

## Solutions

### Task 1

a Gas. Ionized Xe, etc. Absorption in other media.  $540 \text{ eV} - \lambda = \frac{ch}{eE}$

b Frequency doubled NdYAG at 946 nm, Ar ion, Cu- vapor, He-Cd.

c.  $E_m = \Gamma_d l_m^2$ .  $\Gamma(l) = \exp(gl)\Gamma_s$ .

$$l_m^2 = \frac{1}{g^2} \left( \ln \frac{\Gamma_s A}{h\nu} \right)^2$$

### Task 2

a.

b. (ii) Transitional metal lasers, Cr:LiSaF, Alexandrite, Ti:Sapphire, etc.

$$c. \frac{A_{GaAs}}{A_{GaN}} = \frac{\tau_{GaN}}{\tau_{GaAs}} \propto \frac{n_{GaAs} E_{gGaAs}^3}{n_{GaN} E_{gGaN}^3} = 0.065$$

$$\tau_{GaAs} = 1.38 \text{ ns} \quad \tau_{GaN} = 119 \text{ ps}$$

The measured ratio is in fact: 0.106, i. e.  $\tau_{GaN} = 146 \text{ ps}$  which is different from the estimate due to our approximation of equal dipole moment transition elements. Nevertheless the scaling gives rather good approximation.

### Task 3.

a. Ro-vibrational. Mid infrared. R and P branches.

b. He Cd, HeNe, CO<sub>2</sub>-N<sub>2</sub>, Er-Yb, Tm-Ho.

c. Saturated gain:

$$g(I) = \frac{g_0}{1 + I/I_s}$$

Wrong is the description is the implicit assumption that small signal gain  $g_0 = \sigma_e \Delta N$  does not change. The saturated gain remains constant, though.

# Solutions for tasks 4,5,6 (IO2659, 2013)

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## • Task 4

- (a) The optical length:  $L_e = L + (n - 1)l = 58.5\text{cm}$ ; mode spacing  $\Delta\nu = \frac{c}{2L_e} = 2.564 \times 10^8\text{Hz}$ ; the number of modes  $N = \frac{\Delta\nu_0}{\Delta\nu} = 468$ .
- (b) Reason: a FP etalon can have very narrow transmission windows and  $\Delta\nu_{\text{fsr}}$  can be comparable to the gain linewidth. A FP etalon can increase cavity length from  $L \leq \frac{c}{\Delta\nu_0}$  to  $L \leq 2F \frac{c}{\Delta\nu_0}$  for achieving single-longitudinal-mode operation.
- (c) Finesse:  $F = \frac{\Delta\nu_{\text{fsr}}}{\Delta\nu_c} = \frac{3 \times 10^9}{6 \times 10^7} = 50$ .  
Cavity length: From  $\Delta\nu_{\text{fsr}} = \frac{c}{2L}$ , one has  $L = 5\text{cm}$ .  
FWHM spectral width: from  $\Delta\nu_c = \Delta\nu_{\text{fsr}} \frac{1 - \sqrt{R_1 R_2}}{\pi \sqrt{R_1 R_2}}$ , one has  $R^2 - 2.004R + 1 = 0$ , and therefore  $R = 93.91\%$ .  
If mirror is lossy: Compared to the lossless case, the peak transmittance will then be  $\left(\frac{1-T}{1-R}\right)^2 = \left(\frac{1-R-A}{1-R}\right)^2 = 0.5$ . One then has the mirror absorption loss  $A = 1.78\%$ .

## • Task 5

- (a) The stability condition:  $0 < g_1 g_2 < 1$ , where  $g_1 = 1 - \frac{L}{R_1}$ ; from the left side  $g_1 g_2 > 0$ , one has  $0 < L < 1.5\text{m}$ ; from the right side  $g_1 g_2 < 1$ , one has  $L > 0.5\text{m}$ ; in summary  $0.5\text{m} < L < 1.5\text{m}$ .

- (b) Advantage: higher laser power while with reasonably good beam quality (less number of transverse modes).
- (c) Pump efficiency:  $\eta_p = \frac{P_m}{P_p} = 0.04$ ;  $P_m = 2P_{\text{lamp}} \times 0.04 = 80\text{W}$ .  
For uniform pumping,

$$P_m = R_p h\nu_{mp} V_a, \quad (1)$$

where  $\nu_{mp} = \frac{c}{0.94 \times 10^{-6}} \text{Hz}$ , and  $V_a = \pi r^2 l$  ( $r$  is beam radius).

In threshold condition,  $R_{cp} = \frac{P_m}{h\nu_{mp} V_a} = 2.964 \times 10^{26} \text{m}^{-3} \text{s}^{-1}$ .

- (d) ...

• **Task 6**

- (a) Necessary parameters: cavity single-pass logarithmic loss  $\gamma = -\frac{1}{2} \ln(R_1 R_2) + \gamma_i = 0.0609$ ; cavity photon life time  $\tau_c = \frac{L_e}{\gamma c} = 5.15 \text{ns}$ .  
Critical population inversion:  $N_c = \frac{\gamma}{\sigma l} = 4.35 \times 10^{23} \text{m}^{-3}$ .  
Critical pump rate:  $R_{cp} = \frac{\gamma}{\sigma l \tau} = 1.89 \times 10^{27} \text{m}^{-3} \text{s}^{-1}$ .  
Photon number inside cavity:  $\phi_0 = V_a \tau_c (R_p - R_{cp}) = \pi r_b^2 l \tau_c R_{cp} = 3.44 \times 10^9$ .  
Output power:  $P_{out} = \phi_0 \frac{\gamma c}{2L_e} h\nu = 31.7 \text{mW}$ .
- (b) Modulation frequency is inverse of the round trip time, or the longitudinal mode frequency separation  $\nu_m = \Delta\nu = \frac{c}{2L_e} = 256.4 \text{MHz}$ ; pulse separation is the round trip time  $\tau_p = \frac{2L_e}{c} = 3.9 \text{ns}$ ; pulse duration (homogeneously broadened case)  $\Delta\tau_p = \frac{0.45}{\sqrt{\Delta\nu \Delta\nu_0}} = 81.1 \text{ps}$ .
- (c) Mode locking is achieved by the lensing effect (through nonlinear Kerr effect) incurred to the beam in the Ti:sapphire plate, together with the aperture.

**Remarks:**

- Calculator is needed.
- Planck constant should be given:  $h = 6.626 \times 10^{-34} \text{m}^2 \text{kg} \text{s}^{-1}$ .