**Task 1.7:**
**The brightness of a lamp (95W/cm²sr) shall be compared to the brightness of an Ar-ion laser ($\lambda$ = 514.5nm) with an output power of 1W.**

Brightness (in W/m²sr) is a measure of how much power a source emits per unit space; the space given as part of a sphere with a source in the centre. Concluding from the units, one could also call it the intensity (W/m²) per unit solid angle (sr). The solid angle is measured in steradian (sr) and is actually dimensionless. However, from its physical meaning it has a unit of m²/m². It is the relation of the area on the sphere's surface as seen under a given angle from the centre and the distance to that surface squared (i.e. the radius squared). The solid angle of any full sphere is thus $4\pi$ (area: $4\pi r^2$; radius: $r$). A solid angle of one steradian then means an angle covering $1/4\pi$ of a spheres surface. For the unit sphere this would mean 1m².

In mathematical terms, brightness it is defined via the equation:

$$dP = B \cos\theta \, dS \, d\Omega$$

Integration of the right side gives the total power.

Integration over the right sides consists of integration of the source's area ($\cos\theta \, dS$), where the angle $\theta$ is the angle between the light source's surface normal and the sphere's surface normal. For a laser beam this is equal to the beam's divergence, which in turn is small enough to assume $\cos\theta$ to be equal to one. In other words, it is assumed that during integration over the region of interest on the sphere, the light emitting surface in the centre looks essentially the same. This is of course reasonable for the small region in case of a narrow laser beam. For larger integration areas, however, this has to be considered. The expression $\cos\theta \, dS$ can thus be called *effective area*, since it describes the area as seen from the sphere's surface when looking towards it. For this particular task then this is exactly the laser spot size, i.e. a circular area with the size

$$\pi(D/2)^2,$$

where $D$ is the spot's diameter. For a large divergence, the integration on the sphere would have to be performed over a wider area, thus changing the effective area for different positions on the sphere.

The differential $d\Omega$ then denotes the solid angle. The solid angle illuminated by the laser beam is defined by the laser's divergence. The radius of the circular area illuminated by the laser on the sphere's surface is $\sin\theta/r$, which for a unit sphere ($r = 1$) is $\sin\theta$ and for a small divergence reduces to $\theta$. The illuminated area is thus $\pi\theta^2$. To obtain the solid angle in steradian this has to be divided by the radius squared, which in the case of the unit sphere is just 1. The overall solid angle is thus

$$\pi\theta^2.$$

Since the laser beam is assumed to be diffraction limited, the divergence can be substituted by an expression from diffraction theory:

$$\theta = \beta\lambda/D$$

Here, $D$ denotes the spot's diameter at the origin and $\beta$ is a numerical factor from diffraction theory in the order of 1. Putting together all these expression, one obtains for the brightness:

$$B = \left(\frac{2}{\beta\pi\lambda}\right)^2 P$$

With the given values one finds the brightness of the Ar-ion laser to be 740GW/m²sr.

For the task itself, see page 15.
For a description of brightness, the mathematical definition, explanation of the approximations and application to a diffraction-limited source with small divergence, see page 12.