SK2411, IO2659 Examination tasks

VT12, 2012 May 29

Task 1

(a) Higher concentration moves further quasiFermi levens into the bands. This broadens gain spectrum. Due to dependence of joint density of states as $\sim\sqrt{h\upsilon}$ the spectral peak is also moving to higher energies.

(b) Combined effect of carrier confinement and higher gain due to step-wise dependence of the density of states.

(c)

$w := 1 \times 10^{-4}$ [m]

$\tau := 10^{-13}$ [s]

$\varepsilon := 3.14 \times 10^{-6}$ [J]

(d) $c := 3 \cdot 10^{8}$

Photon energy in volume V, with density N:

$E := q \cdot c \cdot N \cdot V$

Photon pressure:

$\rho := \frac{E}{V}$

$\rho := \frac{I}{c}$

$P \cdot V := E$

$I := \frac{\varepsilon}{\pi \cdot w^2 \cdot \tau}$

$P := \frac{I}{c}$

$P = 3.332 \times 10^{6}$ [Pa]

$P := Patm \cdot 3($

Task 2

(a)

$$\tau_{sp} = \frac{3h\varepsilon_0 c^3}{16\pi^3 v_0^3 n|\mu|^2} \qquad (2.3.15)$$

$$P_{th} = \left(\frac{\gamma}{\eta_p}\right)\left(\frac{h\nu_p}{\tau}\right)\left[\frac{\pi(w_0^2 + w_p^2)}{2\sigma_e}\right] \tag{6.3.20}$$

$$P_{th} = \left(\frac{\gamma}{\eta_p}\right)\left(\frac{h\nu_p}{\tau}\right)\left(\frac{\pi a^2}{\sigma_e\{1 - \exp[-(2a^2/w_0^2)]\}}\right) \tag{6.3.21}$$

Due to the fact that pump wavelengths approximately will scale as emission wavelengths Pth$\sim\nu^4$. So for UV laser with 2-times shorter wavelength, one would expect 16-times higher threshold.

(b) Doppler broadening. Inhomogeneous.
(c) Fiber amplifier due to inhomogeneous broadening.

Task 3

(a) Molecular ro-vibrational spectrum contains 3 branches P, R, Q, with $\Delta J = \pm 1{,}0$
(b) $\lambda_Q \approx 4.26\ \mu m$. $\upsilon = \frac{c}{\lambda_Q} = 70\ THz$

Energy position of the narrow ro-vibrational narrow lines is equal:

$$\Delta E(v = 1, J \rightarrow v = 2, J + 1) = E_{v1} + B(J + 2)(J + 1) - E_{v2} - B(J + 1)J$$
$$= \Delta E_v + 2B(J + 1)$$
$$\Delta E(v = 1, J \rightarrow v = 2, J - 1) = \Delta E_v - 2BJ$$

Distance between two ro-vibrational lines then is independent on J and is equal to 2B, where $B = \hbar^2/2I$. Here I is the moment of inertia of the molecule.

Solutions for tasks 4,5,6 (IO2659, 2012)

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- **Task 4**
  (a) ...
  (b) Divergence angle: $\theta_d = \frac{\lambda}{\pi w_0}$; hence $w_0 = \frac{\lambda}{\pi \theta_d} = 0.388$ mm.
  Rayleigh range $z_R = \frac{\pi w_0^2}{\lambda} \simeq 0.445$ m.
  Curvature at Rayleigh range: $R = z\left[1 + \left(\frac{z_R}{z}\right)^2\right] = 2z_R \simeq 0.89$ m.
  (c) The lens is at Rayleigh range.
  Beam parameters before lens: $R_1 = 0.89$ m, $w_1 = \sqrt{2}w_0 \simeq= 0.549$ mm.
  ABCD matrix for lens $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$.
  The complex Gaussian beam parameters before lens q_1 and after lens q_2 are related by

$$\frac{1}{q_2} = \frac{C + D/q_1}{A + B/q_1}, \tag{1}$$

  or after considering the ABCD elements, $\frac{1}{q_2} = -\frac{1}{f} + \frac{1}{q_1}$. Since q is defined as in $\frac{1}{q} = \frac{1}{R} - j\frac{\lambda}{\pi w^2}$, effectively ones has the beam parameters after the lens $w_2 = w_1 = 0.549$ mm and $\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f}$ corresponding to $R_2 \simeq -5.3$ cm.

- **Task 5**

  (a) Gaussian mode has $M^2 = 1$.
  Reasons for deteriorated M^2 factor: unstable cavity may be used; high-order transverse modes might be present due to heavier pumping.

  (b) Logrithmic cavity loss: $\gamma = \ln(R_1 R_2) \simeq -0.0202$.
  Cavity life time: $\tau_c = \frac{L}{c\gamma} = \simeq 82.5$ ns.
  FWHM of resonance in frequency: $\Delta\nu_c = \frac{1}{2\pi\tau_c} \simeq 1.93$ MHz.
  Q factor: $Q = \nu/\Delta\nu_c = 1.46 \times 10^8$.
  E field depends on time as: $E(t) = E_0 \exp(-\frac{t}{\tau_c} + j\omega t)$; time for the photon number (intensity, or E^2) to decreases to $1/e$ is $\tau_c/2 =$ 41.25ns. One round trip takes $t_r = 2L/c = 3.33$ns. The number of round trips: $41.25/3.33 \simeq 12$.

  (c) Laser pumping; electrical pumping.

- **Task 6**

  (a) To make it Q-switched, e.g. by addding a saturable absorber.

  (b) From the rate eq.: $\frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau}$. Let $\phi = 0$ (shutter closed), one has $\frac{dN}{dt} = R_p - \frac{N}{\tau}$. It has the solution $N(t) = R_p\tau \left[1 - \exp\left(-\frac{t}{\tau}\right)\right]$. Max N happens when $t = \infty$, at $N_\infty = R_p\tau$.

  (c) It is the "natural oscillation frequency" of the laser. The oscillation frequency (angular) is $\omega = \sqrt{\frac{x-1}{\tau_c\tau}}$ with $x = R_p/R_{CP}$. Therefore it depends on pump rate, cavity life time, and the upper-level life time.

  (d) ...