1. Basic concepts

$$N_c = -\frac{[\ln R_1 R_2 + 2\ln(1 - L_i)]}{2\sigma l} \qquad (1.2.3)$$

$$\gamma_1 = -\ln R_1 = -\ln(1 - T_1) \qquad (1.2.4a)$$

$$\gamma_2 = -\ln R_2 = -\ln(1 - T_2) \qquad (1.2.4b)$$

$$\gamma_i = -\ln(1 - L_i) \qquad (1.2.4c)$$

$$N_c = \frac{\gamma}{\sigma l} \qquad (1.2.5)$$

$$\gamma = \gamma_i + \frac{(\gamma_1 + \gamma_2)}{2} \qquad (1.2.6)$$

2. Electromagnetic-wave-atom and ion interaction

$$\rho_\nu = \frac{8\pi\nu^2}{c_n^3}\frac{h\nu}{\exp(h\nu/kT) - 1} \qquad (2.2.22)$$

$$\tau_{sp} = \frac{3h\varepsilon_0 c^3}{16\pi^3\nu_0^3 n|\mu|^2} \qquad (2.3.15)$$

$$A = \frac{16\pi^3\nu_0^3 n|\mu|^2}{3h\varepsilon_0 c^3} \qquad (2.3.19)$$

$$g(\nu - \nu_0) = \frac{2}{\pi\Delta\nu_0}\frac{1}{1 + [2(\nu - \nu_0)/\Delta\nu_0]^2} \qquad (2.4.8)$$

$$g(0) = \frac{2}{\pi\Delta\nu_0} = \frac{0.637}{\Delta\nu_0} \qquad (2.4.9b)$$

$$W_{12}^{sa} = \frac{2\pi^2}{3n\varepsilon_0 ch^2}|\mu_{21}|^2 I g(\nu - \nu_0) \qquad (2.4.11)$$

$$I = \frac{c\rho}{n} \qquad (2.4.10)$$

$$\sigma_h = \frac{2\pi^2}{3n\varepsilon_0 ch}|\mu|^2 \nu g(\nu - \nu_0) \qquad (2.4.18)$$

$$f_{2j} = \frac{g_{2j}\exp[-(E_{2j}/kT)]}{\sum_{1}^{g_2}{}_m g_{2m}\exp[-(E_{2m}/kT)]} \qquad (2.7.16a)$$

$$g^*(\nu_0' - \nu_0) = \frac{2}{\Delta\nu_0^*}\left(\frac{\ln 2}{\pi}\right)^{1/2}\exp-\left[\frac{4(\nu_0' - \nu_0)^2}{\Delta\nu_0^{*2}}\ln 2\right] \qquad (2.4.24)$$

$$f_{1i} = \frac{g_{1i}\exp[-(E_{1i}/kT)]}{\sum_{1}^{g_1}{}_l g_{1l}\exp[-(E_{1l}/kT)]} \qquad (2.7.16b)$$

$$\sigma_{in} = \frac{2\pi^2}{3n\varepsilon_0 ch}|\mu|^2 \nu g_t(\nu - \nu_0) \qquad (2.4.25)$$

$$g_t = \int_{-\infty}^{+\infty} g^*(x) g[(\nu - \nu_0) - x]dx \qquad (2.4.26)$$

$$\sigma_{ml}^e = \frac{W_{ml}^e}{F} = f_{2m}\sigma_{ml} \qquad (2.7.21a)$$

$$\sigma_{lm}^a = \frac{W_{lm}^a}{F} = f_{1l}\sigma_{lm} \qquad (2.7.21b)$$

$$g = \sigma(N_2 - N_1) \qquad (2.4.35)$$

$$\Delta\nu_0 = \frac{1}{2\pi\tau_{sp}} \qquad (2.5.13)$$

$$\frac{A}{B} = \frac{8\pi h\nu_0^3 n^3}{c^3} \qquad (2.4.42)$$

$$g^*(\nu_0' - \nu_0) = \frac{1}{\nu_0}\left(\frac{Mc^2}{2\pi kT}\right)^{1/2}\exp-\left[\frac{Mc^2}{2kT}\frac{(\nu_0' - \nu_0)^2}{\nu_0^2}\right] \qquad (2.5.17)$$

$$\Delta\nu_0^* = 2\nu_0\left(\frac{2kT\ln 2}{Mc^2}\right)^{1/2} \qquad (2.5.18)$$

$$\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}} \qquad (2.6.18)$$

$$\phi = \frac{\int (P(t)/h\nu_0)dt}{N_2(0)V} = \frac{\tau}{\tau_r} \qquad (2.6.22)$$

$$I_s = \frac{h\nu}{2\sigma\tau} \quad (2.8.11)$$

$$\alpha = \frac{\alpha_0}{1 + (I/I_s)} \quad (2.8.12)$$

$$\Gamma_s = \frac{h\nu}{2\sigma} \quad (2.8.17)$$

$$g = \frac{g_0}{1 + (I/I_s)} \quad (2.8.25)$$

$$g = g_0 \exp\{-[\Gamma(t)/\Gamma_s]\} \quad (2.8.30)$$

$$I_s = \frac{h\nu}{\sigma\tau} \quad (2.8.24)$$

3. Interactions with molecules and solids

$$E_r = BJ(J+1) \qquad B = \frac{\hbar}{2I} \quad (3.1.7)$$

$$(2J+1)_m = \left(\frac{2kT}{B}\right)^{1/2} \quad (3.1.10)$$

$$f_c(E_c) = \frac{1}{1 + \exp[(E_c - E_{F_c})/kT]} \quad (3.2.10a)$$

$$f_v(E_v) = \frac{1}{1 + \exp[(E_{F_v} - E_v)/kT]} \quad (3.2.10b)$$

$$g = \alpha_0[f_c(E_2') - f_v(E_1')] \quad (3.2.37)$$

$$E_2' - E_1' < E_{F_c}' - E_{F_v}' \quad (3.2.38)$$

$$E_g \le h\nu \le E_{F_c}' - E_{F_v}' \quad (3.2.39)$$

$$g = \sigma(N - N_{tr}) \quad (3.2.\text{}$$

4. Ray and wave propagation

$$\begin{vmatrix} r_2 \\ r_2' \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \begin{vmatrix} r_1 \\ r_1' \end{vmatrix} \quad (4.2.2)$$

$$\begin{bmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{bmatrix}$$

$$E_t = E_0 e^{j\phi'} \frac{t_1 t_2}{1 - (r_1 r_2)\exp(2j\phi)} \quad (4.5.4)$$

$$T = \frac{t_1^2 t_2^2}{1 - 2r_1 r_2 \cos(2\phi) + r_1^2 r_2^2} \quad (4.5.5)$$

$$F = \frac{\pi (R_1 R_2)^{1/4}}{1 - (R_1 R_2)^{1/2}} \quad (4.5.14a)$$

$$\Delta\nu_m = \frac{\Delta\nu_{fsr}}{F} \quad (4.5.17)$$

$$u(x, y, z) = \frac{1}{A + (B/q_1)} \exp\left[-jk\left(\frac{x^2 + y^2}{2q}\right)\right] \quad (4.7.3)$$

$$q = \frac{Aq_1 + B}{Cq_1 + D} \quad (4.7.4)$$

$$\frac{1}{q} = \frac{1}{R} - j\left(\frac{\lambda}{\pi w^2}\right) \quad (4.7.8)$$

$$w^2(z) = w_0^2\left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right] \quad (4.7.13a)$$

$$z_R = \frac{\pi w_0^2}{\lambda} \quad (4.7.16)$$

$$R(z) = z\left[1 + \left(\frac{\pi w_0^2}{\lambda z}\right)^2\right] \quad (4.7.13b)$$

$$\theta_d = \frac{\lambda}{\pi w_0} \quad (4.7.19)$$

5. Passive optical resonators

$$\tau_c = \frac{L}{c\gamma} \quad (5.3.9)$$

$$\Delta\nu = \frac{c}{2L} \quad (5.1.3)$$

$$\tau_c = -\frac{2L}{c\ln[R_1R_2(1-T_i)^2]} \quad (5.3.7)$$

$$\Delta\nu_c = \frac{1}{2\pi\tau_c} \quad (5.3.10)$$

$$g_1 = 1 - \left(\frac{L}{R_1}\right) \quad (5.4.10a)$$

$$g_2 = 1 - \left(\frac{L}{R_2}\right) \quad (5.4.10b)$$

$$Q = \frac{\nu}{\Delta\nu_c} \quad (5.3.13)$$

$$0 < g_1g_2 < 1 \quad (5.4.11)$$

$$N = \frac{a^2}{L\lambda} \quad (5.5.26)$$

6. Pumping processes

$$\eta_p = \eta_r\eta_t\eta_a\eta_{pq} \quad (6.2.5)$$

$$\eta_a = [1 - \exp -(\alpha l)] \quad (6.3.11)$$

$$\langle R_p\rangle = \eta_p\left(\frac{P_p}{h\nu_p}\right)\frac{2}{\pi(w_0^2+w_p^2)l} \quad (6.3.12)$$

$$\eta_p = \eta_r\eta_t\eta_a.$$

$$\phi_0 = V_a\tau_c\lfloor R_p - R_{cp}\rfloor \quad (7.3.4b)$$

$$P_{th} = \left(\frac{\gamma}{\eta_p}\right)\left(\frac{h\nu_p}{\tau}\right)\left[\frac{\pi(w_0^2+w_p^2)}{2\sigma_e}\right] \quad (6.3.20)$$

$$P_{th} = \left(\frac{\gamma}{\eta_p}\right)\left(\frac{h\nu_p}{\tau}\right)\left(\frac{\pi a^2}{\sigma_e\{1-\exp[-(2a^2/w_0^2)]\}}\right) \quad (6.3.21)$$

$$P_{th} = \left(\frac{\sigma_a N_t l + \gamma}{\eta_p}\right)\left(\frac{h\nu_p}{\tau}\right)\left[\frac{\pi(w_0^2+w_p^2)}{2(\sigma_e+\sigma_a)}\right] \quad (6.3.25)$$

7. CW laser behavior

$$L_e = L + (n-1)l \quad (7.2.11)$$

$$\tau_c = \frac{L_e}{\gamma c} \quad (7.2.14)$$

$$V = \left(\frac{L_e}{l}\right)V_a \quad (7.2.15)$$

$$P_{out} = \left(\frac{\gamma_2 c}{2L_e}\right)(h\nu)\phi \quad (7.2.18)$$

$$R_{cp} = \frac{N_c}{\tau} = \left(\frac{\gamma}{\sigma l\tau}\right) \quad (7.3.3)$$

$$P_{out} = (A_bI_s)\left(\frac{\gamma_2}{2}\right)\left(\frac{P_p}{P_{th}} - 1\right) \quad (7.3.9)$$

$$\eta_s = \frac{A_bh\nu}{\sigma\tau}\frac{\gamma_2}{2}\frac{1}{P_{th}} \quad (7.3.11)$$

$$P_{th} = \frac{\gamma(1+B)}{\eta_p}\left(\frac{h\nu_p}{\tau}\right)\left(\frac{A}{\sigma_e+\sigma_a}\right) \quad (7.4.4)$$

$$P_{out} = \left[\frac{A_b(1+B)}{\sigma_e+\sigma_a}\right]\left(\frac{h\nu}{\tau}\right)\left(\frac{\gamma_2}{2}\right)\left(\frac{P_p}{P_{th}} - 1\right) \quad (7.4.8)$$

$$P_{op} = \left[A_bI_s\left(\gamma_i + \frac{\gamma_1}{2}\right)\right][(x_m)^{1/2} - 1]^2 \quad (7.5.6)$$

8. Transient laser behavior

$$\omega' = \left[\omega^2 - \left(\frac{1}{t_0}\right)^2\right]^{1/2} \quad (8.2.11)$$

$$t_0 = \frac{2\tau}{x} \quad (8.2.14)$$

$$\omega = \left[\frac{(x-1)}{\tau_c\tau}\right]^{1/2} \qquad x = \frac{R_p}{R_{cp}} \quad (8.2.15)$$

$$\frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau} \quad (7.2.16a)$$

$$P_p = \frac{\gamma_2}{2}\left(\frac{A_b}{\sigma}\right)\left(\frac{h\nu}{\tau_c}\right)\left[\frac{N_i}{N_p} - \ln\left(\frac{N_i}{N_p}\right) - 1\right] \quad (8.4.15)$$

$$E = \left(\frac{\gamma_2}{2}\frac{N_i}{N_p}\right)\eta_E\left(\frac{A_b}{\sigma}\right)h\nu \quad (8.4.20)$$

$$\tau_d = \frac{\tau_c}{x-1}\ln(\phi_p/10) \quad (8.4.23)$$

$$\Delta\tau_p = \frac{2\ln 2}{\pi\Delta\nu_L} = \frac{0.441}{\Delta\nu_L} \quad (8.6.12)$$

$$\Delta\tau_p \cong \frac{0.45}{(\nu_m\Delta\nu_0)^{1/2}} \quad (8.6.19)$$

9. Solid state lasers

$$J_{th} = \left(\frac{ed}{\eta_i\tau_r}\right)N_{th} \quad (9.4.3)$$

$$N_{th} = \left(\frac{\gamma}{\sigma L\Gamma}\right) + N_{tr} \quad (9.4.9)$$

11. Properties of laser beams

$$r \cong 0.16\left(\frac{\lambda z}{d}\right) \quad (11.3.43)$$

$$\theta_d = \frac{1.22\lambda}{D} \quad (11.4.6)$$

$$\theta_{dx} = \frac{W_x(z)}{(z - z_{0x})} = M_x^2\left(\frac{\lambda}{\pi W_{x0}}\right) \quad (11.4.20)$$

$$d_g \cong 2\delta x = \frac{2\lambda L}{D} \quad (11.5.2)$$

$$d_{ag} = \frac{2\lambda L}{D'} \quad (11.5.4)$$

12. Laser beam transformation, amplification, nonlinear optics

$$\Gamma_s = \left(\frac{h\nu}{\sigma}\right) \quad (12.3$$

$$\frac{d\Gamma}{dz} = g\Gamma_s[1 - \exp(-\Gamma/\Gamma_s)] - \alpha\Gamma \quad (12.3.11)$$

$$\Gamma(l) = \Gamma_s \ln\{1 + [\exp(\Gamma_{in}/\Gamma_s) - 1]G_0\} \quad (12.3.12)$$

$$\Gamma(l) = G_0\Gamma_{in} \quad (12.3.13)$$

$$\Gamma(l) = \Gamma_{in} + gl\Gamma_s \quad (12.3.14)$$

$$\Gamma_s = \frac{h\nu}{(\sigma_e + \sigma_a)} \quad (12.3.15)$$

$$P_i^{2\omega} = \sum_{j,k=1,2,3} \varepsilon_0 d_{ijk}^{2\omega} E_j^{\omega} E_k^{\omega} \quad (12.4.14)$$

$$\hbar\omega_3 = \hbar\omega_1 + \hbar\omega_2 \quad (12.4.19a)$$

$$\hbar\mathbf{k}_3 = \hbar\mathbf{k}_1 + \hbar\mathbf{k}_2 \quad (12.4.19b)$$

$$\frac{d|A_1|^2}{dz} = \frac{d|A_2|^2}{dz} = -\frac{d|A_3|^2}{dz} \quad (12.4.35)$$

$$g^2 = \alpha_1\alpha_2 = 4\left(\frac{\gamma_1\gamma_2}{l^2}\right) \quad (12.4.40)$$

$$g^2 = \frac{4\alpha_1}{l} = \frac{8\gamma_1}{l^2} \quad (12.4.44)$$

$$|E'_{2\omega}| = E'_{\omega}(0)\tanh(z/l_{SH}) \quad (12.4.58a)$$

$$|E'_{\omega}| = E'_{\omega}(0)\,\mathrm{sech}(z/l_{SH}) \quad (12.4.58b)$$

$$E_m = \frac{\Gamma_d\,(gl)^2}{g^2} \quad (12.3.19)$$