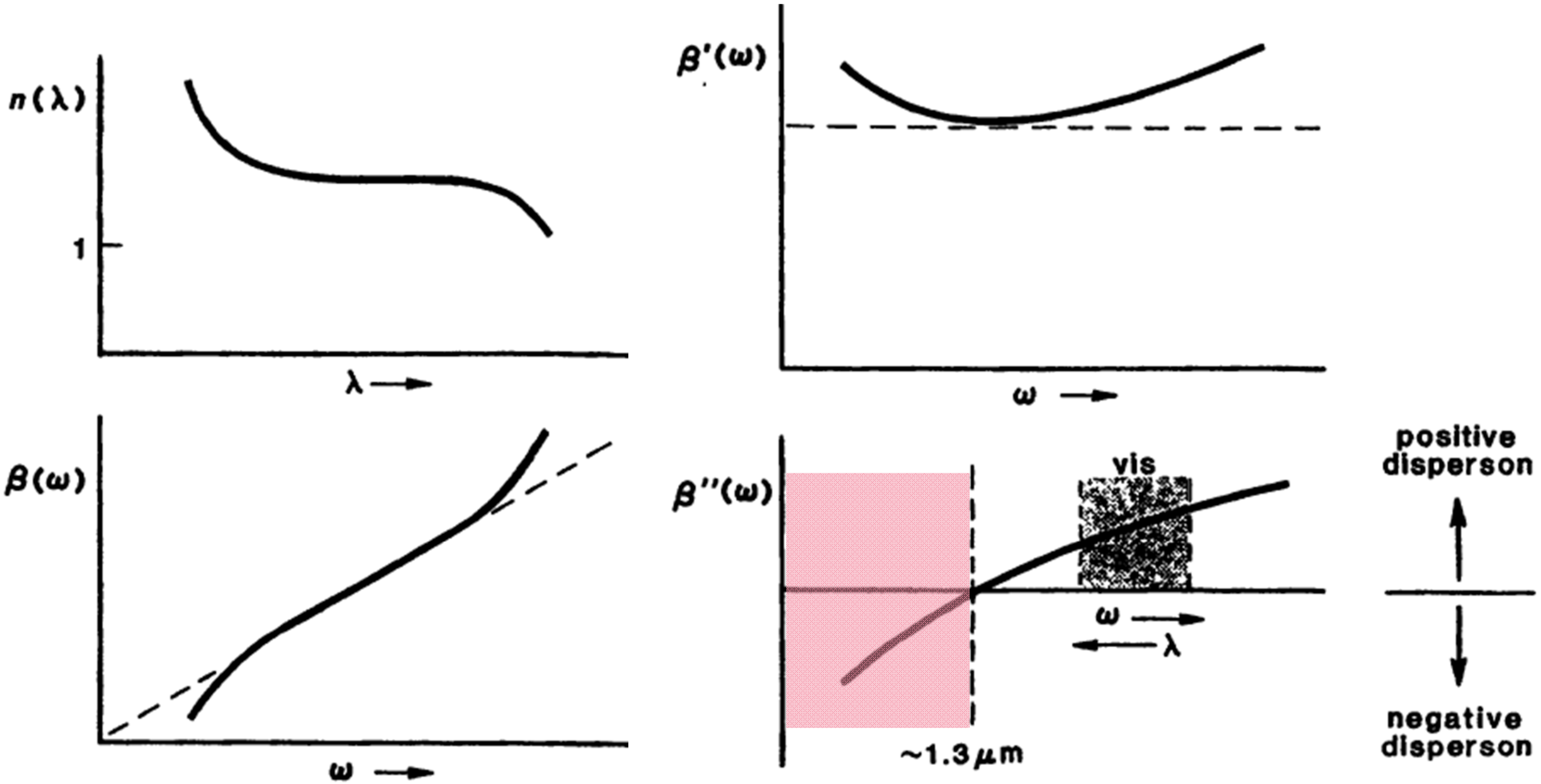
FIGURE 10.15
Self-broadening of an initial 6-ps 100-W pulse after propagation through 30 m of single-mode fiber. (a) Output pulse intensity versus time. (b) Output frequency chirp. (c) Output pulse spectrum. (d) Result of linear dispersive compression of this chirped pulse.

Pulse propagation in nonlinear dispersive systems

Application: Pulse compression



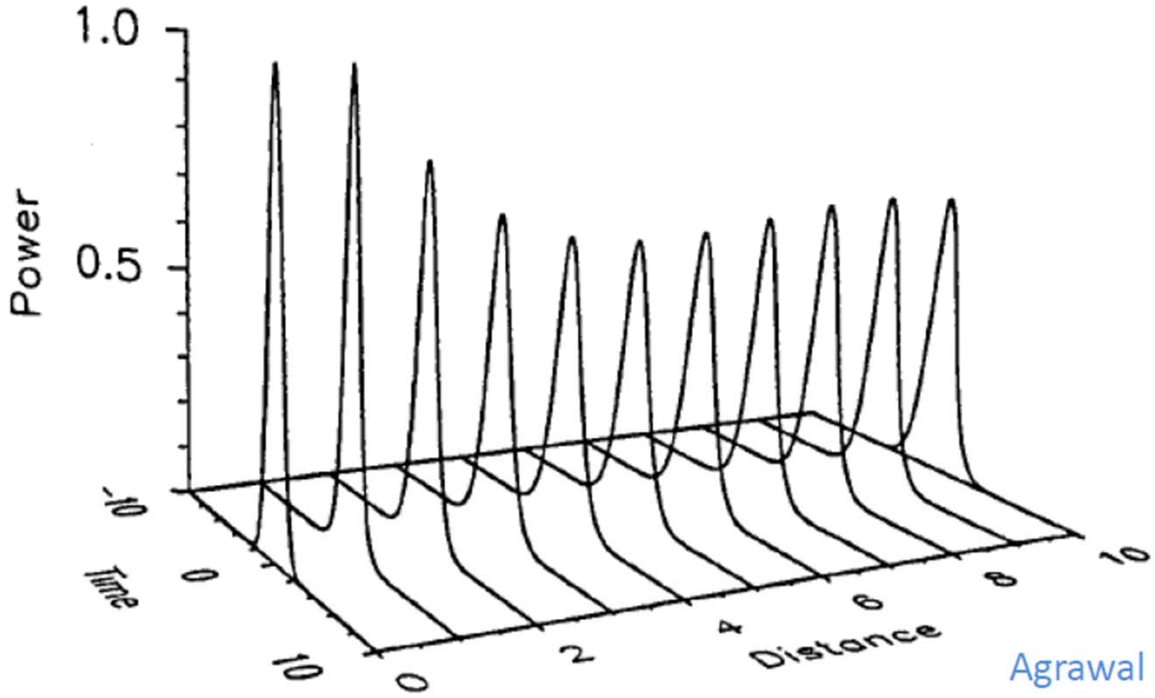
Soliton in optical fiber



Soliton in optical fiber

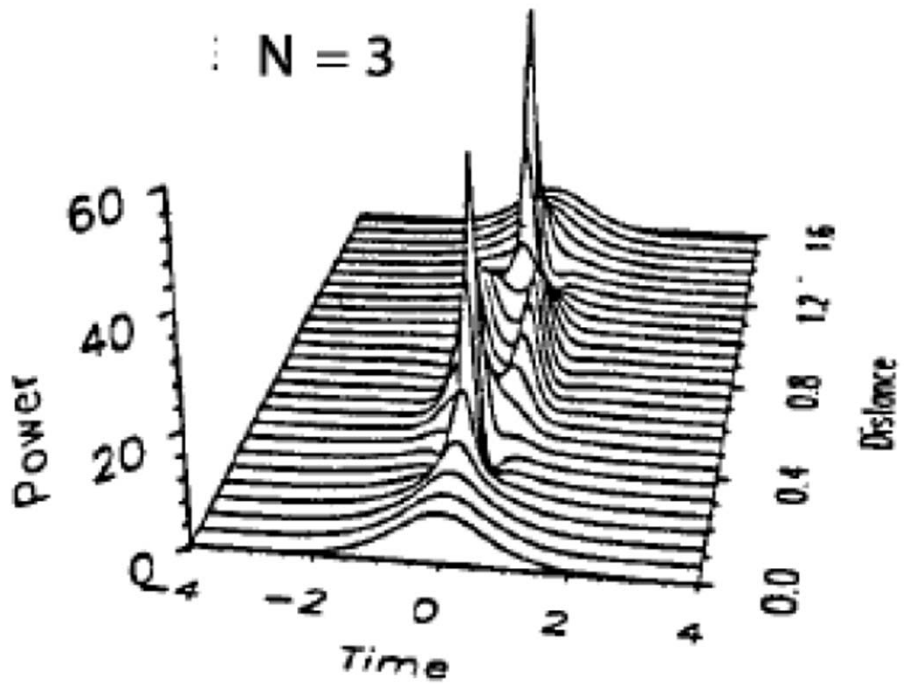
Solitons are a solution to the NLSE, and at lower order, N=1, has the form

$$E(t) = E_0 \operatorname{sech}\left(\frac{t - t_0 - z/v_g{}'}{\tau_0}\right) \exp(j(\Omega t - \kappa z))$$



Soliton in optical fiber

Higher order solitons, N>1, reproduce themselves at periodic distances along the fiber



Agrawal