SK2411, IO2659 Examination tasks Solutions.
VT11, 2011 May 23

Task 1

(a) Homoatomic, not vibrational transitions in MIR.

(b) Far-infrared: Rotational. 3 vibrational, 3 rotational.

(c) $E_r = BJ(J+1)$, $B \propto 1/I$, from mechanics moment of inertia $I = \sum_i m_i r_i^2$, where r is the distance to rotation axis (going through center of mass, i.e. oxygen molecule). Hydrogen mass is two times lower than deuterium, the expected frequency is two times higher, and expected laser wavelength is 142.5 µm. In reality the observed wavelength is pretty close, 143 µm.

Task 2

(a) Nd:glass, He-Cd.

(b) In all of them.

(c) why the gain maxima are shifting: increasing joint density of states, why at the photon energy of 1.424 eV, the gain is always very close to zero even at high carrier densities: density of states is zero at the bandgap energy, why the gain spectra are broadening with increasing carrier density: Occupation of parabolic bands as quasi-Fermi levels are pushed further into their respective bands with increasing carrier density.

Task 3

(a) From 2.3.19, 2.4.18 and 2.4.24:

$\sigma\tau_{sp} = \dfrac{c^2}{8\pi^2\nu^2\Delta\nu\, n^2}$, From here the radiative relaxation time is 9.65 ns.

From 2.618, 2.6.22, the nonradiative decay time:

$\tau_{NR} = \dfrac{\phi\tau_{sp}}{1-\phi}$ and thus is 4.136 ns for the fluorescence yield $\phi = 0.3$.

(b) ...

Solutions for tasks 4,5,6 (IO2659)

Min Yan
Laboratory of Photonics and Microwave Engineering
School of Information and Communication Technology
Royal Institute of Technology, Sweden

May 18, 2011

- **Task 4**

  (a) Frequency spacing $\Delta\nu = \frac{c}{2L}$;
  Single mode condition: from $\Delta\nu \geq \Delta\nu_0/2$, therefore $L \leq \frac{c}{\Delta\nu_0}$.

  (b) According to the general stability condition $0 < g_1 g_2 < 1$, where $g_1 = 1 - \frac{L}{R_1}$ and $g_2 = 1 - \frac{L}{R_2}$, one has

  $$\left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) > 0 \tag{1}$$

  $$\left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) < 1, \tag{2}$$

  $$\tag{3}$$

  which lead to solutions

  $$0 < L < R_1 \tag{4}$$

  $$R_2 < L < R_1 + R_2, \tag{5}$$

  respectively

  (c) Yes.
  Transverse intensity profile: $I = I_{\max} \exp\left[-\frac{2(x^2+y^2)}{w^2}\right]$.

Beam width evolves in z direction as $w^2(z) = w_0^2(1 + \frac{z}{z_R})$, where w_0 is the beam waist, and z_R is the Rayleigh range.

(d) ...

- **Task 5**

(a) Four level rate equation:

$$\frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau} \tag{6}$$

$$\frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c} \tag{7}$$

(b) From Eq. 7, take $\frac{d\phi}{dt} = 0$, one has $N_c = \frac{1}{BV_a\tau_c} = \frac{\gamma}{\sigma l}$.
From Eq. 6, take $\frac{dN}{dt} = 0$, one has $R_{cp} = \frac{N_c}{\tau} = \frac{\gamma}{\sigma l \tau}$.

(c) Steady state: $N_0 = N_c$; $\phi_0 = V_a\tau_c(R_p - R_{cp})$.

(d) Pump efficiency: radiative, transfer, absorption, and power quantum efficiencies.
Additional: output coupling efficiency, laser quantum efficiency, transverse efficiency.

- **Task 6**

(a) ...

(b) For single-longitudinal-mode operation, one should have

$$\frac{\Delta\nu_c}{2} \leq \Delta\nu, \tag{8}$$

$$\Delta\nu_{fsr} \geq \frac{\Delta\nu_0}{2}. \tag{9}$$

Notice $\Delta\nu_c = \Delta\nu_{fsr}/F$. From the above two equations, one obtains

$$\frac{\Delta\nu_0}{2} \leq 2F\Delta\nu. \tag{10}$$

Remember that $\Delta\nu = \frac{c}{2L}$. One then has

$$L \leq \frac{2Fc}{\Delta\nu_0}. \tag{11}$$

(c) ...

(d) ...