Lecture 4
Ray and Wave Propagation*

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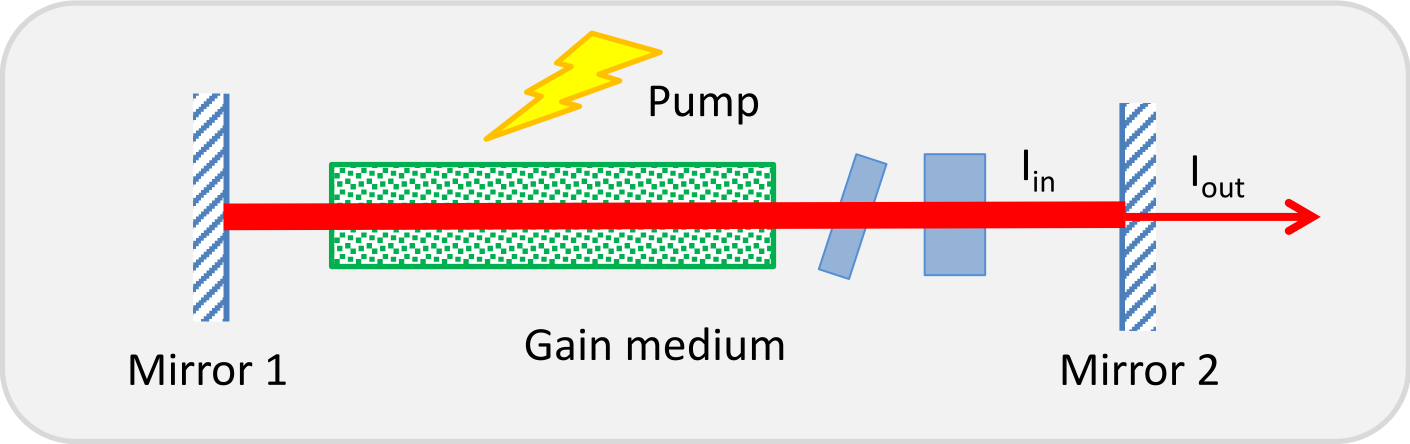
* Some figures and texts belong to: O. Svelto, *Principles of Lasers*, 5th Ed., Springer.

Reading

- *Principles of Lasers* (5th Ed.): Chapter 4.
- Skip: Subsection 4.5.2.
- Warning: Sections 4.6 and 4.7 can be mind-twisting.

Laser



- Mathematical tool for ray/wave behavior (not yet in cavity)
- Single-interface refl./refr. (Brewster angle)
- Anti-reflection coating
- High-reflectivity dielectric mirrors
- Fabry-Pérot cavity

Contents

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1. ABCD matrix formulation	15′
2. Reflection and transmission - Single interface - Multiple interfaces	15′
3. Fabry-Pérot interferometer	15′
4. Diffraction optics	10′
5. Gaussian beams	25′
Total:	**80′**

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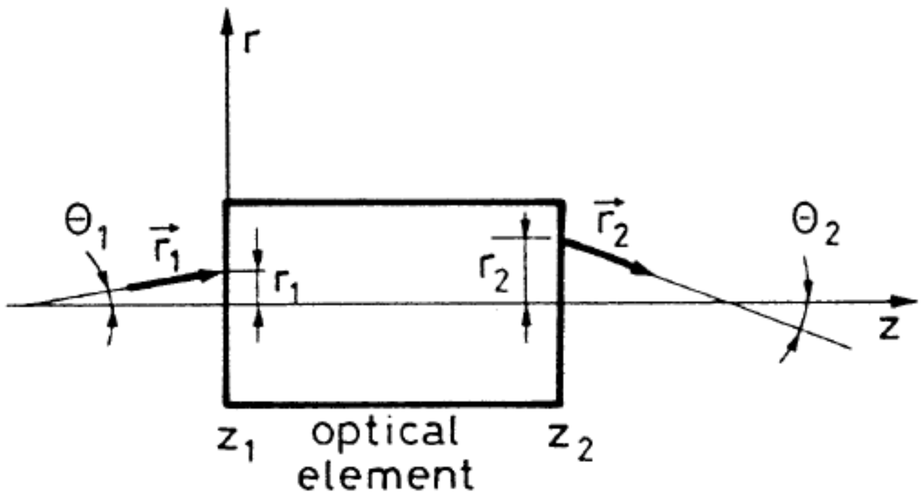
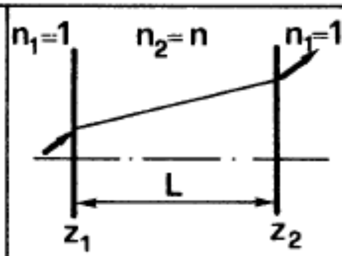
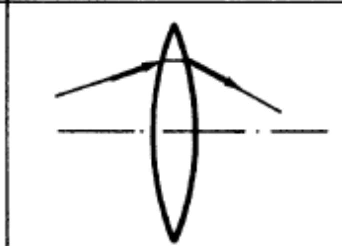
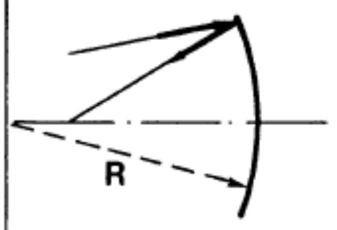
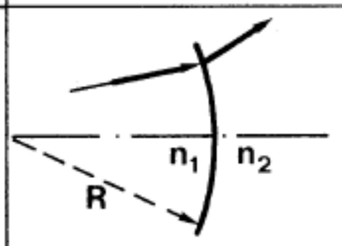
Ray propagation



$$\begin{vmatrix} r_2 \\ r_2' \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \begin{vmatrix} r_1 \\ r_1' \end{vmatrix}$$

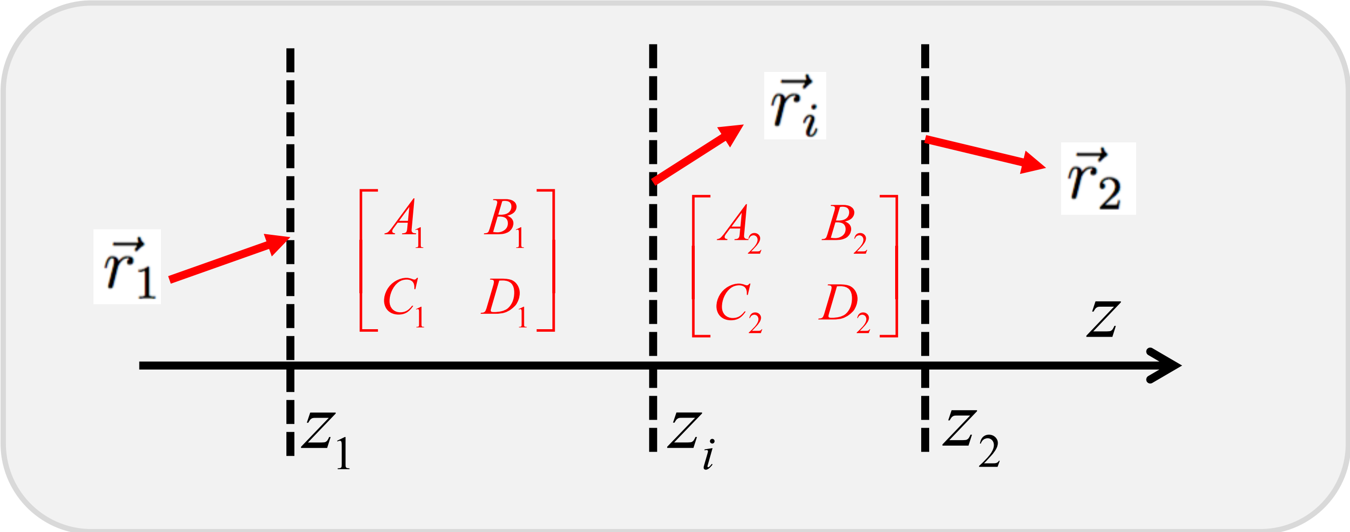
Assumptions:
- Geometrical optics
- Paraxial propagation

Common ray matrices

Free space propagation		$\begin{bmatrix} 1 & \frac{L}{n} \\ 0 & 1 \end{bmatrix}$
Thin lens		$\begin{bmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{bmatrix}$
Spherical mirror		$\begin{bmatrix} 1 & 0 \\ \frac{-2}{R} & 1 \end{bmatrix}$
Spherical dielectric interface		$\begin{bmatrix} 1 & 0 \\ \frac{n_2-n_1}{n_2}\frac{1}{R} & \frac{n_1}{n_2} \end{bmatrix}$

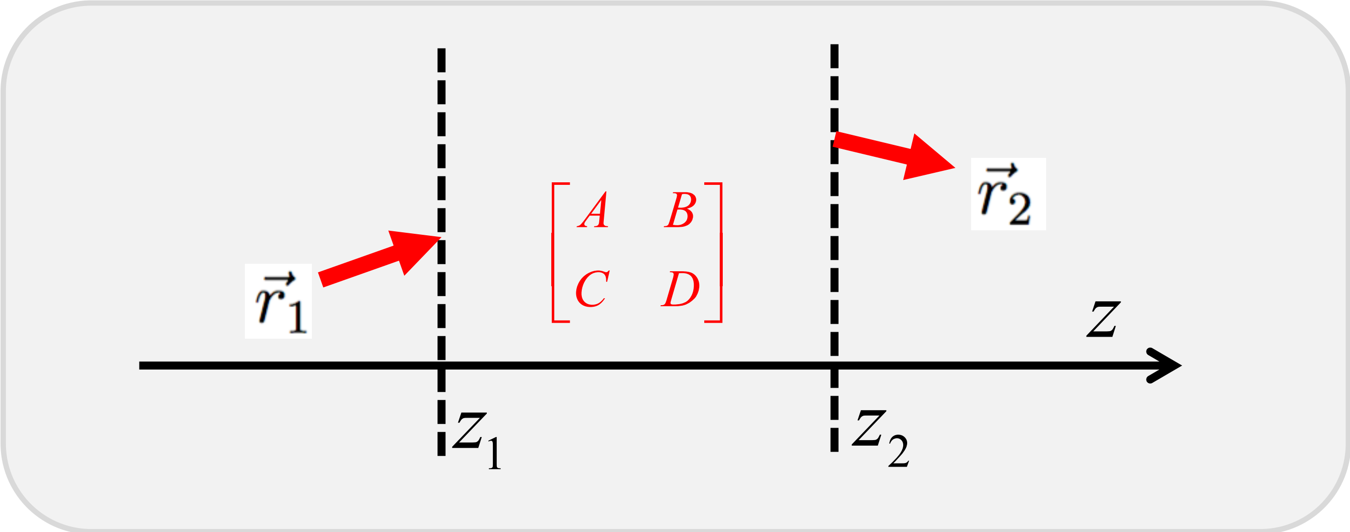
AD-BC=1

Cascading of ABCDs



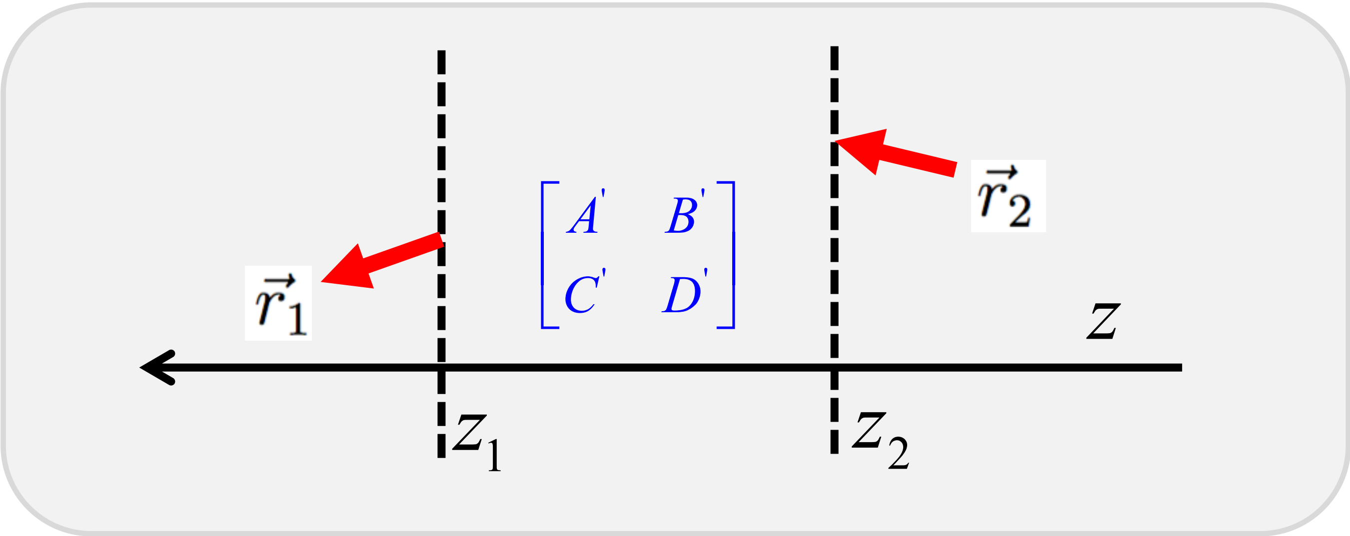
$$\begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_1' \end{bmatrix}$$

Forward v.s. reverse propagation



$$\begin{bmatrix} r_2 \\ r_2' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_1 \\ r_1' \end{bmatrix}$$

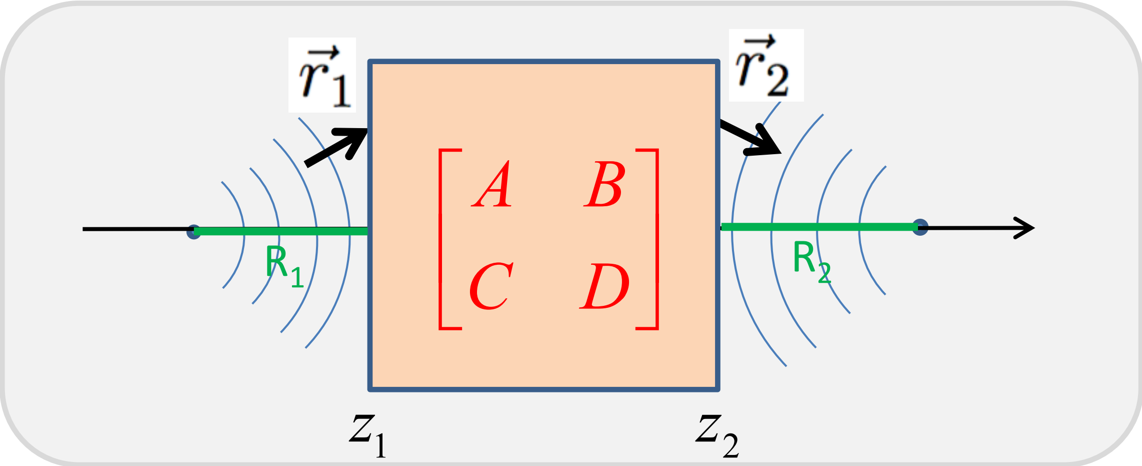
Forward v.s. reverse propagation



$$\begin{bmatrix} r_1 \\ r_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} r_2 \\ r_2' \end{bmatrix}$$

where, $$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} D & B \\ C & A \end{bmatrix}$$

Spherical wave



$$r_2 = Ar_1 + Br'_1$$

$$r'_2 = Cr_1 + Dr'_1$$

$$r_1 = R_1 r'_1$$

$$r_2 = R_2 r'_2$$

$$R_2 = \frac{AR_1 + B}{CR_1 + D}$$

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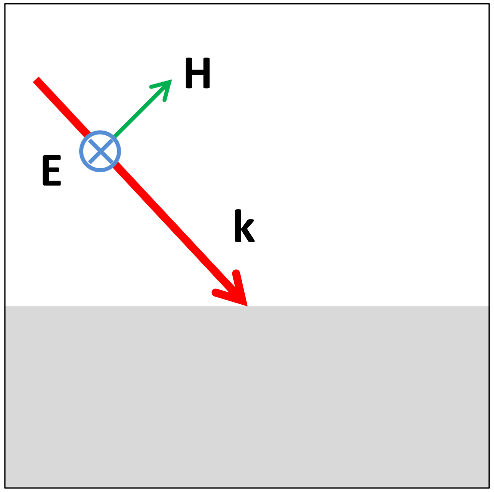
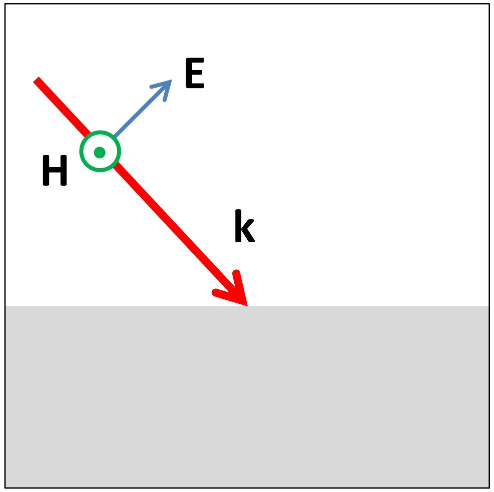
Polarizations

TE: Transverse-electric

TM: Transverse-magnetic





s-polarized

p-polarized

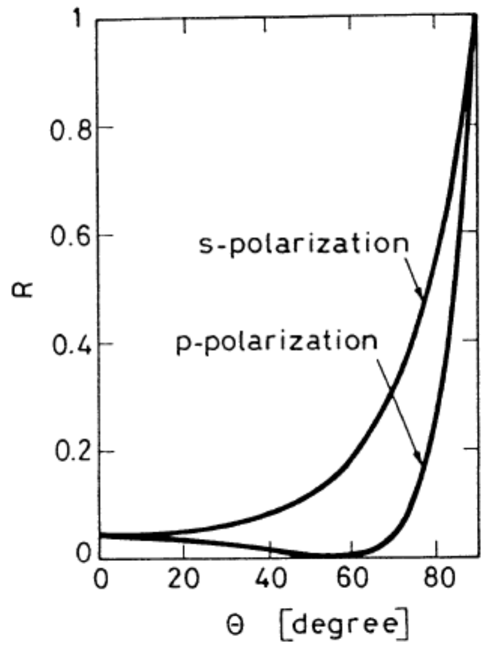
”s” for senkrecht

Single dielectric interface

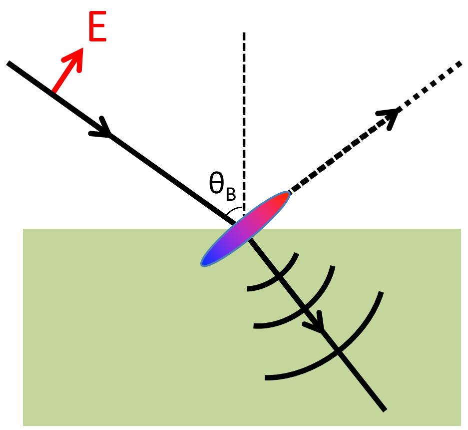
Normal incidence:

$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2}$$

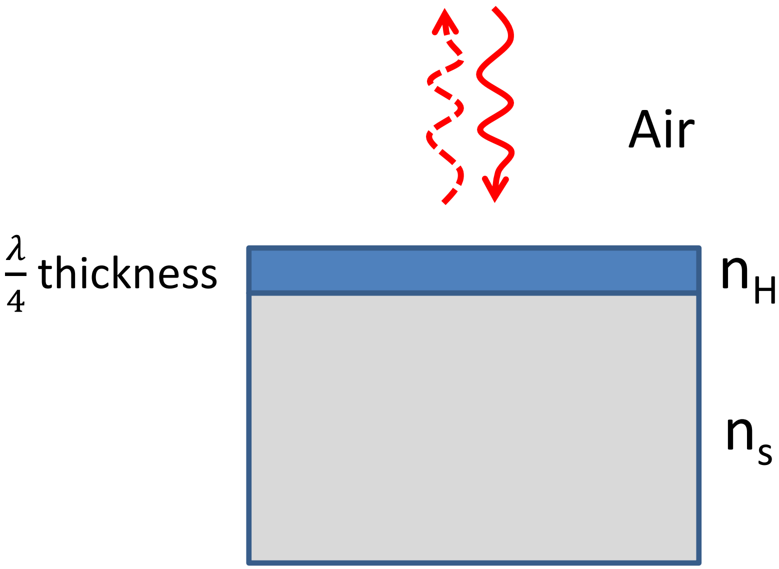
Oblique incidence: Angle- and polarization-dependent



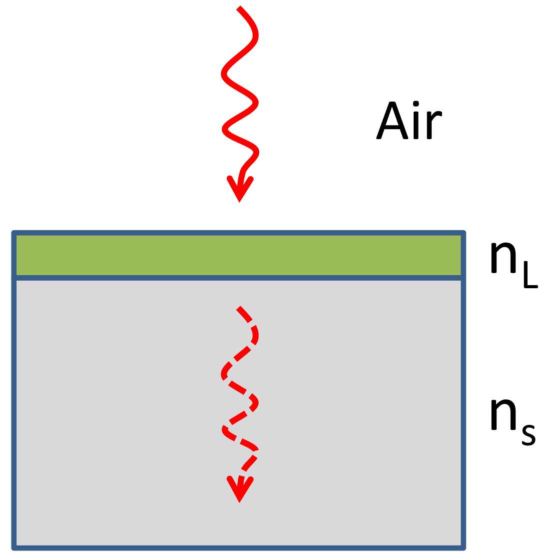
- Possible π phase shift
- Brewster angle (bi-directional)



1 coating: “pro-” and anti-reflection



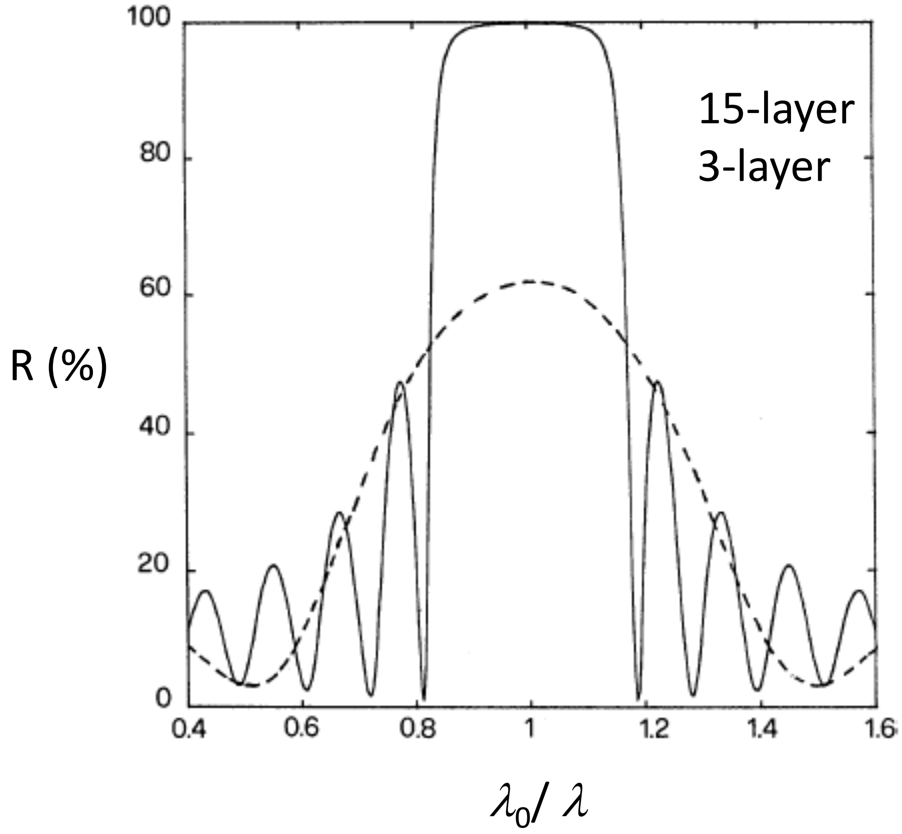
$n_H > n_s$

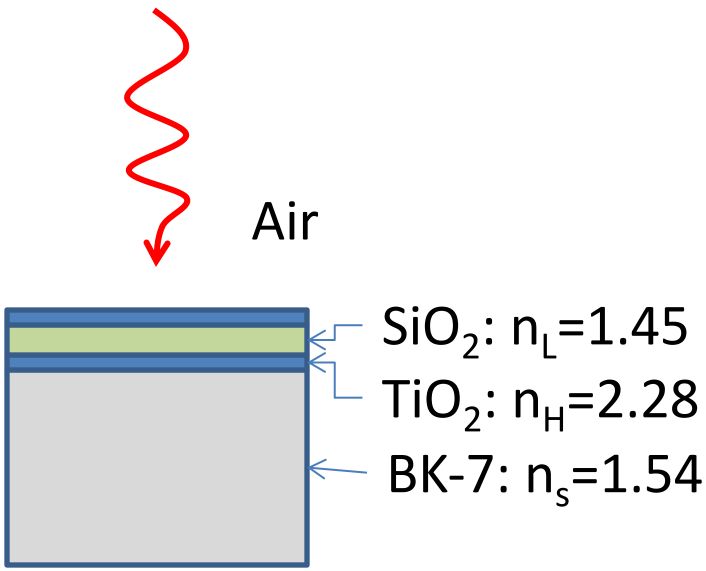


$n_L < n_s$

R_{min} occurs at $n_L = \sqrt{n_s}$

Multiple dielectric coatings





Quarter-wave stack

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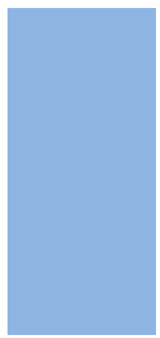
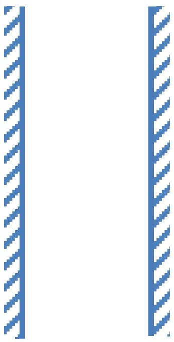
Fabry-Pérot interferometer

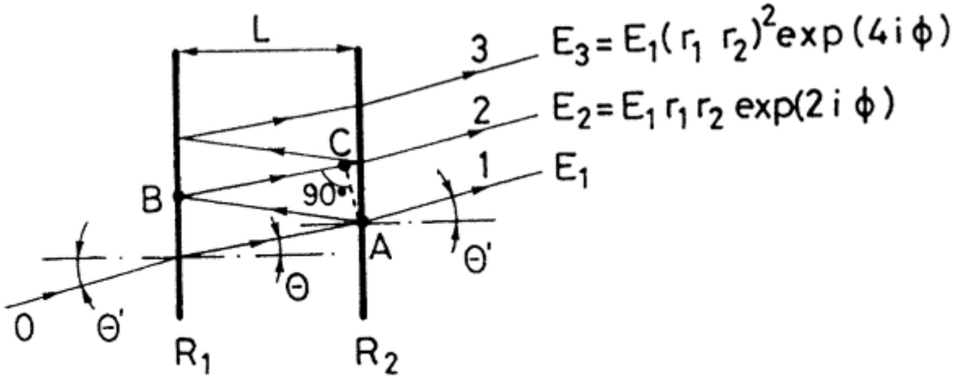


Etalon



FP interferometer

Properties



$$E_1 = E_0 t_1 t_2 \exp(j\phi')$$

$$\phi = \frac{2\pi\nu}{c} L'$$

$$L' = n_r \frac{L}{\cos\theta}$$

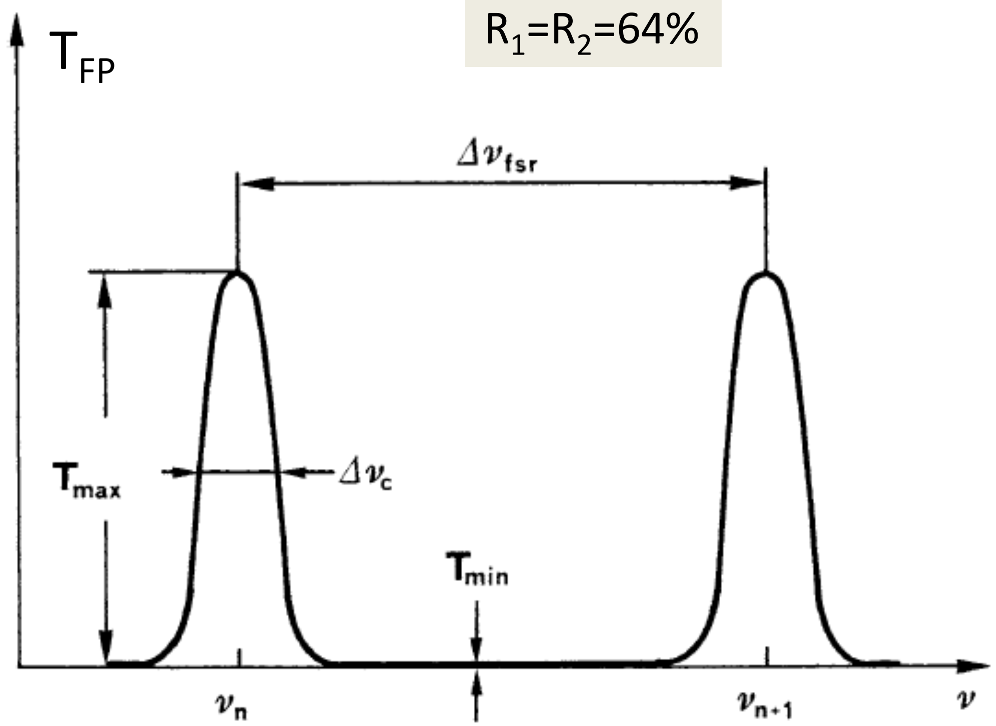
$$E_t = \sum_{1}^{\infty}{}_l E_l = [E_0 t_1 t_2 \exp(j\phi')] \sum_{0}^{\infty}{}_m (r_1 r_2)^m \exp(2mj\phi)$$

$$E_t = E_0\, e^{j\phi'} \frac{t_1 t_2}{1 - (r_1 r_2)\exp(2j\phi)}$$

$$T_{FP} = |E_t|^2 / |E_0|^2$$

$$T_{FP} = \frac{(1 - R_1)(1 - R_2)}{[1 - (R_1 R_2)^{1/2}]^2 + 4(R_1 R_2)^{1/2} \sin^2\phi}$$

Properties



$$\nu_n = mc/2L'$$

$$\Delta\nu_{fsr} = c/2L'$$

$$T_{\max} = \frac{(1-R_1)(1-R_2)}{[1-(R_1R_2)^{1/2}]^2}$$

$$T_{\min} = \frac{(1-R_1)(1-R_2)}{[1+(R_1R_2)^{1/2}]^2}$$

$$\Delta\nu_c = \frac{c}{2L'}\frac{1-(R_1R_2)^{1/2}}{\pi(R_1R_2)^{1/4}}$$

$$F = \Delta\nu_{fsr}/\Delta\nu_c$$ Finesse

$$T_{FP} = \frac{(1-R_1)(1-R_2)}{[1-(R_1R_2)^{1/2}]^2 + 4(R_1R_2)^{1/2}\sin^2\phi}$$

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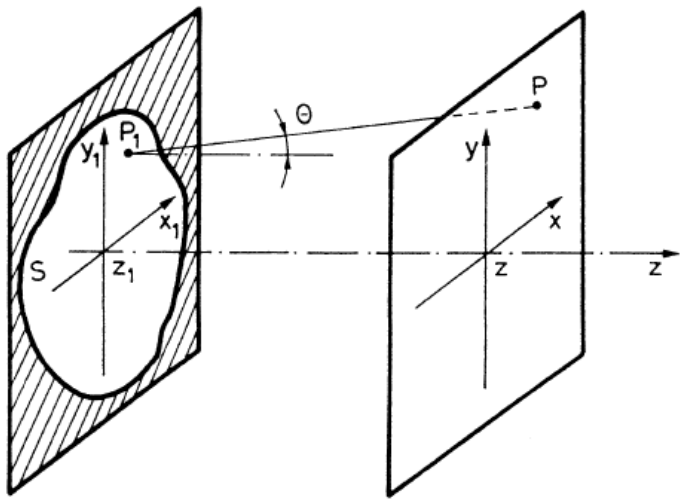
Wave equation, diffraction optics

In a homogeneous medium

$$\left(\nabla^2 + k^2\right) \tilde{E}(x, y, z) = 0$$ $\tilde{E}$ is scalar

Problem:



Solution:

$$\tilde{E}(x, y, z) = \frac{j}{\lambda} \iint_S \tilde{E}(x_1, y_1, z_1) \frac{\exp(-jkr)}{r} \cos\theta dx_1 dy_1$$

Fresnel-Kirchhoff integral (Huygen's principle)

Paraxial beam

If propagation angle is small

$$\tilde{E}(x, y, z) = u(x, y, z) \exp -(jkz)$$

u is slowly varying along z (*i.e.* $d^2u/dz^2=0$), and satisfies

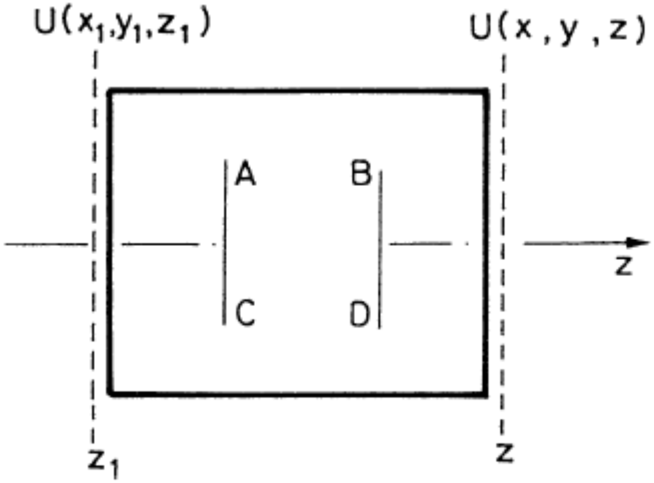
$$\nabla_\perp^2 u - 2jk\frac{\partial u}{\partial z} = 0$$

Paraxial wave equation

In integral form

$$u(x, y, z) = \frac{j}{\lambda L} \iint_S u(x_1, y_1, z_1) \exp\left[-jk\frac{(x-x_1)^2 + (y-y_1)^2}{2L}\right] dx_1 dy_1$$

Paraxial beam: ABCD system



$$u(x,y,z)=\frac{j}{B\lambda}\iint_S u(x_1,y_1,z_1)\exp\left[-jk\frac{A(x_1^2+y_1^2)+D(x^2+y^2)-2x_1x-2y_1y}{2B}\right]dx_1dy_1$$

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Gaussian beam - derivation

Fresnel-Kirchoff integral form of wave propagation (general ABCD system)

$$u(x,y,z) = \frac{j}{B\lambda} \iint_S u(x_1,y_1,z_1) \exp\left[-jk\frac{A(x_1^2+y_1^2)+D(x^2+y^2)-2x_1x-2y_1y}{2B}\right] dx_1 dy_1$$

One eigen-solution: $u(x,y,z) \propto \exp\left(-jk\frac{x^2+y^2}{2q}\right)$

q(z): complex beam parameter

Proof: If $u(x_1,y_1,z_1) \propto \exp\left(-jk\frac{x_1^2+y_1^2}{2q_1}\right)$

One has $u(x,y,z) = \frac{1}{A+B/q_1}\exp\left(-jk\frac{x^2+y^2}{2q}\right)$ (★)

$$q = \frac{A\,q_1+B}{C\,q_1+D}$$

ABCD law of Gaussian beam propagation

Electric field distribution

From u, one has E field: $$\tilde{E} \propto \exp\left[-jk\left(z+\frac{x^2+y^2}{2q}\right)\right]$$

Complex beam parameter: $$\frac{1}{q} = \frac{1}{R} - j\frac{\lambda}{\pi w^2}$$

Radius of curvature (R) — Beam spot size (w)

$$\tilde{E} \propto \exp\left(-\frac{x^2+y^2}{w^2}\right)\exp\left[-jk\left(z+\frac{x^2+y^2}{2R}\right)\right]$$

Transverse pattern — Phase information (paraboloid equiphase surface)

Evolutions of w and R are governed by ABCD law (slide before)

Free-space case

A=D=1, C=0, B=z

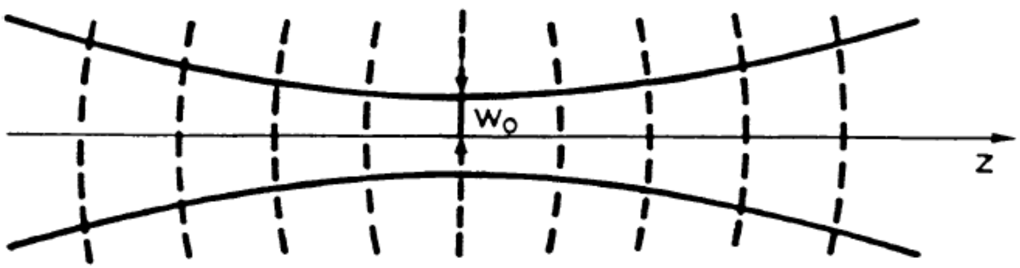
Assume R=∝ and $w=w_0$ at z=0 (q_1 is known).

One can calculate q values (therefore R and w) at all z coordinates.

$$w^2(z) = w_0^2 \left[1 + (z/z_R)^2\right]$$

$$R(z) = z \left[1 + (z_R/z)^2\right]$$

with $z_R = \pi w_0^2 / \lambda$

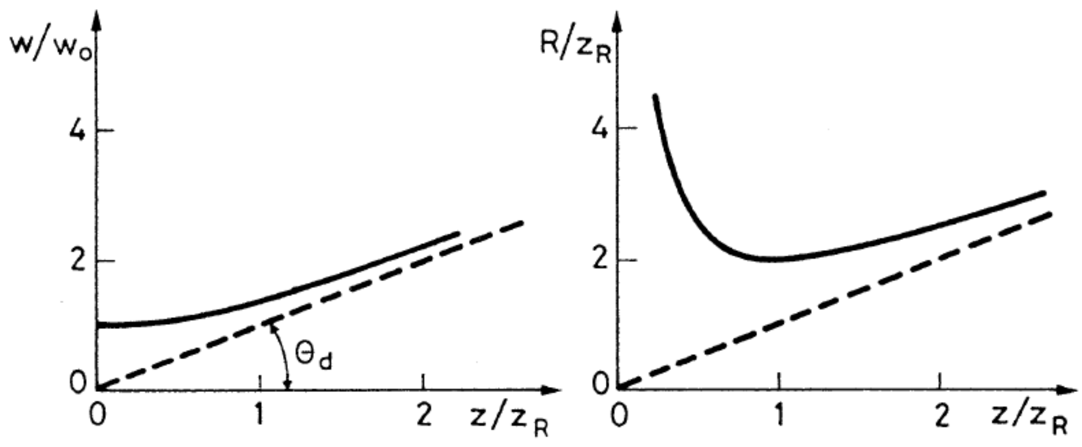


z=0

w and R

$$\boxed{\theta_d = \lambda / \pi\, w_0}$$

Diffraction angle

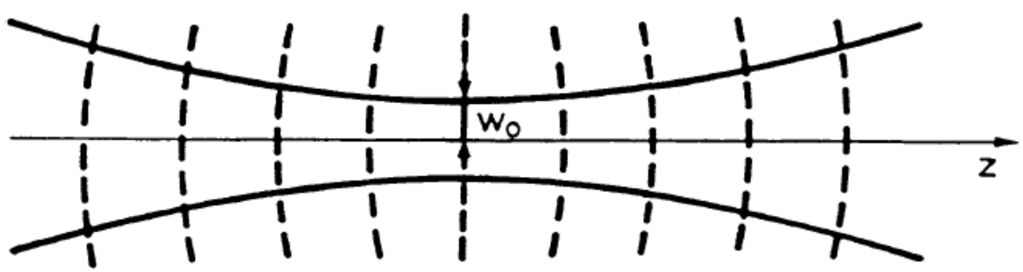


(a) (b)

$$w^2(z) = w_0^2\left[1 + (z/z_R)^2\right]$$

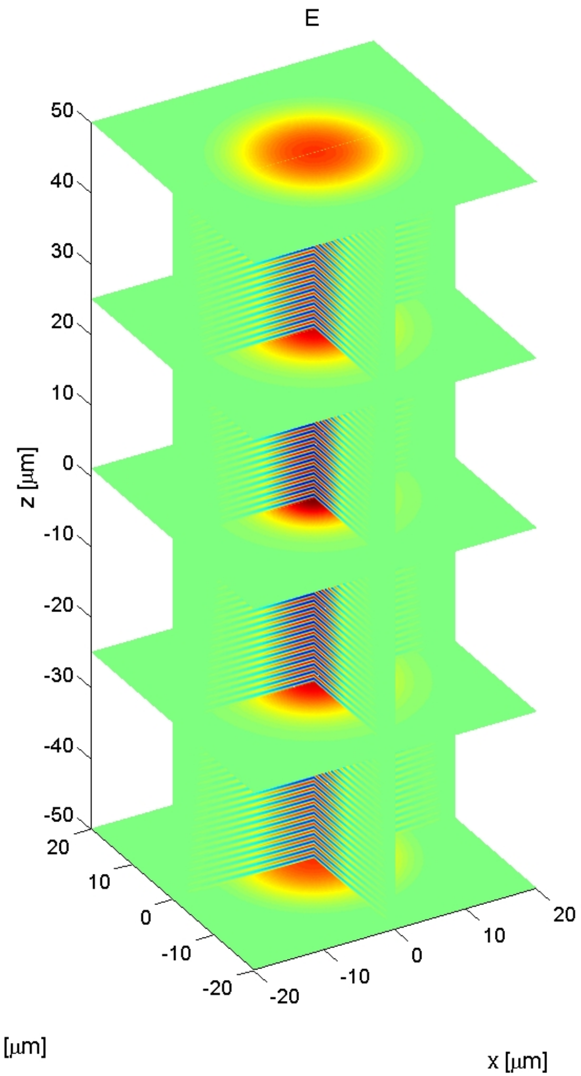
$$R(z) = z\left[1 + (z_R/z)^2\right]$$

with $\boxed{z_R = \pi\, w_0^2 / \lambda}$

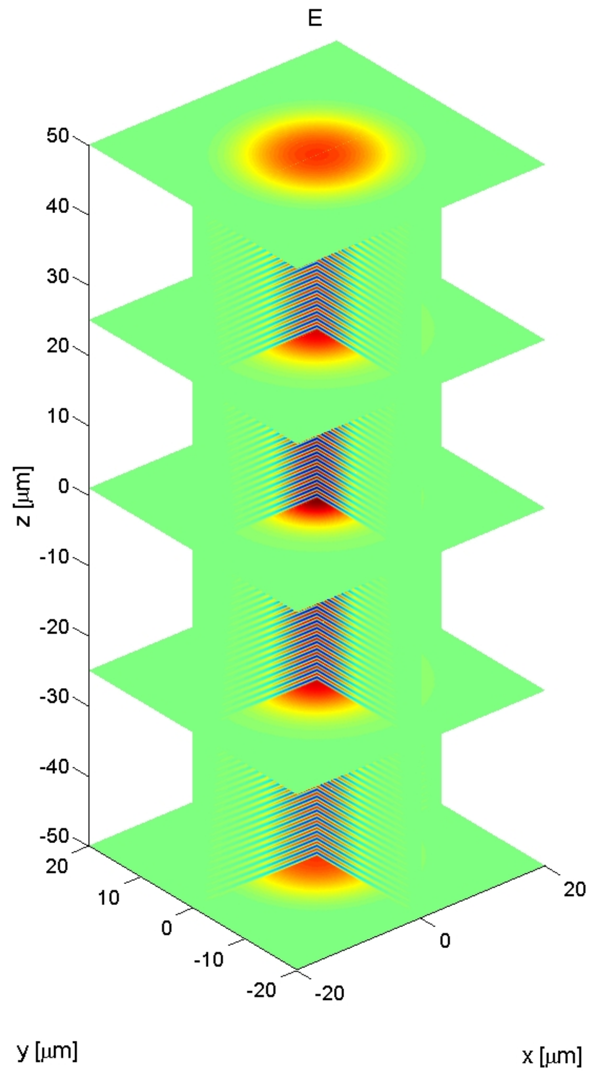


z=0

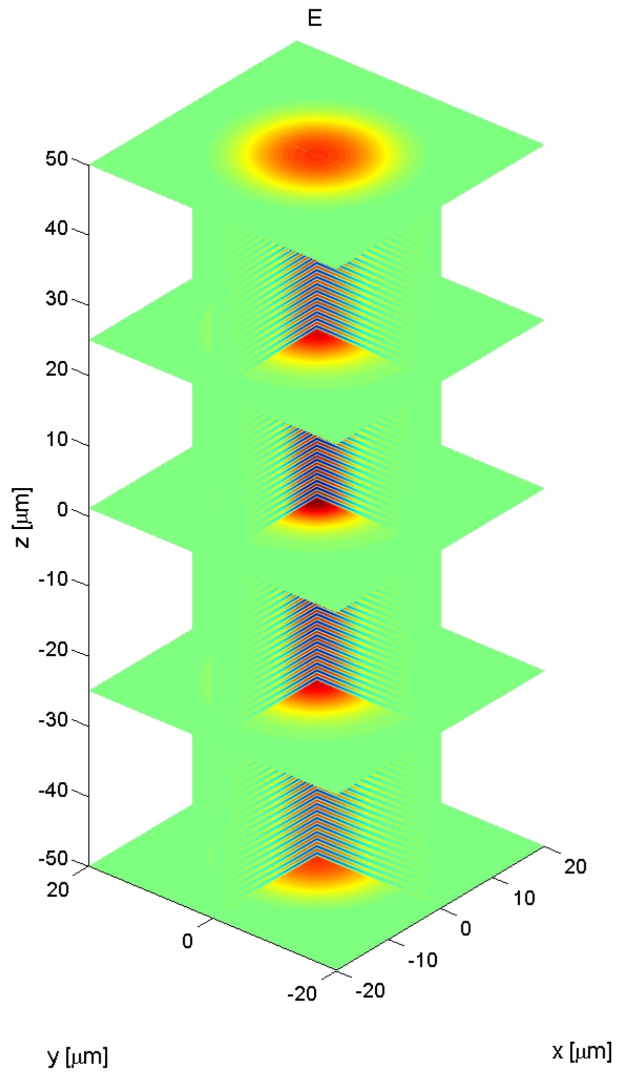
Lowest-order mode: TEM_{00}



Lowest-order mode: TEM_{00}



Lowest-order mode: TEM_{00}



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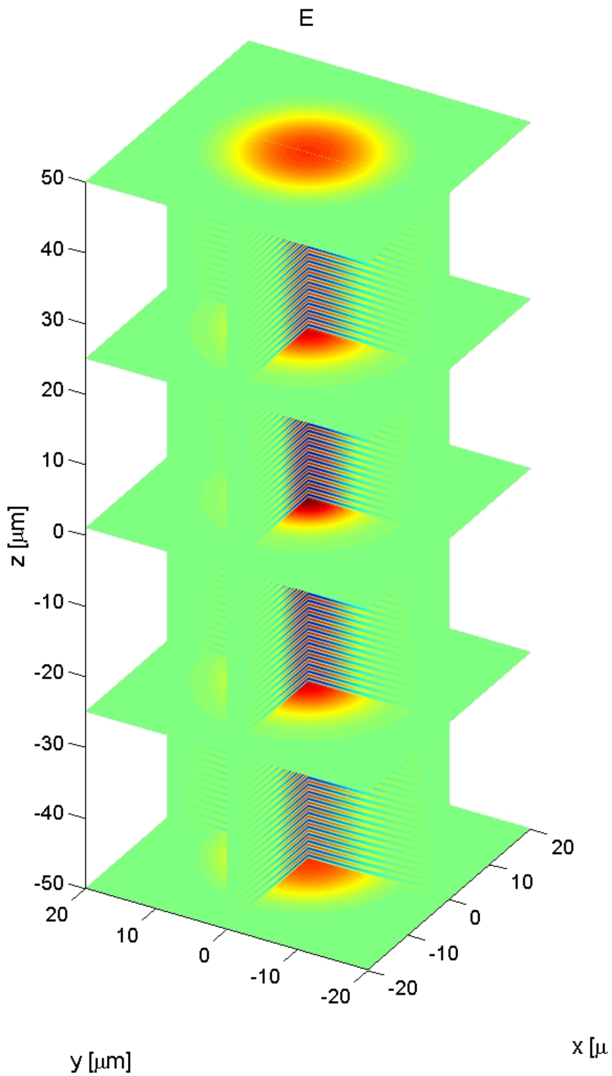

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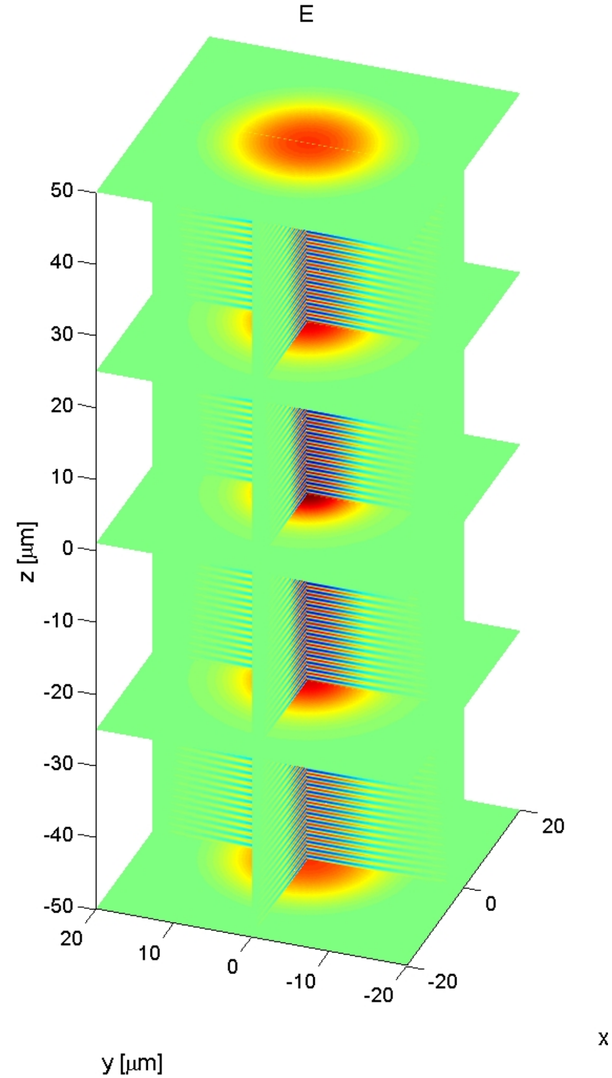

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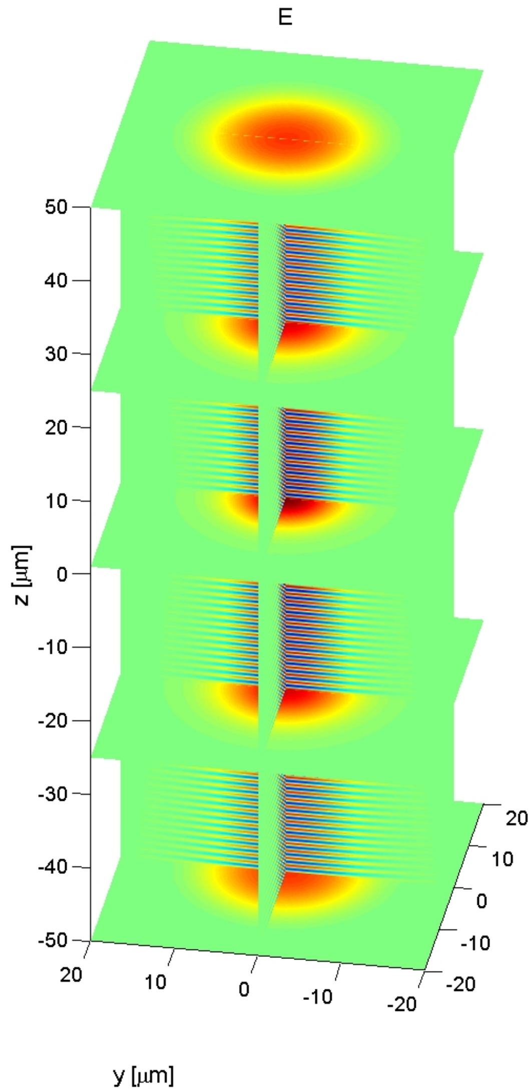

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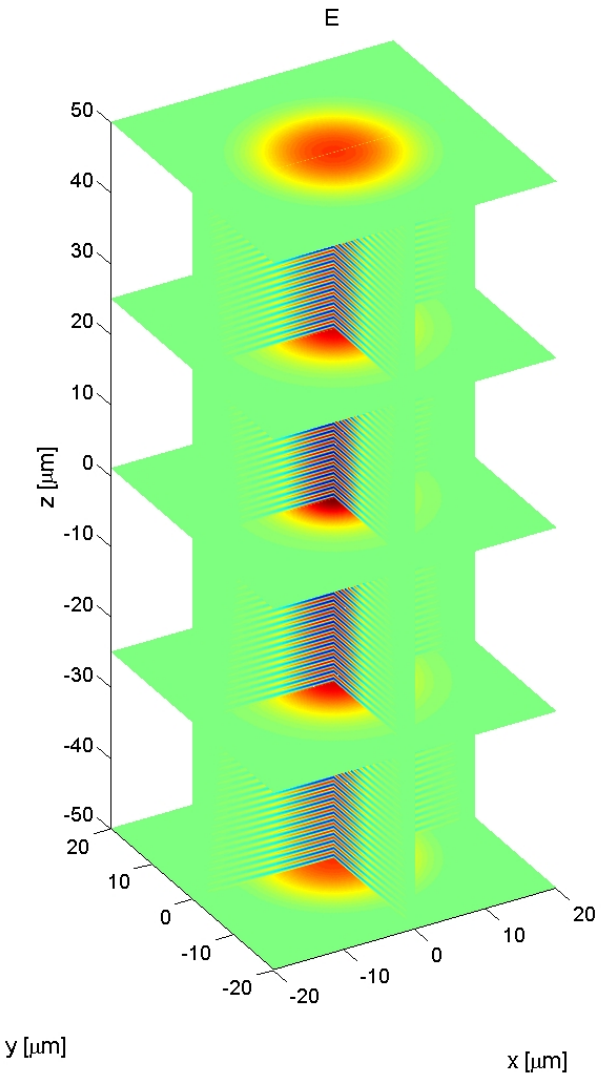
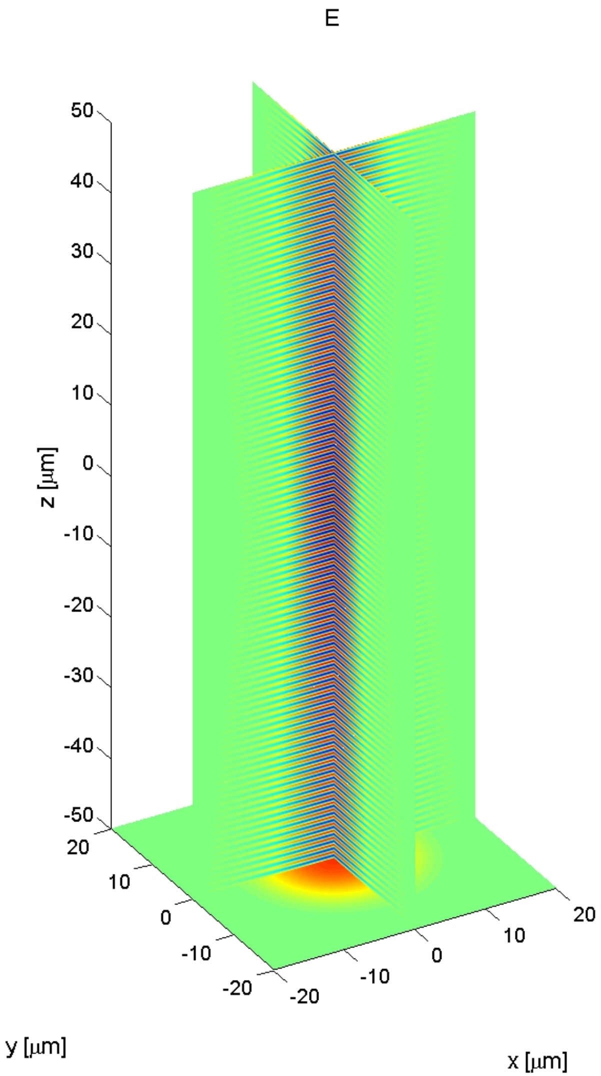
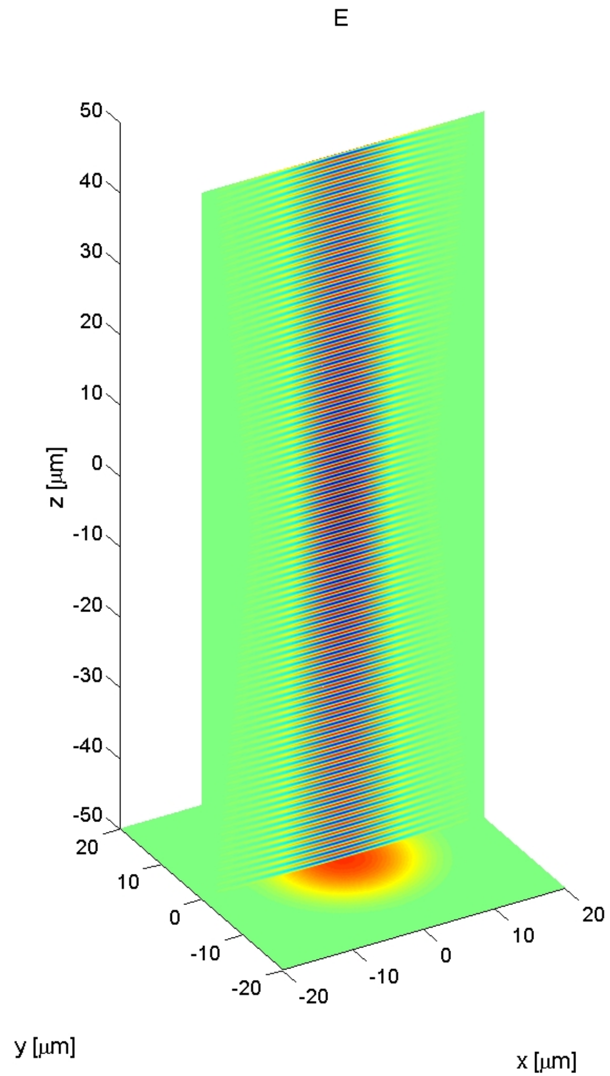


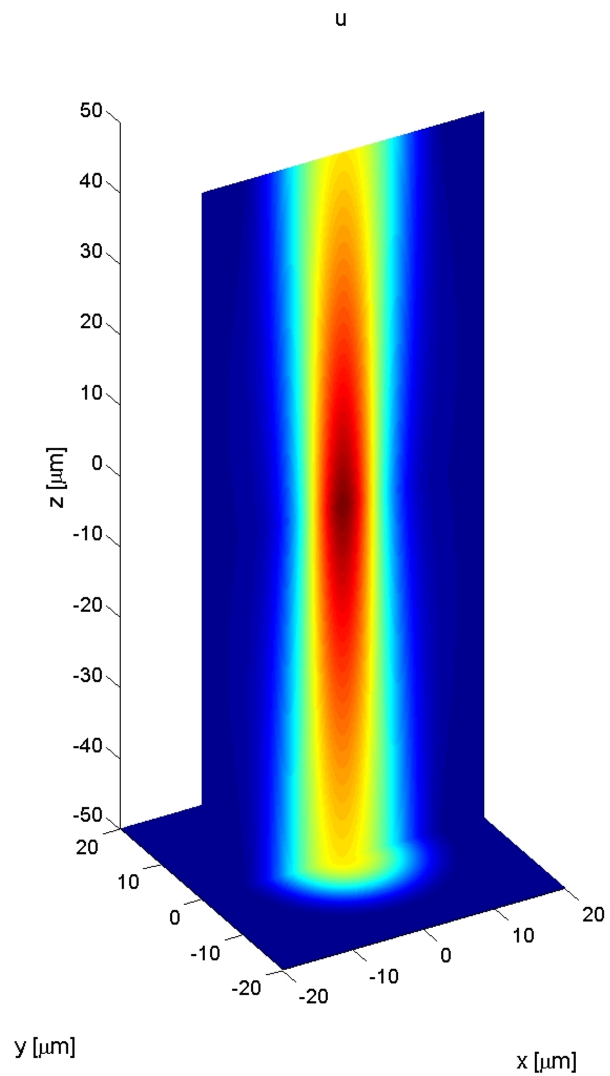
Lowest-order mode: TEM_{00}



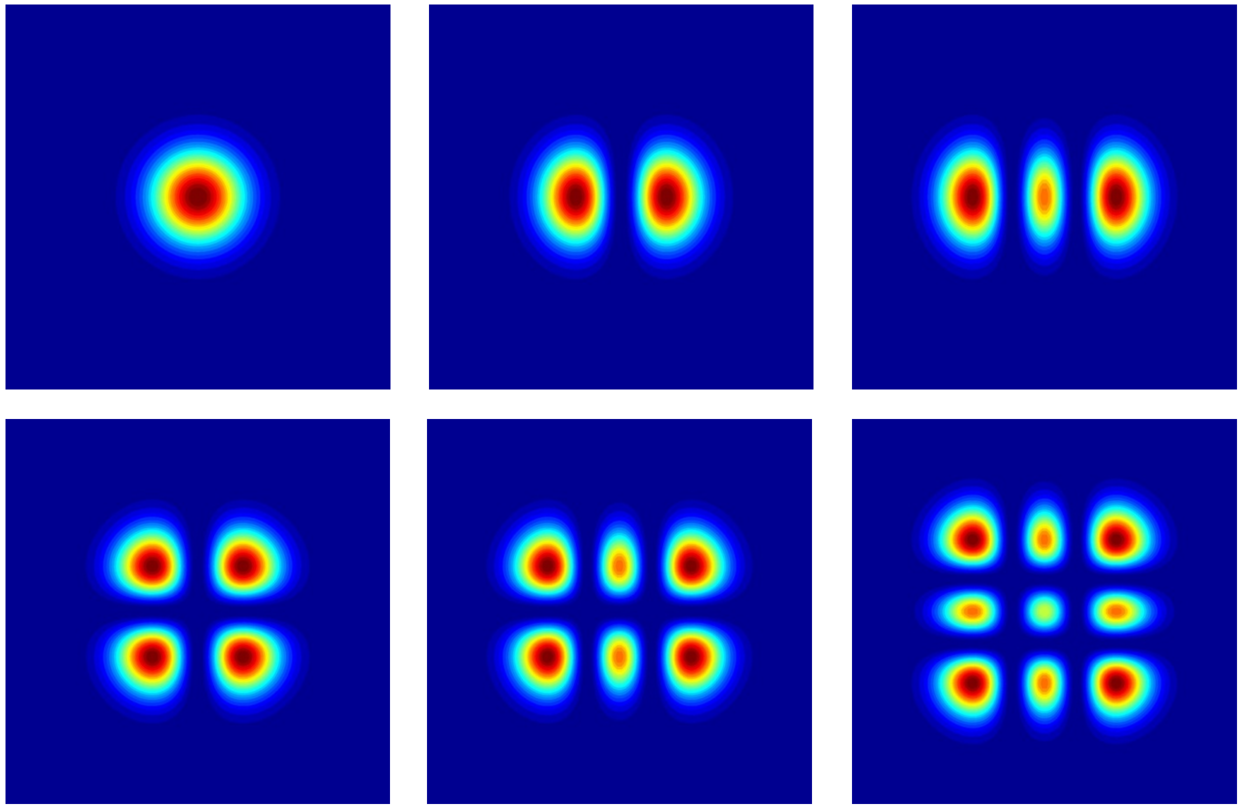
Lowest-order mode: TEM_{00}



Lowest-order mode: TEM_{00}



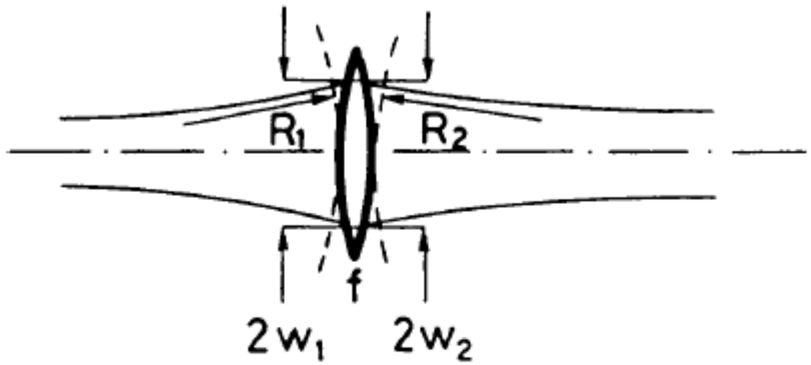
High-order modes



$$\begin{aligned} u_{l,m}(x,y,z) &= (w/w_0)H_l\left[2^{1/2}x/w\right]H_m\left[2^{1/2}y/w\right]\exp\left[-\left(x^2+y^2\right)/w^2\right] \\ &\quad \times \exp\left\{-j\left[k\left(x^2+y^2\right)/2R\right]+j(1+l+m)\phi\right\} \end{aligned}$$

ABCD law for Gaussian beams

A test for a thin lens



A=1, B=0
C=-1/f, D=1

$$\frac{1}{q_2} = \frac{C + (D/q_1)}{A + (B/q_1)}$$

$$\frac{1}{q_2} = -\frac{1}{f} + \frac{1}{q_1}$$

$$w_2 = w_1$$

$$\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f}$$

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