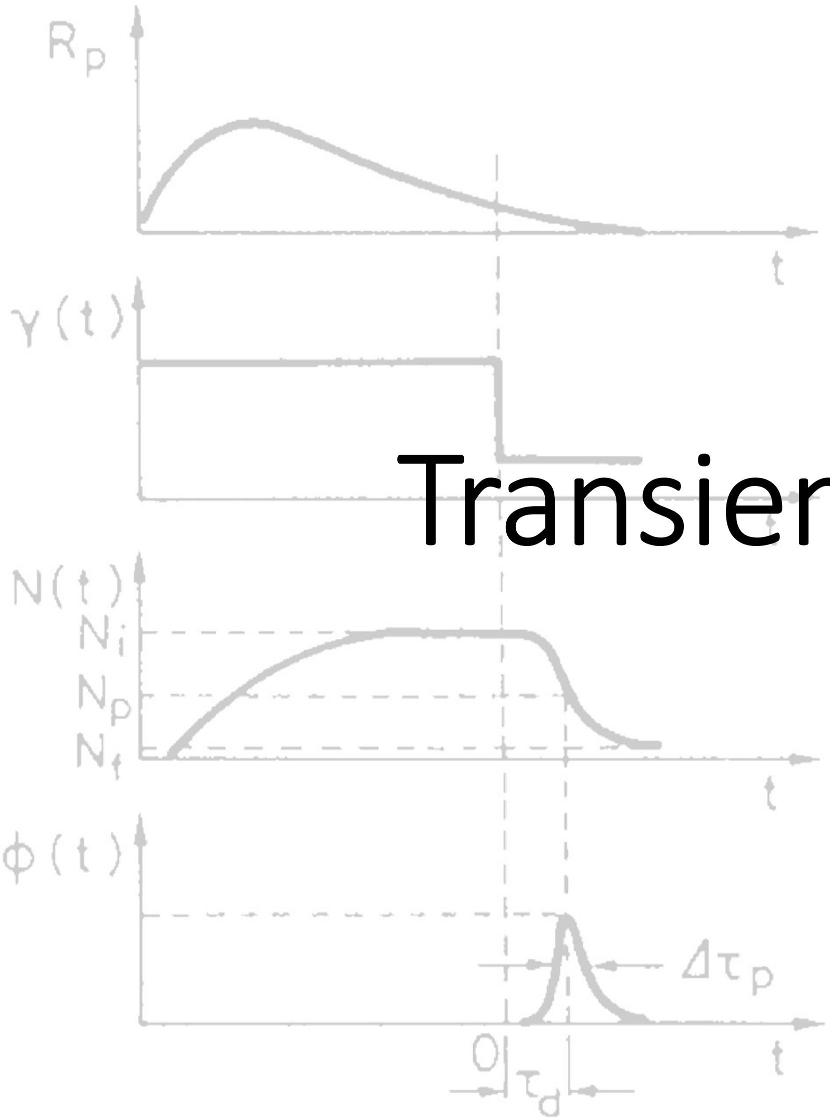
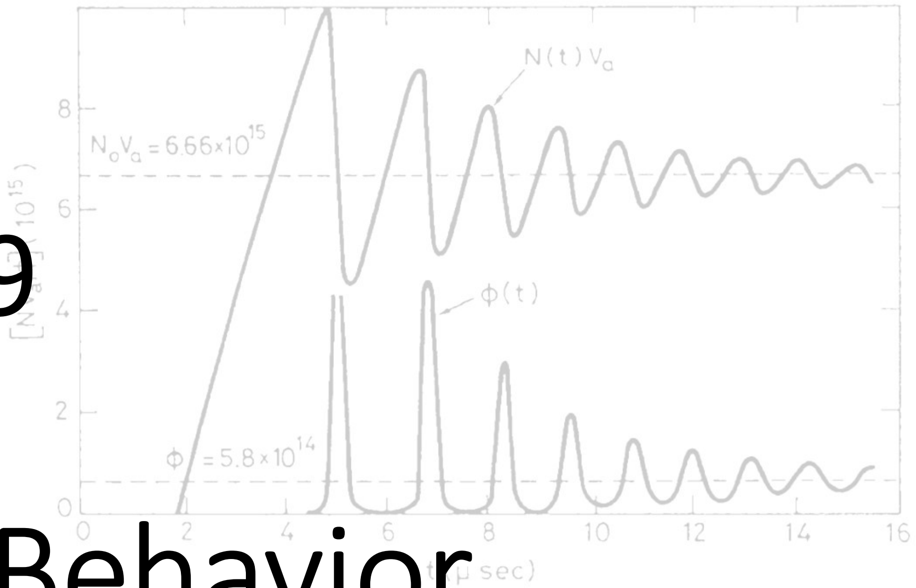
Lecture 9

Transient Laser Behavior

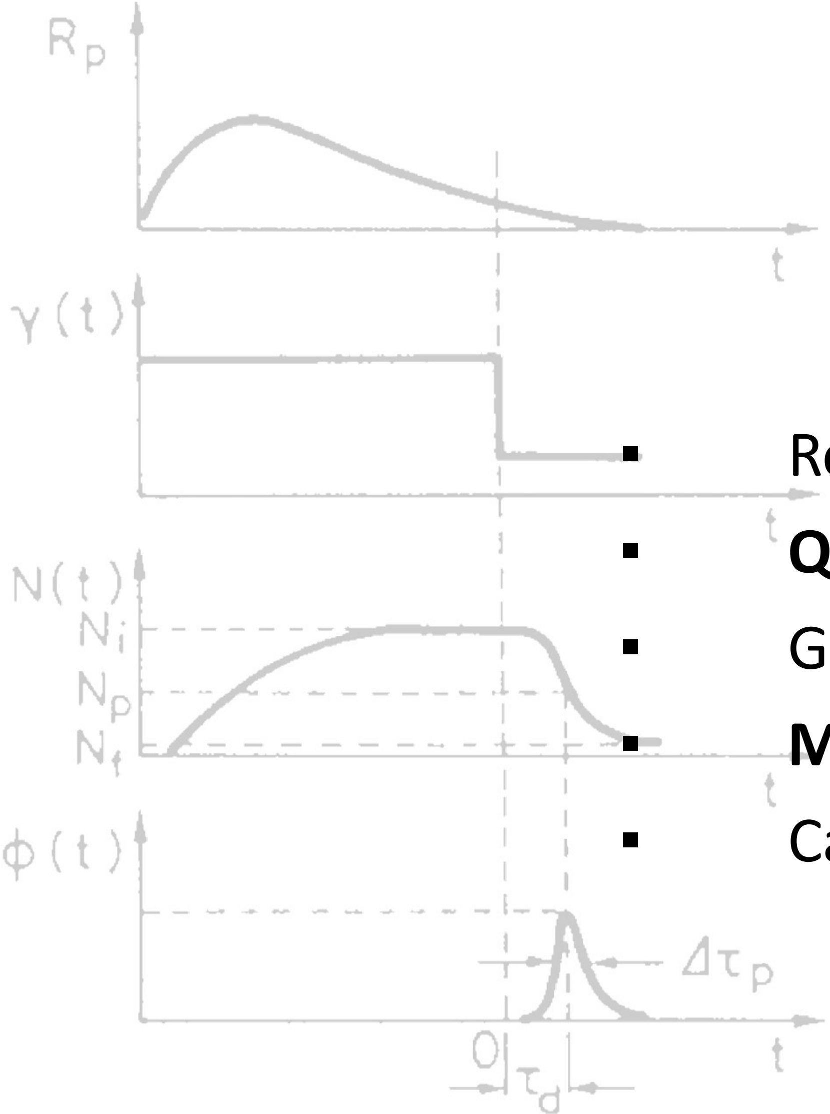


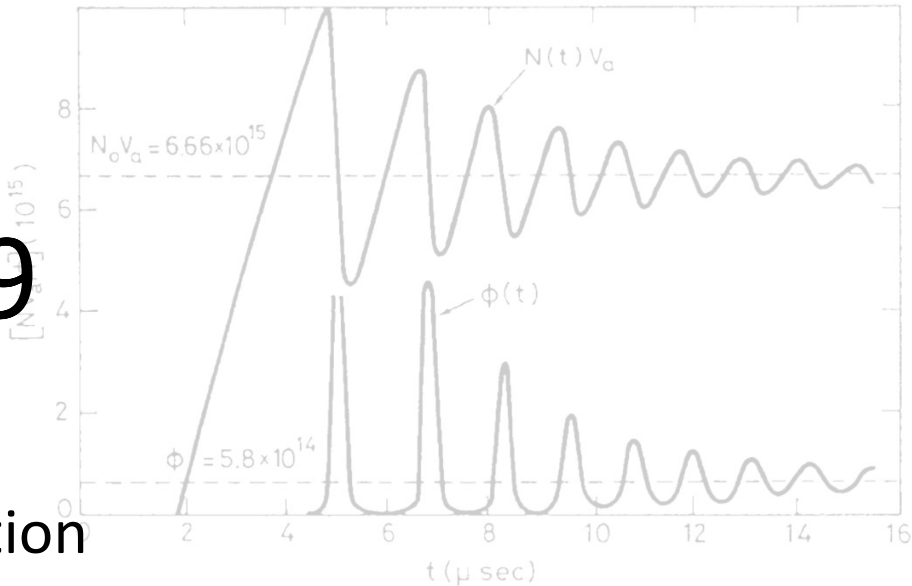


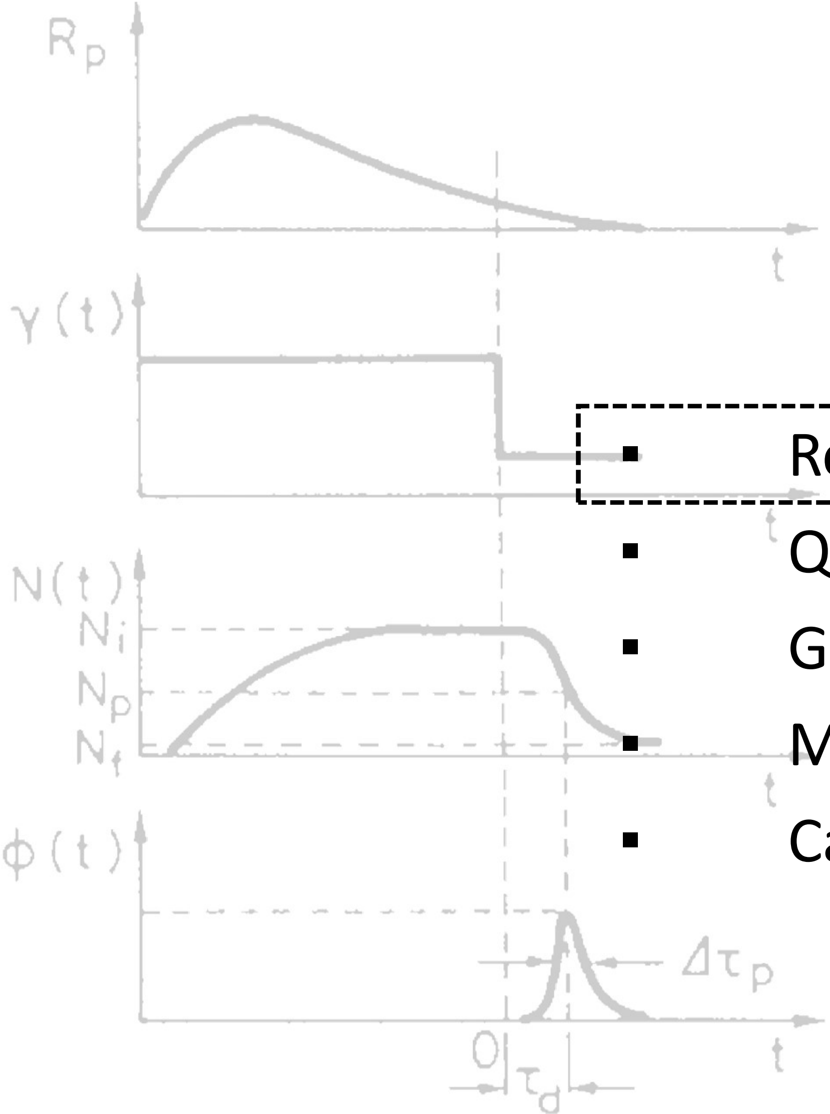
Hoon, KTH

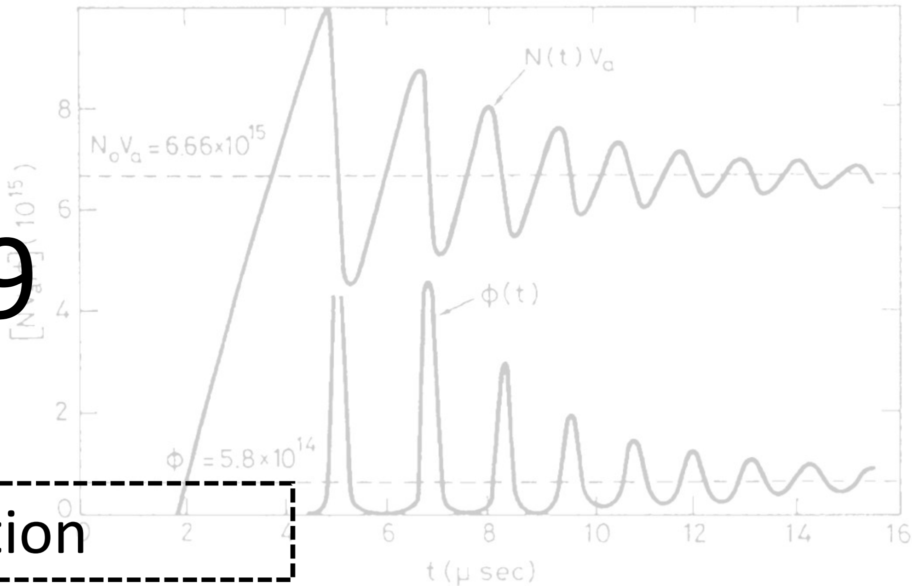
Lecture 9

- Relaxation Oscillation
- **Q-switching**
- Gain-switching
- **Mode-locking**
- Cavity dumping







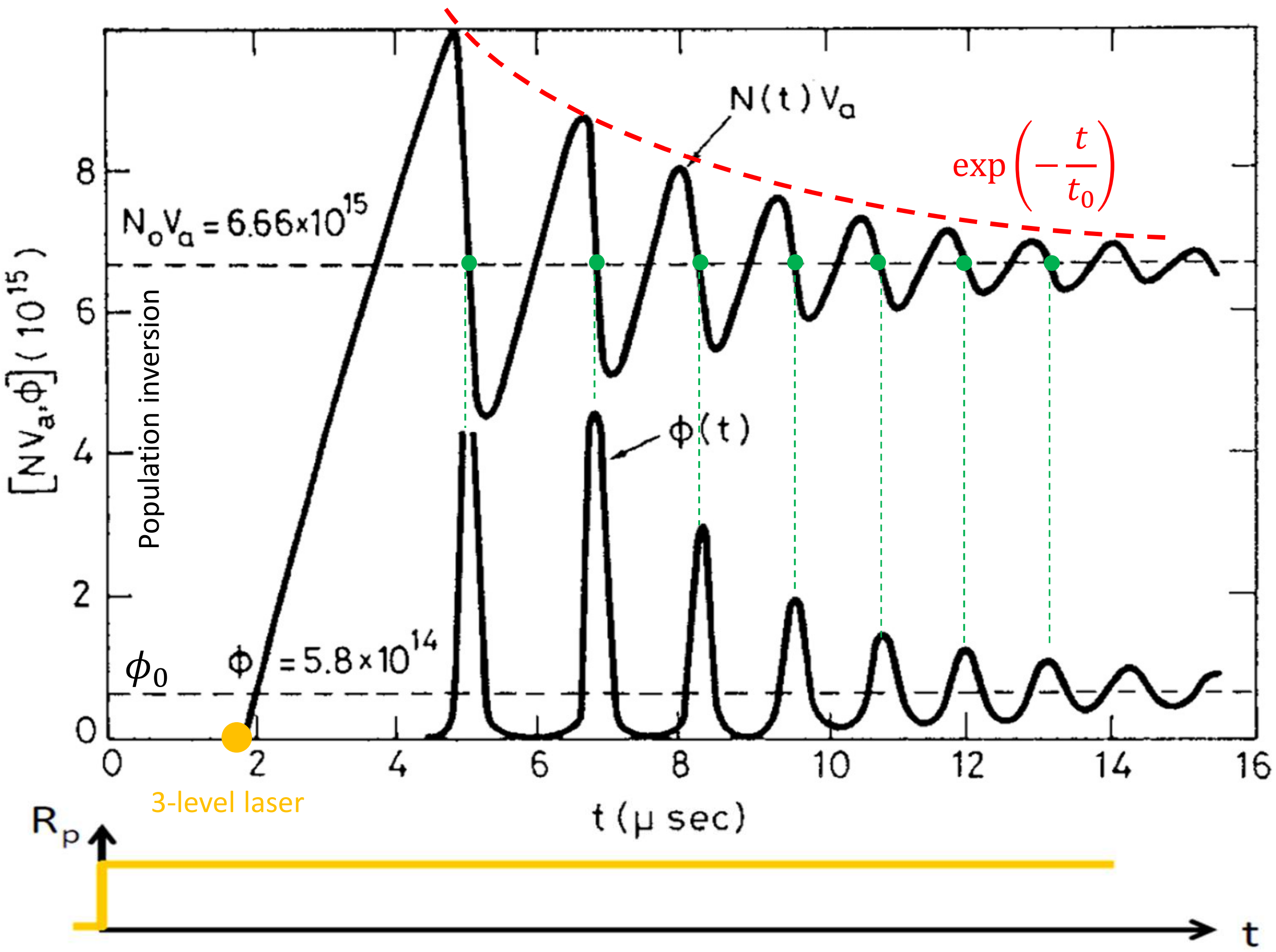


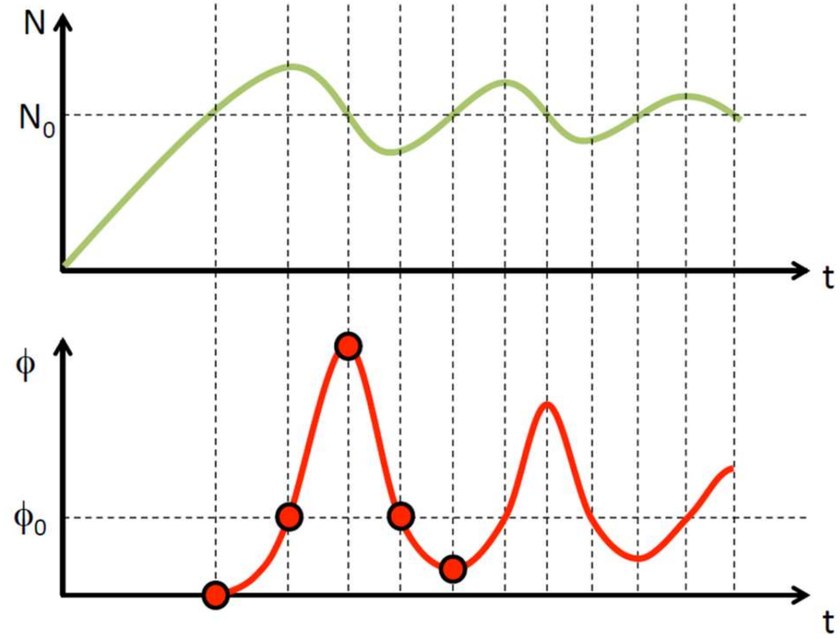
Lecture 9

- Relaxation Oscillation
- Q-switching
- Gain-switching
- Mode-locking
- Cavity dumping

- ## Relaxation Oscillation

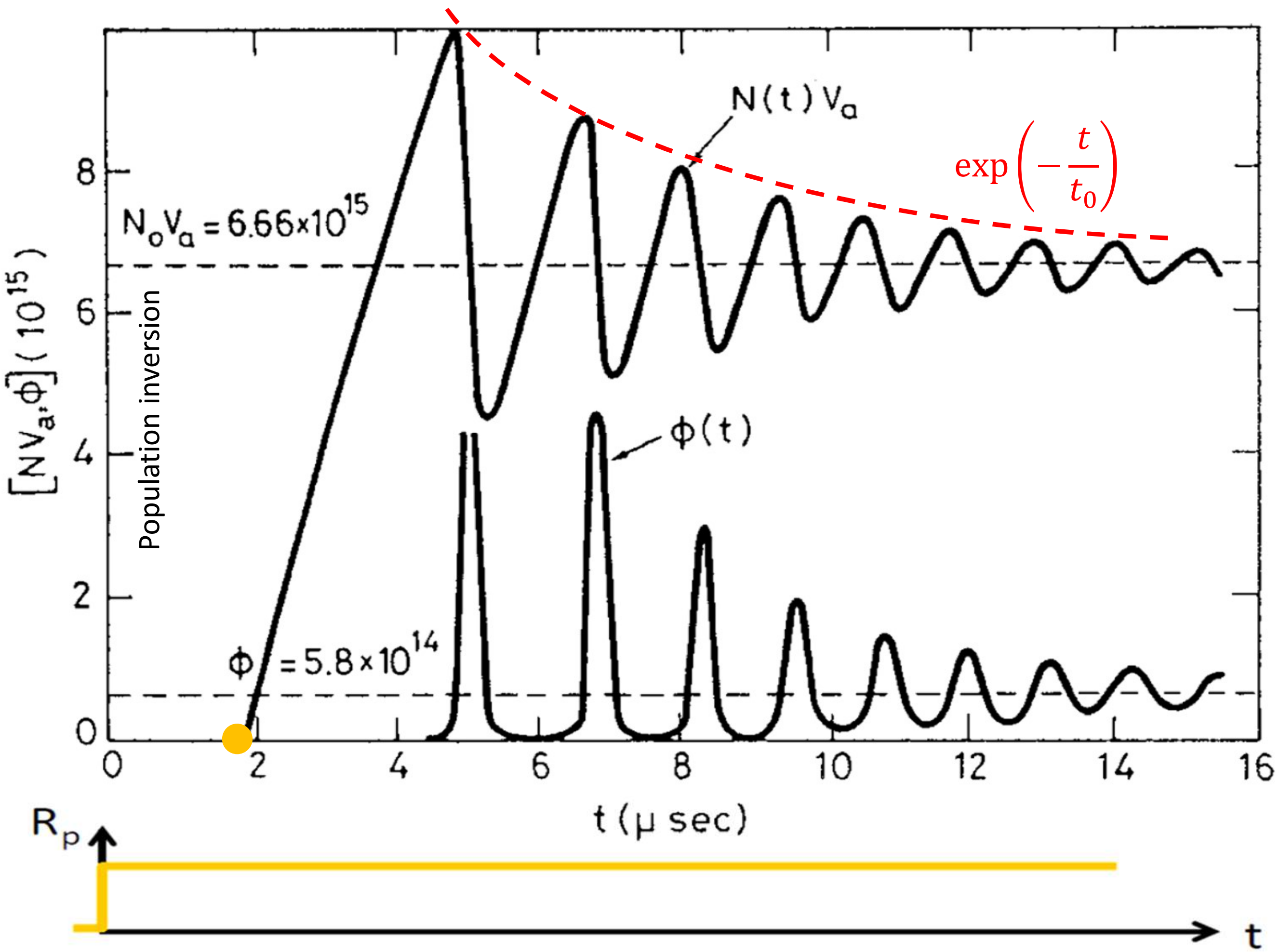
Two-level Rabi-Oscillation (Damped)





▪ Relaxation Oscillation

Two-level Rabi-Oscillation (Damped)



$$N(t) = N_0 + \delta N(t)$$

$$\phi(t) = \phi_0 + \delta\phi(t)$$

rate equations

If $1/t_0 \ll \omega'$ (this is the case e.g. when $R_p \gg R_{cp}$)

$$\delta\phi \propto \exp\left(-\frac{t}{t_0}\right)\sin(\omega' t + \phi)$$

$$\delta N \propto \exp\left(-\frac{t}{t_0}\right)\cos(\omega' t + \phi)$$

- $\frac{1}{t_0} = \frac{1}{2}\left(B\phi_0 + \frac{1}{\tau}\right) \qquad \omega' = \left(\omega^2 - \frac{1}{t_0^2}\right)^{\frac{1}{2}}$

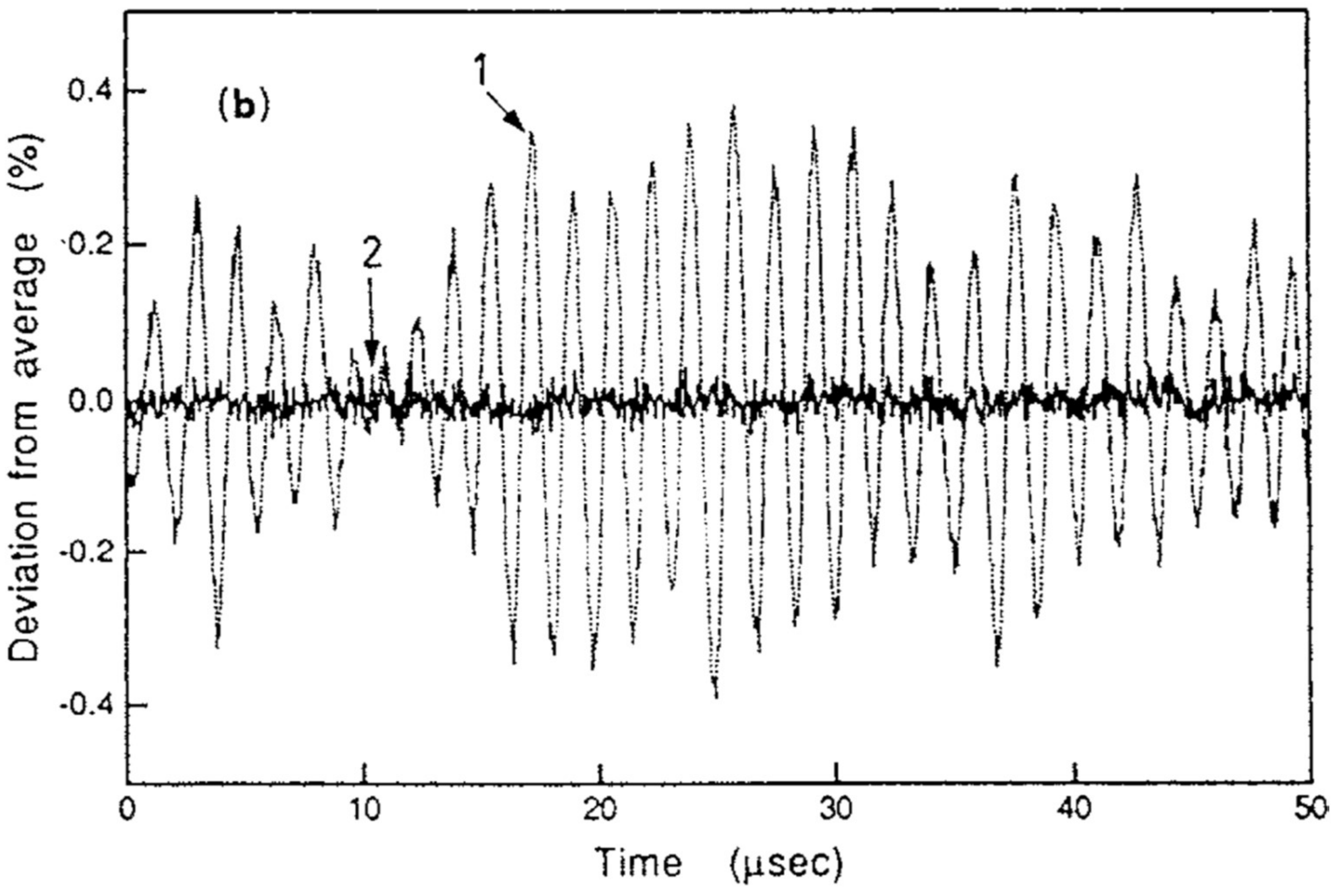
- $t_0 = \frac{2\tau}{x} \qquad \omega = \left(\frac{x-1}{\tau\tau_c}\right)^{1/2} \qquad x = \frac{R_p}{R_{cp}}$

No oscillation when $t_0 < 1/\omega$

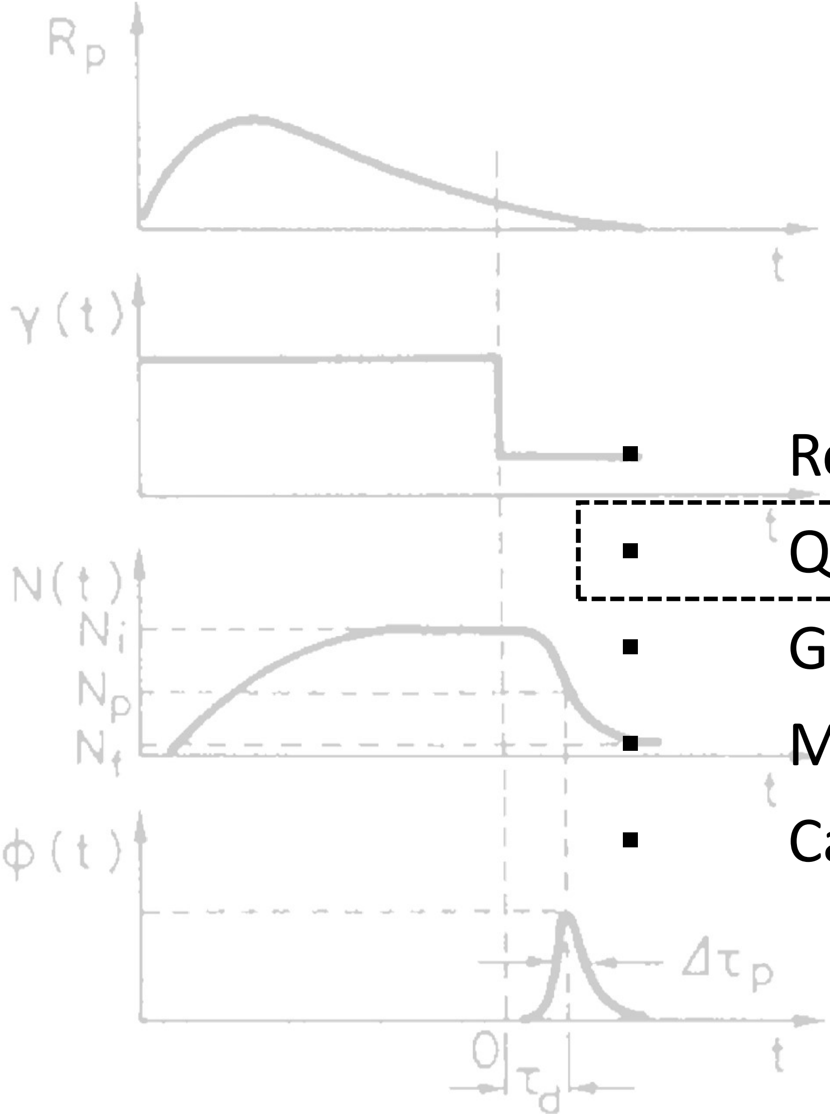
$\left(\frac{\tau_c}{\tau}\right) > 1 \geq \frac{4(x-1)}{x^2}$ Typical in gas lasers

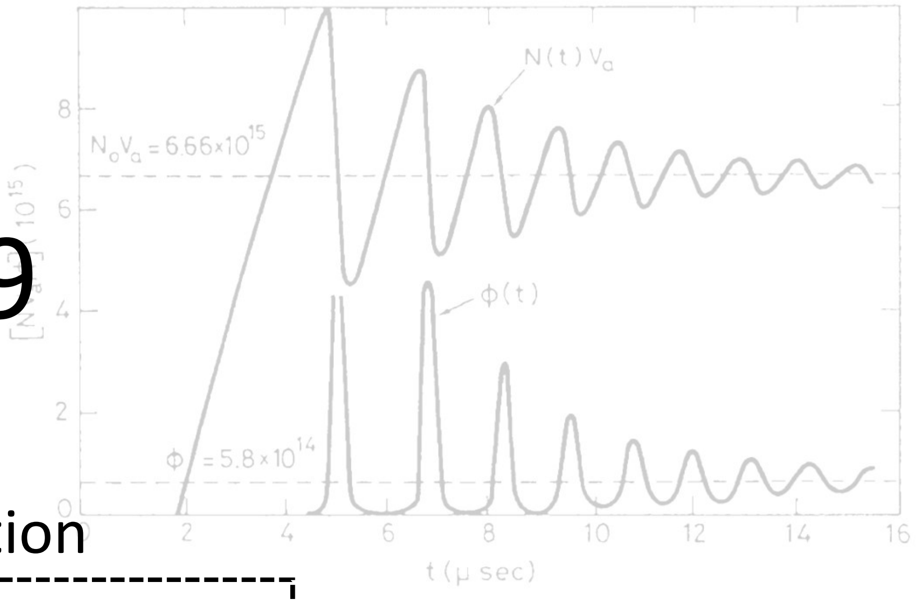
▪ Relaxation Oscillation: Pulsation

Perturbations in pumping level or cavity loss result in “intensity noise”



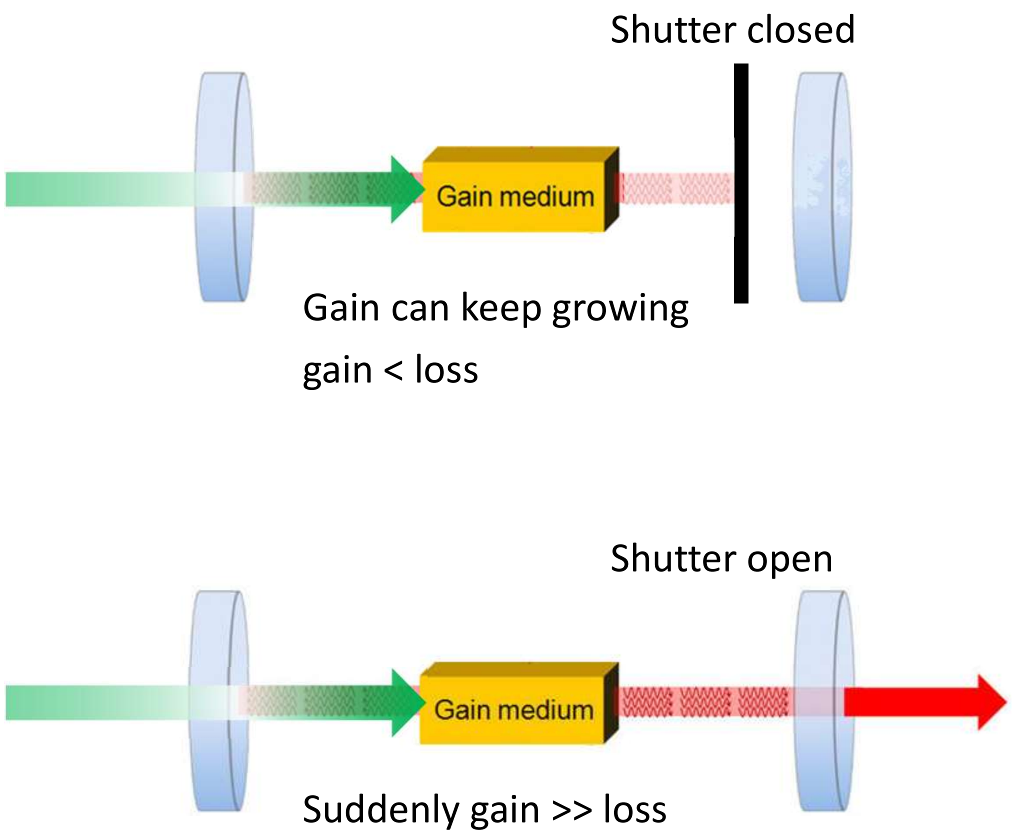
Lecture 9

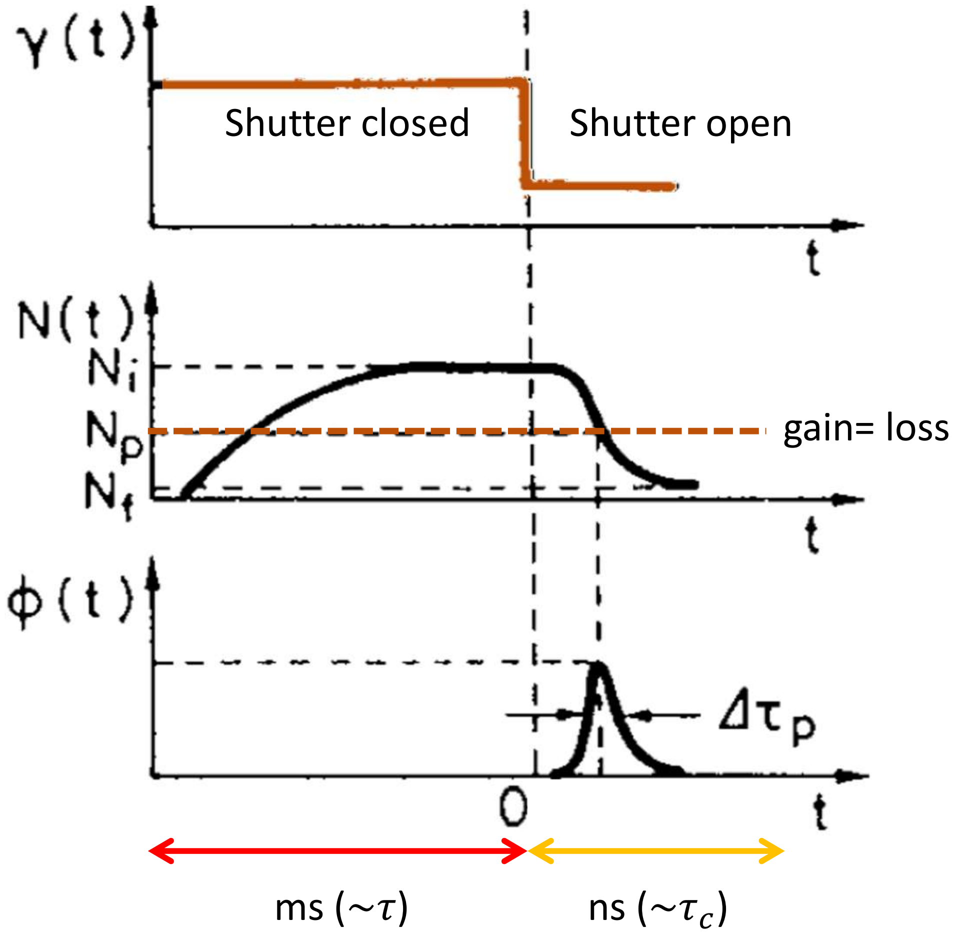




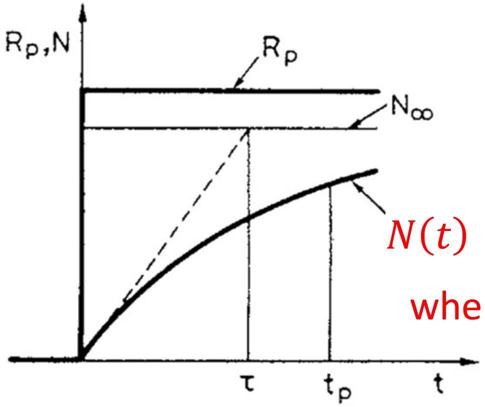
- Relaxation Oscillation
- $\boxed{\text{Q-switching}}$
- Gain-switching
- Mode-locking
- Cavity dumping

- Q-switching


Shutter closed
Gain medium
Gain can keep growing
gain < loss
Shutter open
Gain medium
Suddenly gain >> loss

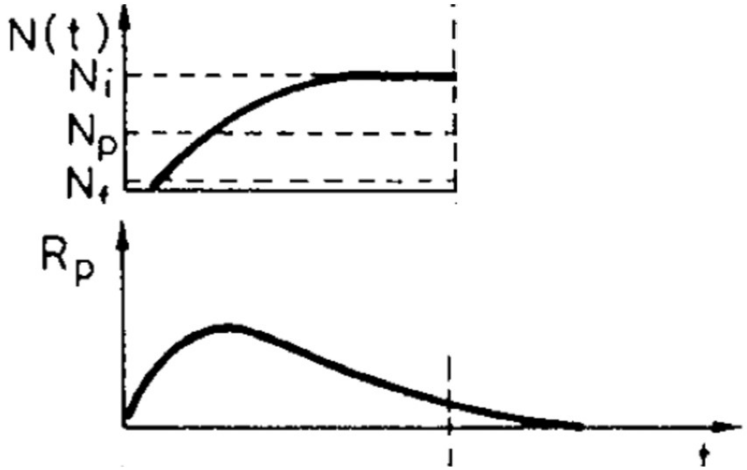

γ(t)
Shutter closed
Shutter open
t
N(t)
N_i
N_p
N_f
gain= loss
t
φ(t)
Δτ_p
0
t
ms (∼τ)
ns (∼τ_c)

▪ Q-switching: Principle (four-level lasers)



$N(t) = N_\infty \left[1 - \exp\left(-\frac{t}{\tau}\right)\right]$

where $N_\infty = R_p \tau$



$\phi \approx 0$ before lasing

$$\frac{dN}{dt} = R_p - \cancel{B\phi N_2} - \frac{N}{\tau}$$

$$\frac{dN}{dt} = R_p - \frac{N}{\tau}$$

Longer τ can provide larger gain N_∞

For pump druation $t_p \gg \tau$,

longer pumping does not increase N

Instead, energy goes wasted through $-\frac{N}{\tau}$

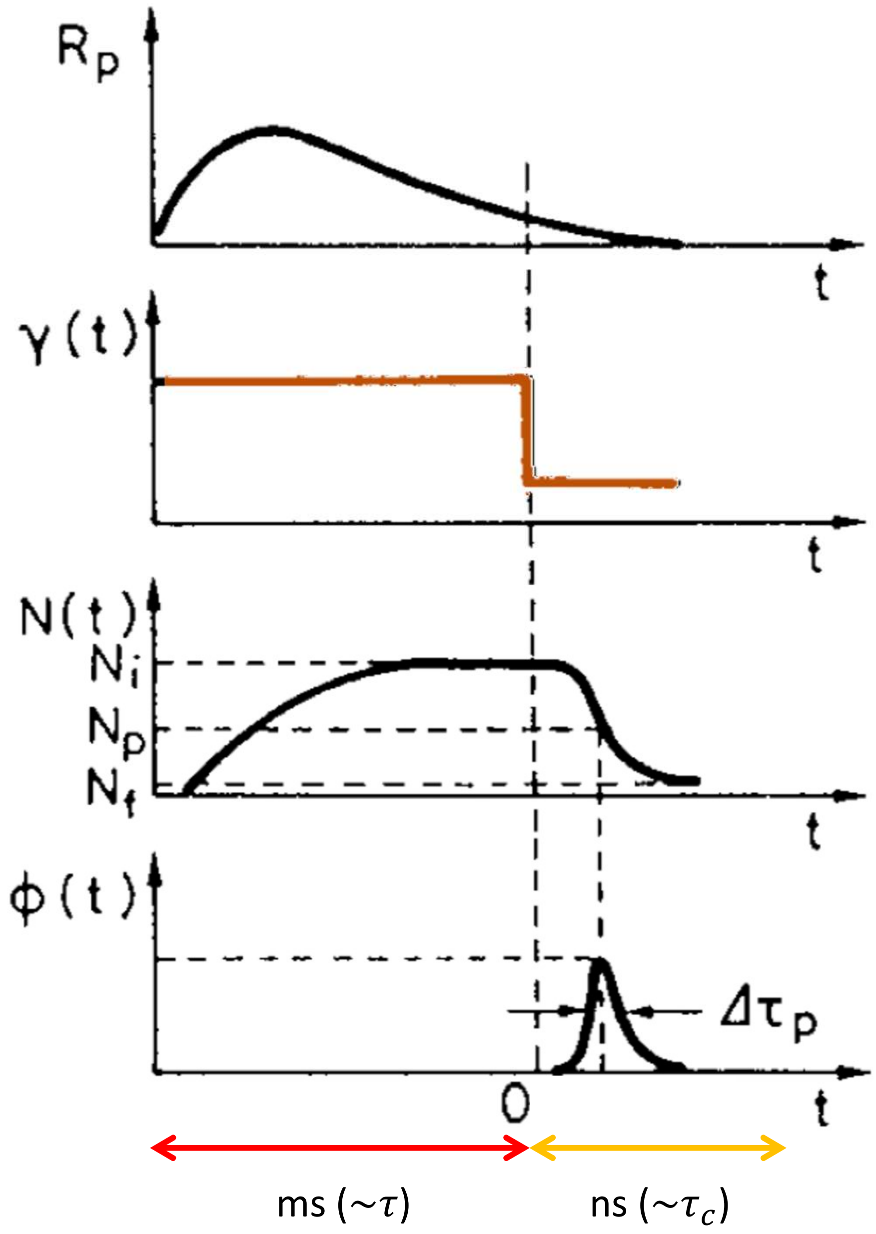
Most solid-state lasers (Nd, Yb, Eb, Ho) and some gas lasers (CO_2, I_2) have a longer τ ~ms → Q-switching

Semiconductor lasers and many gas lasers (He-Ne, Ar, Excimers) has a short τ ~ns

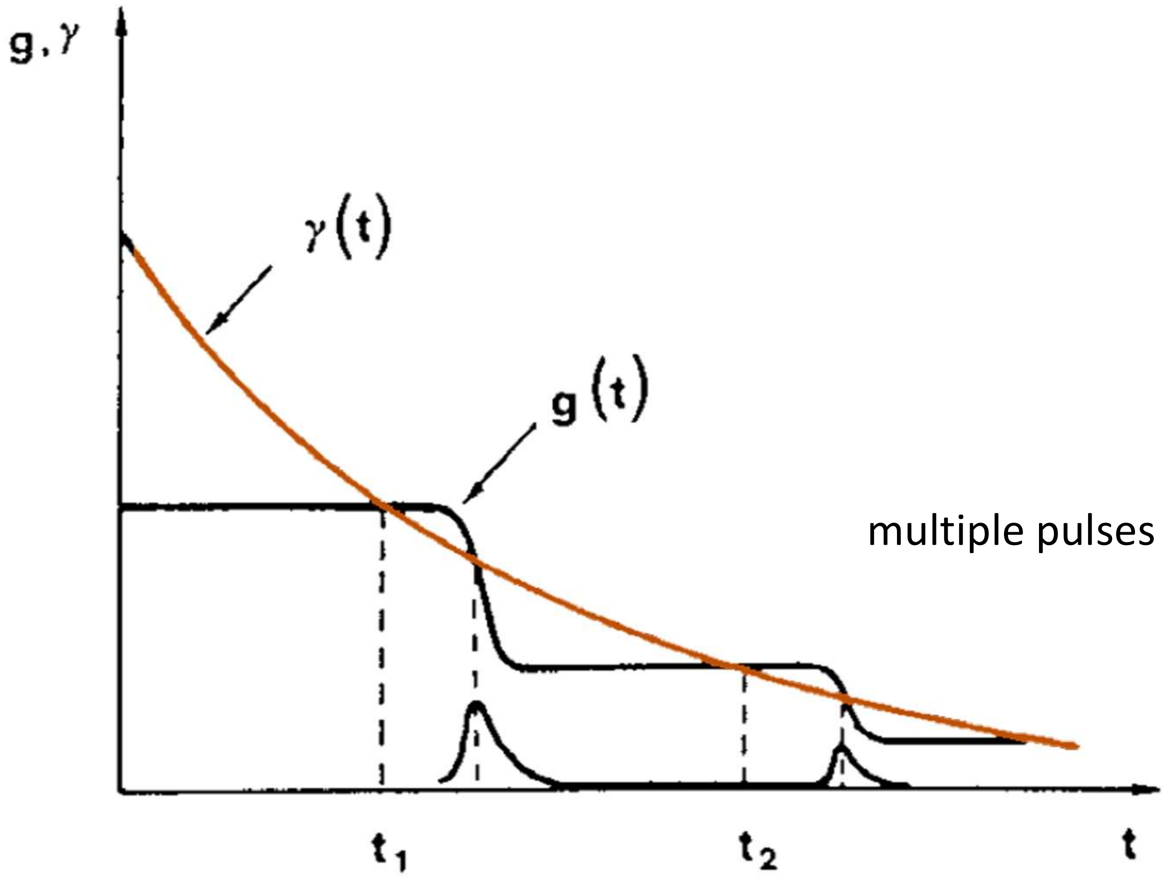
- Q-switching: Fast

Fast switching of γ ~a few τ_c

e.g. Electro-optical switching $t_S < 20\ ns$



- Q-switching: Slow



multiple pulses

How to Q-switch? : **Active** vs. Passive

Active

- Electro-optical

    Pockels cell Q-switches widely used

    Fast (typically t_s< 20 ns)

    High voltage (1~5 kV)

- Rotating prism

    Common mechanical Q-switching

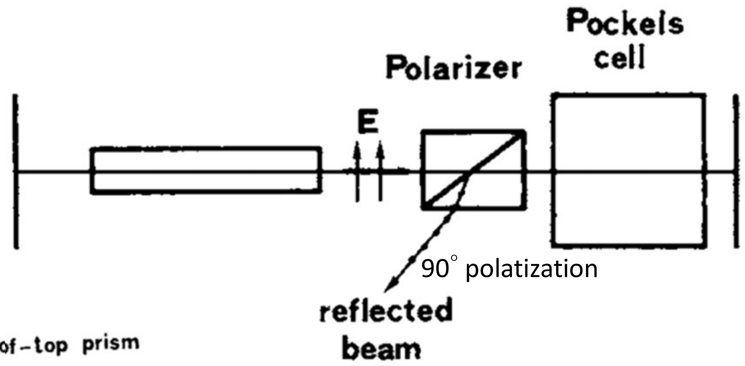
    Wavelength independent

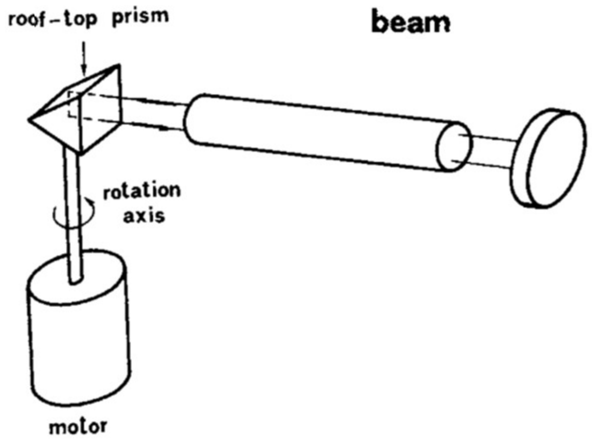
    Slow (400 Hz) and noisy

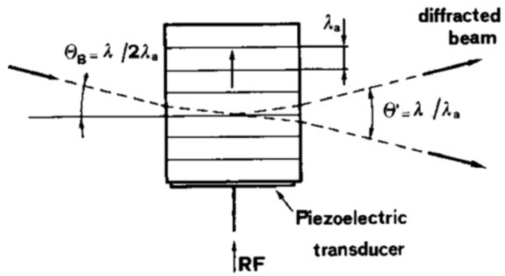
- Acousto-optic

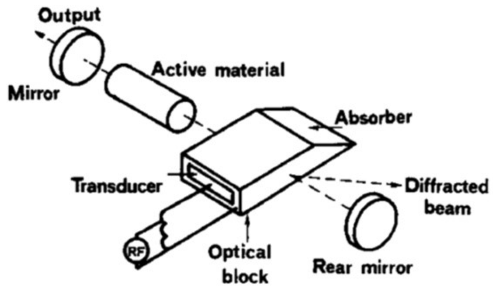
    Low insertion loss, high repetition (kHz)

    Limited loss by diffraction, Low gain









▪ How to Q-switch? : Active vs. **Passive**

Passive

▪ Saturable Absorber

Before saturation: absorption → low Q

After saturation: Transparent → High Q

Single mode operation

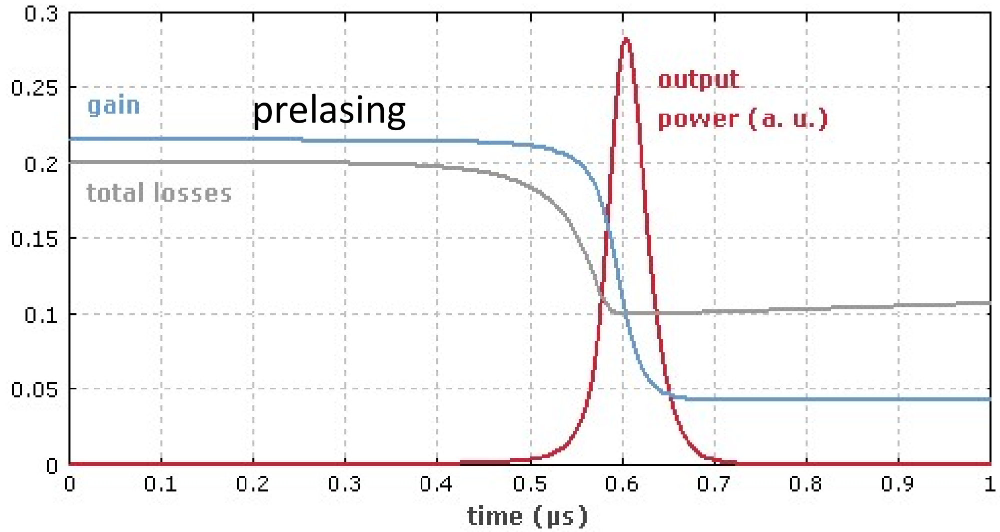
$$\frac{I_1}{I_2} = \left[\frac{\exp(g_1 - \gamma_1)}{\exp(g_2 - \gamma_2)}\right]^n = \exp n[(g_1 - \gamma_1) - (g_2 - \gamma_2)]$$

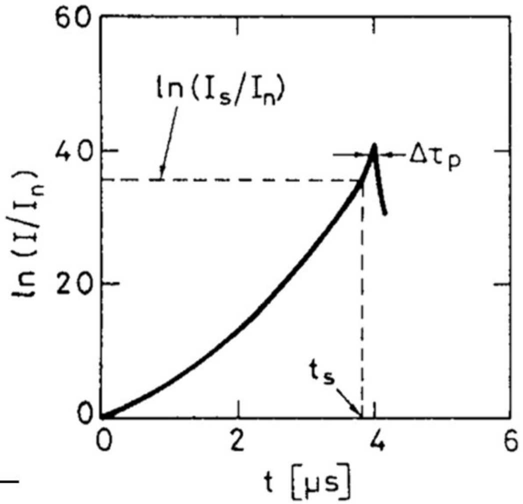
$\exp 2300 \times 0.001 \approx 10$ times large discrimination

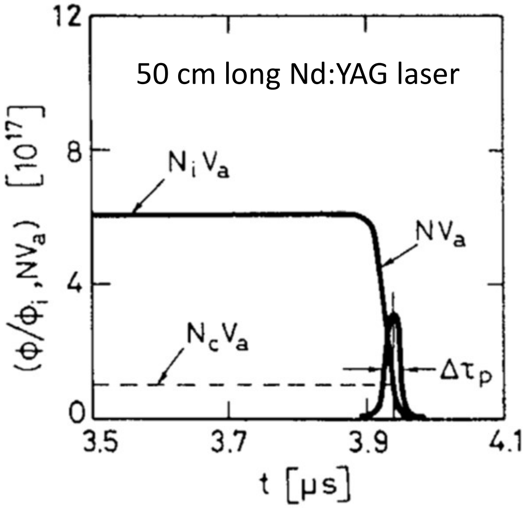
for 50 cm long cavity.

In many cases, the pulse energy and duration are then fixed.

Changes of the pump power would influence the pulse repetition rate.



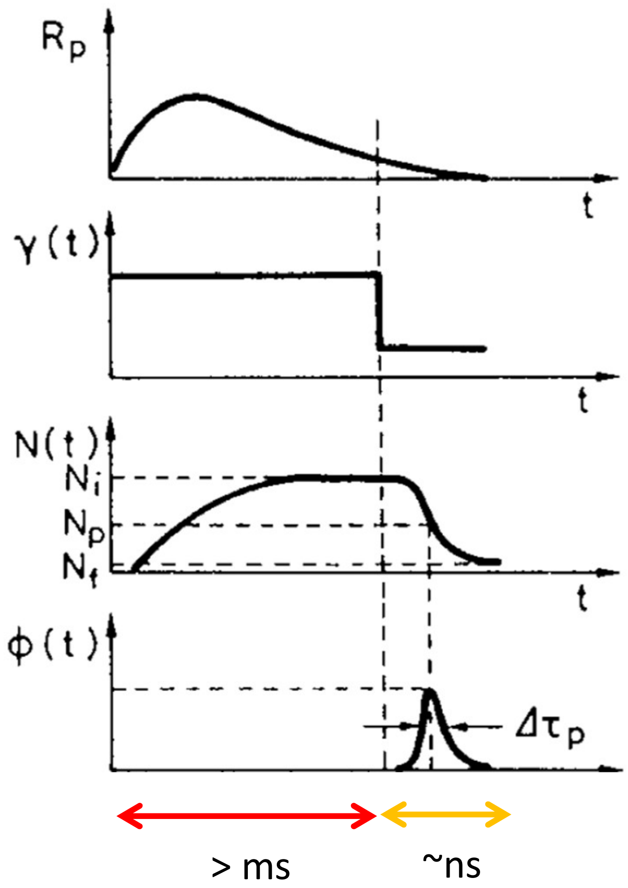




- ## Pumping schemes for Q-switching

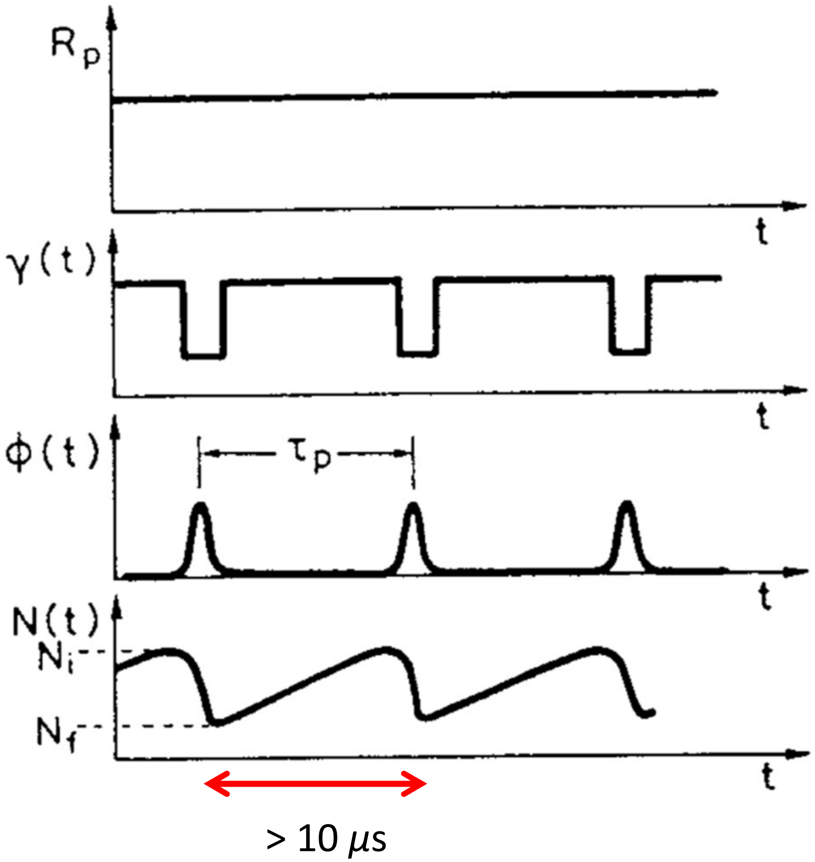
Pulsed pumping R_p < 100 Hz

High Gain, Large output pulse energy

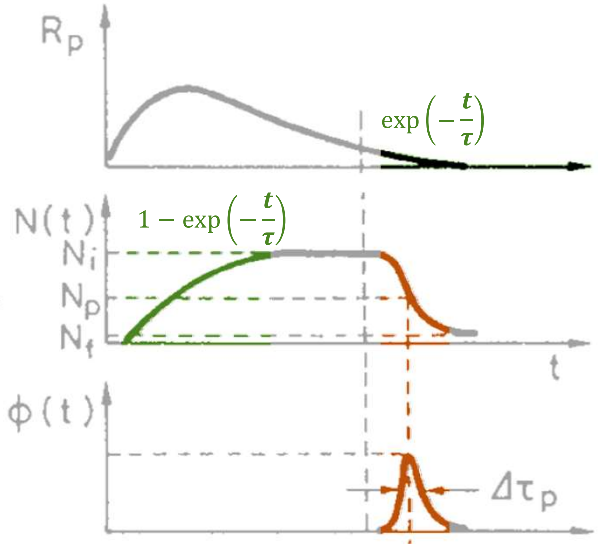


Continuous pumping

Q-switching at < 100 kHz, Low gain



- Theory of Active Q-switching



$$\frac{dN}{dt} = R_p - B\phi N_2 - \frac{N}{\tau}$$

$$\frac{d\phi}{dt} = V_a B\phi N_2 - \frac{\phi}{\tau_c}$$

$$\frac{dN}{dt} = \cancel{R_p} - B\phi N \cancel{- \frac{N}{\tau}}$$

$$\frac{d\phi}{dt} = \left(V_a BN - \frac{1}{\tau_c}\right)\phi$$

at the peak

$$N_p = \frac{1}{V_a B\tau_c} = \frac{\gamma}{\sigma l}$$

$$\frac{d\phi}{dN} = -V_a\left(1 - \frac{N_p}{N}\right) \xrightarrow{\int} \phi = V_a\left[N_i - N - N_p \ln\left(\frac{N_i}{N}\right)\right]$$

neglecting $\phi_i \approx 0$

at the peak

$$\phi_p = V_a N_p\left[\frac{N_i}{N_p} - \ln\left(\frac{N_i}{N_p}\right) - 1\right]$$

- ## Theory of Active Q-switching: Peak power, Pulse Energy, Pulse duration

Peak power $P_p = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu\phi_p$ Eq. 7.2.18 $\Rightarrow$ $P_p = \left(\frac{\gamma_2}{2}\right)\left(\frac{A_b}{\sigma}\right)\left(\frac{h\nu}{\tau_c}\right)\left(\frac{N_i}{N_p} - \ln\frac{N_i}{N_p} - 1\right)$

$$E = \int_0^\infty P(t)dt = \left(\frac{\gamma_2 c}{2L_e}\right) h\nu \int_0^\infty \phi\, dt$$

From the 2nd rate equation

$$\int_0^\infty \frac{d\phi}{dt} dt = \phi\Big|_0^\infty \cong 0 \Rightarrow \int_0^\infty \left(V_a B\phi N_2 - \frac{\phi}{\tau_c}\right) dt = 0$$

Pulse Energy

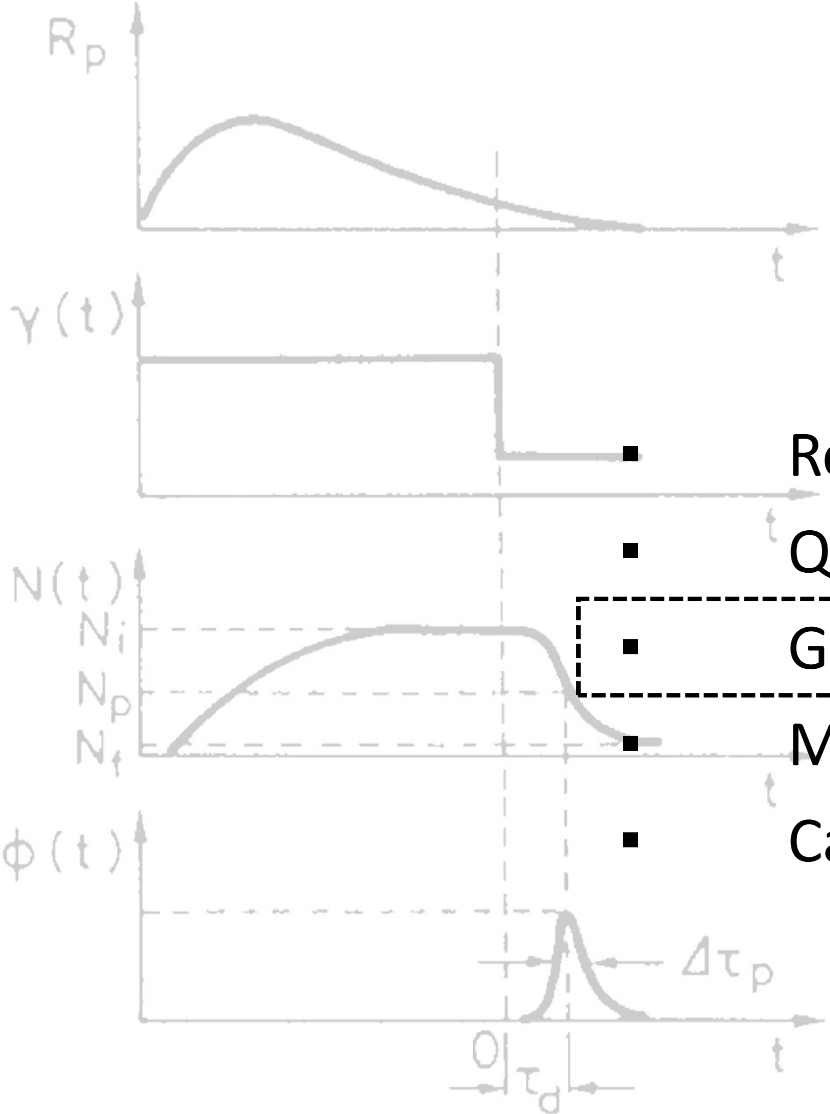
$$\int_0^\infty \phi\, dt = V_a\tau_c \int_0^\infty B\phi N\, dt = V_a\tau_c(N_i - N_f)$$

$$E = \left(\frac{\gamma_2}{2\gamma}\right)(N_i - N_f)(V_a h\nu) = \left(\frac{\gamma_2}{2}\frac{N_i}{N_p}\right)\eta_E\left(\frac{A_b}{\sigma}\right)h\nu$$

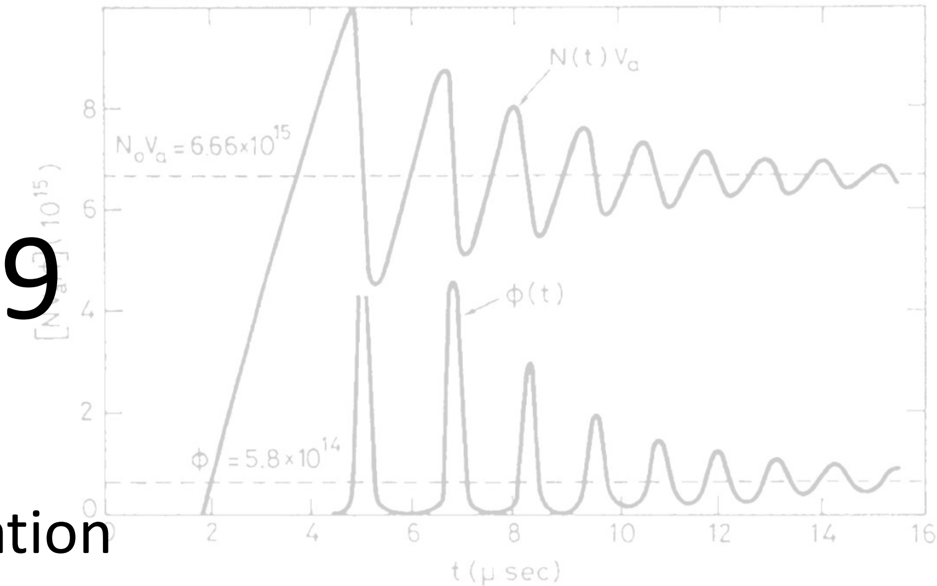
where $\eta_E \equiv \frac{N_i - N_f}{N_i}$ Energy utilization factor

$\Rightarrow$ Pulse duration $\Delta\tau_p \equiv \frac{E}{P_p} = \tau_c \frac{(N_i/N_p)\eta_E}{[(N_i/N_p) - \ln(N_i/N_p) - 1]}$

$\frac{\Delta\tau_p}{\tau_c}$ = Function (N_i/N_p)

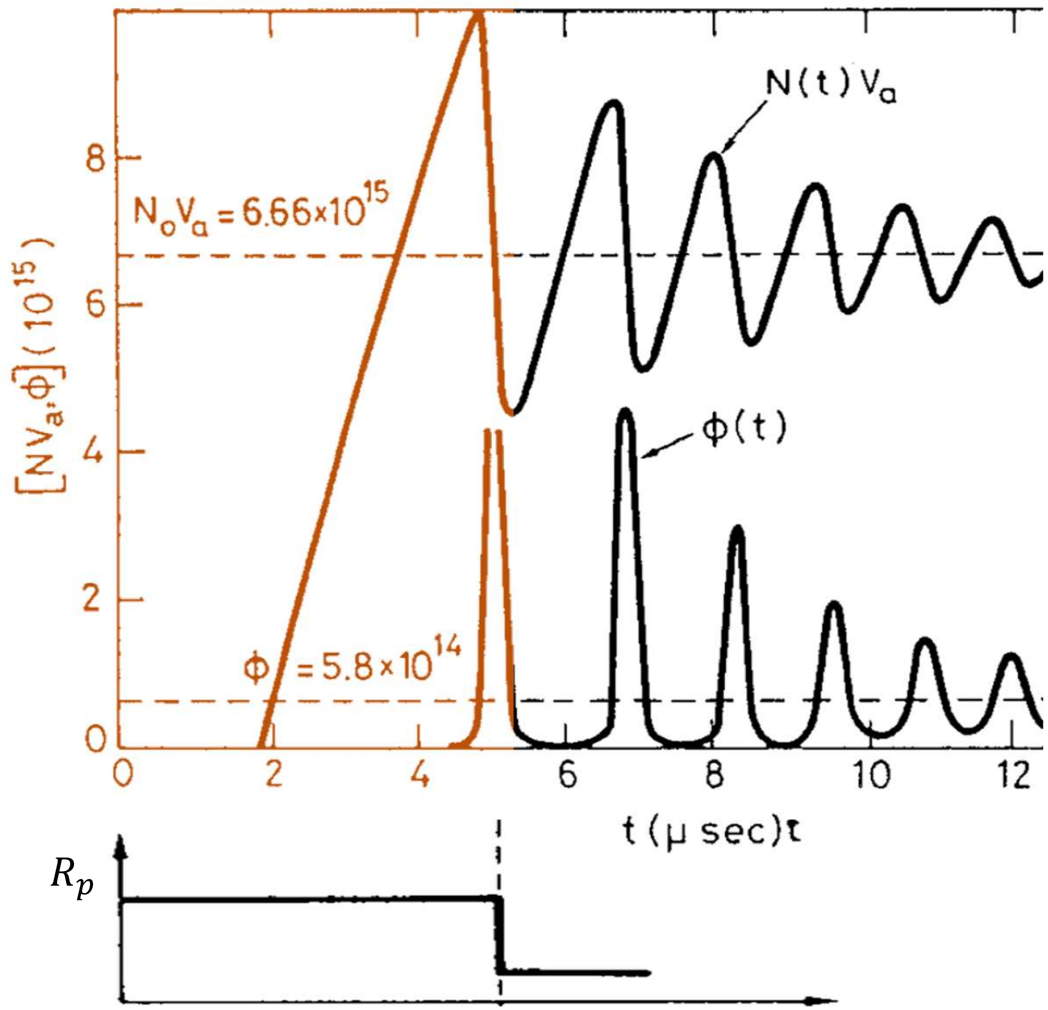


Lecture 9

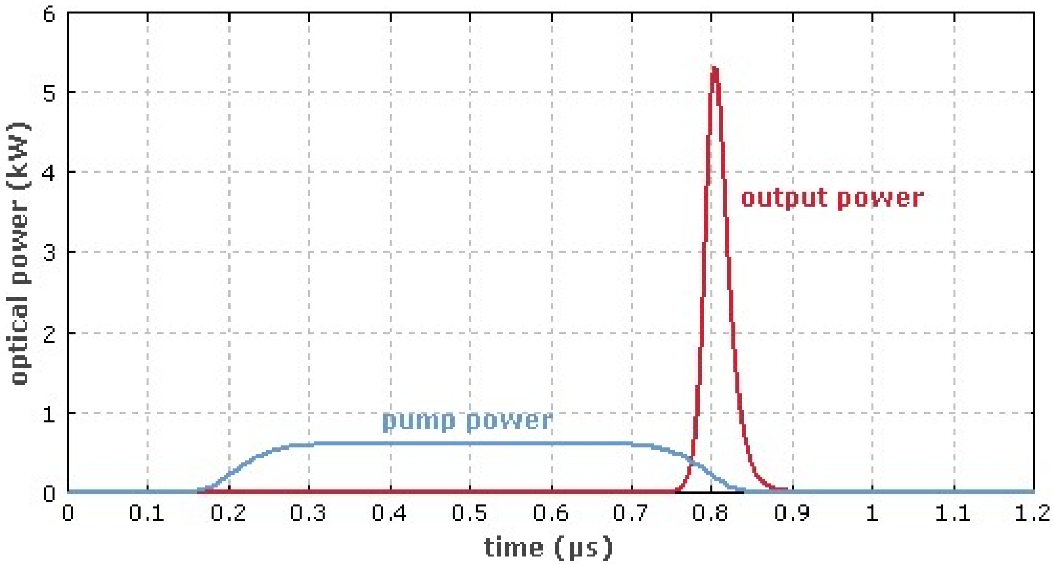


- Relaxation Oscillation
- Q-switching
- Gain-switching
- Mode-locking
- Cavity dumping

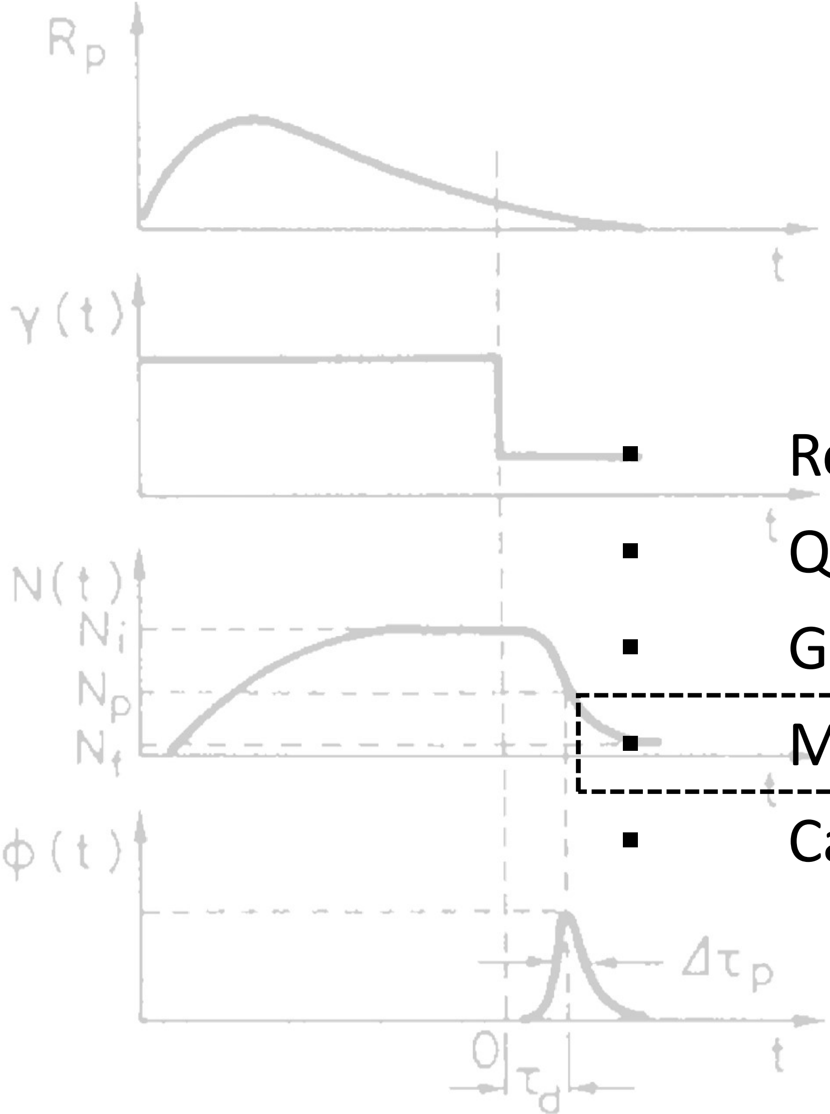
- Gain-switching

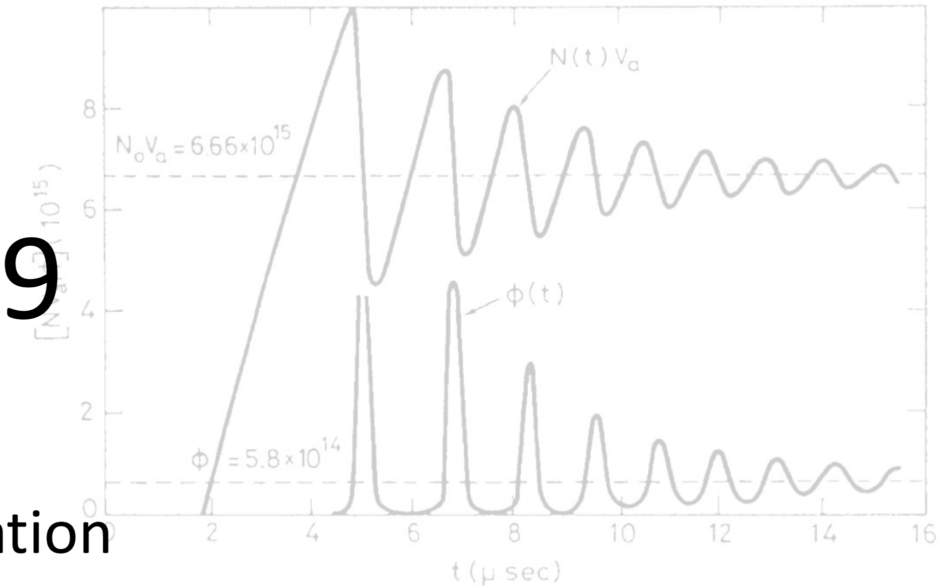


- Fixed Q (loss), Rapid switching Gain on & off
- $R_p \gg R_{cp}$: Large peak inversion 4~10 times N_c
- Pumping duration 5~20 τ_c <1 μs
- No need to worry about Gain saturation!



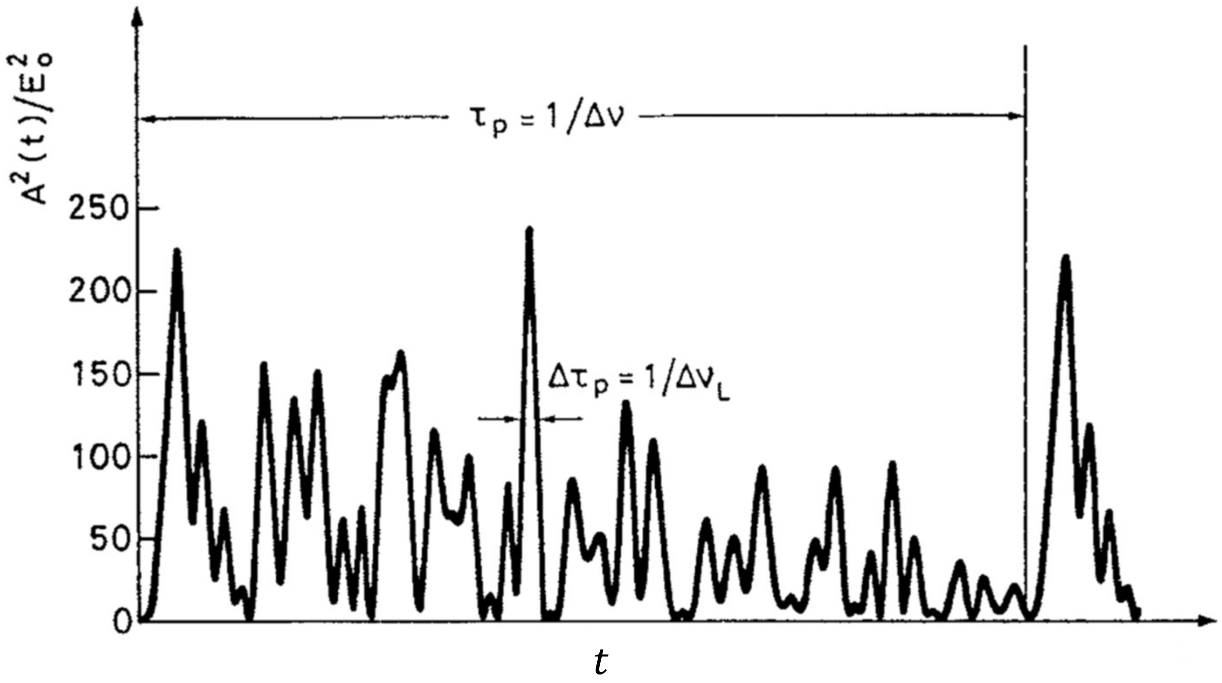
Lecture 9





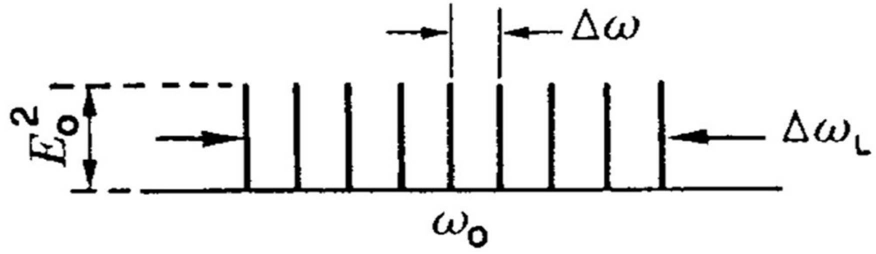
- Relaxation Oscillation
- Q-switching
- Gain-switching
- Mode-locking
- Cavity dumping

- Before mode-locking:



CW laser with large number of **incoherent** longitudinal modes

$\Delta\tau_p = 1/\Delta\nu_L$



$$E(t) = \sum_{l=-n}^{n} E_0 \exp\{j[(\omega_0 + l\Delta\omega)t + \varphi_l]\}$$

φ_l: Random phase

- Mode-locking principle: description in frequency domian

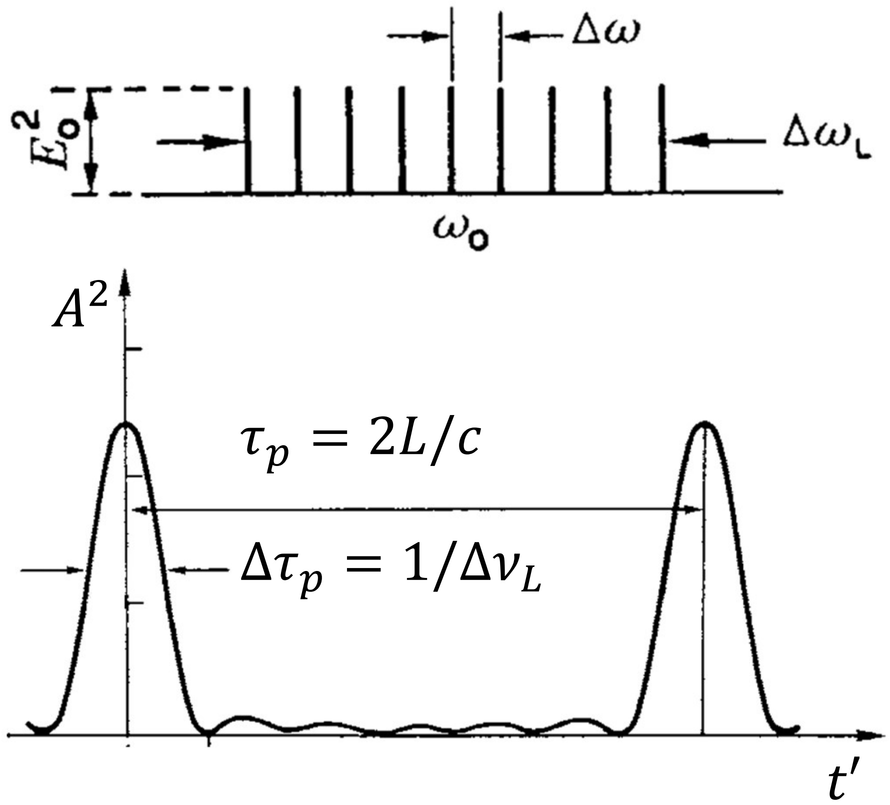
Mode-locking = phases of longitudinal modes are correlated → constructively interfered

Let $\varphi_l - \varphi_{l-1} = 0$

$$E(t) = \sum_{l=-n}^{n} E_0 \exp\{j[(\omega_0 + l\Delta\omega)t]\}$$

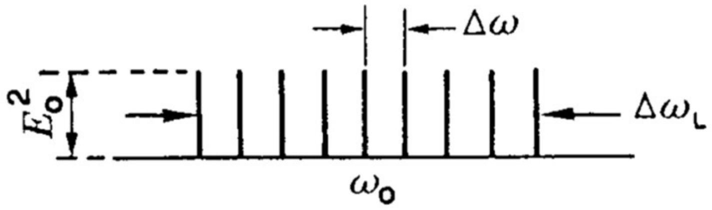
$$= \sum_{l=-n}^{n} E_0 \exp(jl\Delta\omega t) \exp(j\omega_0 t)$$

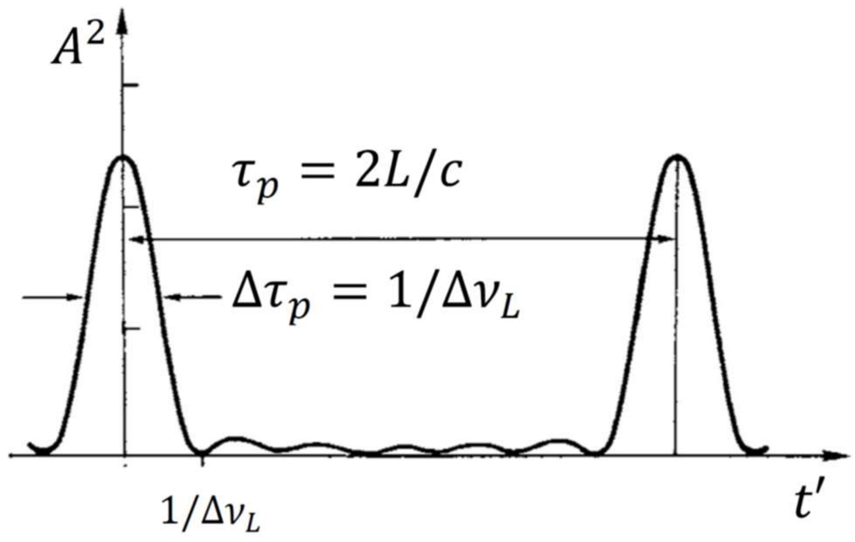
$$= A(t) \exp(j\omega_0 t)$$



- Mode-locking principle: description in frequency domian

Mode-locking = phases of longitudinal modes are correlated → constructively interfered





$$A(t') = \sum_{l=-n}^{n} E_0 \exp(jl\Delta\omega t) \quad = E_0' \frac{\sin\left[\frac{(2n+1)\Delta\omega t}{2}\right]}{\sin\left(\frac{\Delta\omega t}{2}\right)}$$

Max: $A_{max} = (2n+1)E_0$ when $\frac{\Delta\omega t}{2} = m\pi \rightarrow t = m\frac{2\pi}{\Delta\omega} = m\frac{1}{\Delta\nu} = m\frac{2L}{c}$ FSR = Rep. Rate

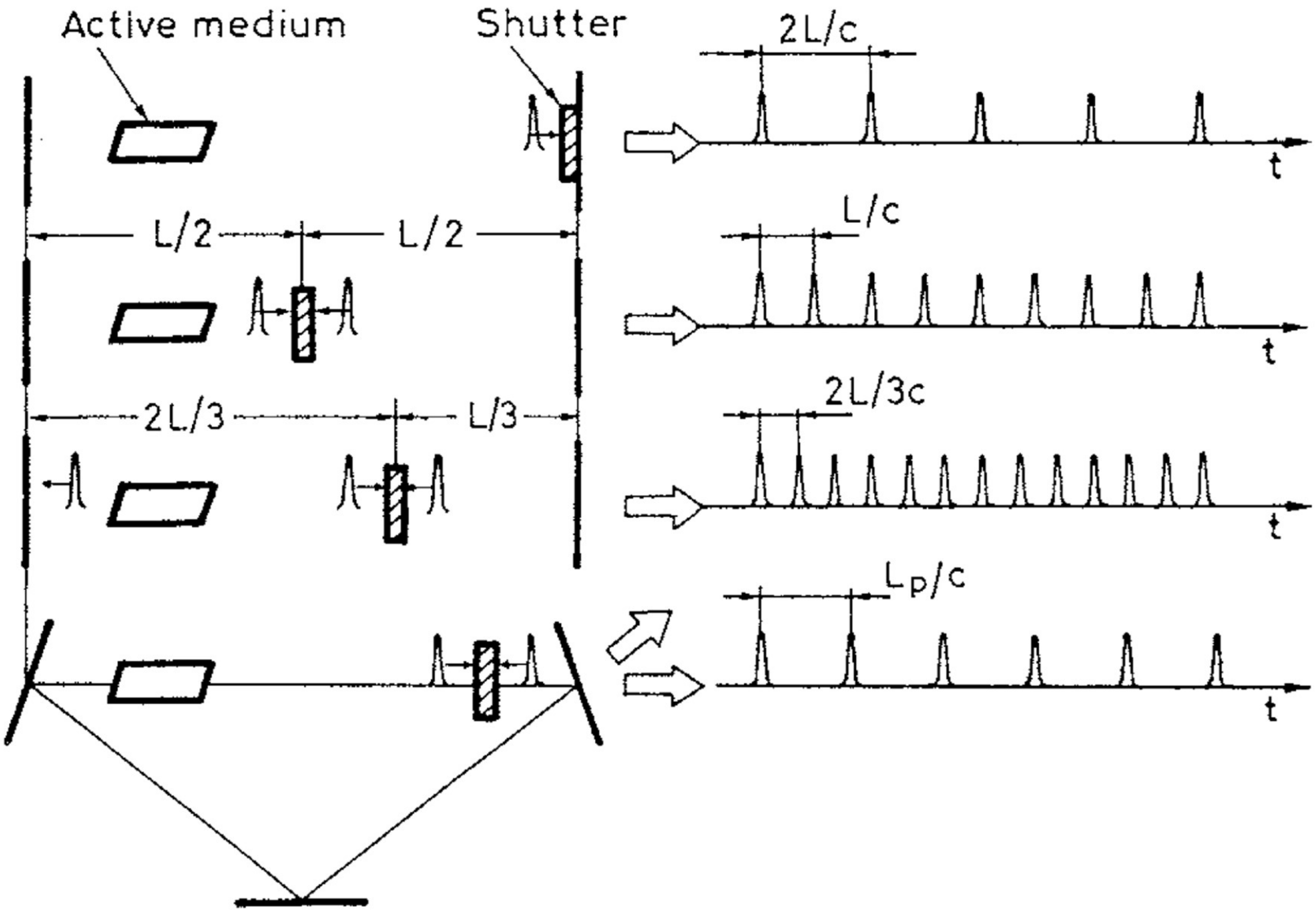
min: $A_{min} = 0$ when $\frac{(2n+1)\Delta\omega t}{2} = \pi \rightarrow t = \frac{2\pi}{(2n+1)\Delta\omega} = \frac{2\pi}{\Delta\omega_L} = \frac{1}{\Delta\nu_L}$

- Mode-locking principle: description in time domian

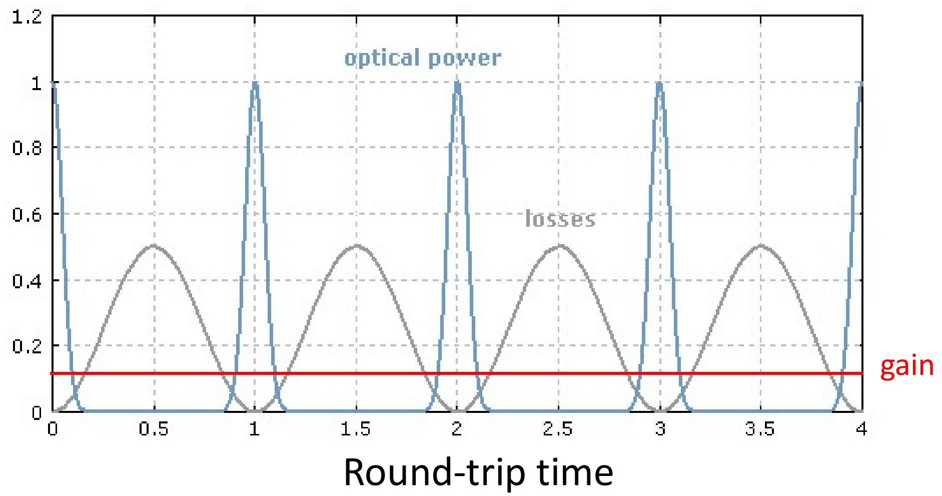
Fundamental

Harmonic

Fundamental



- How to Mode-lock? : **Active**



- Amplitude-modulation (AM)

  Principle: Direct modulation of cavity loss γ at $\nu_m = c/2L$

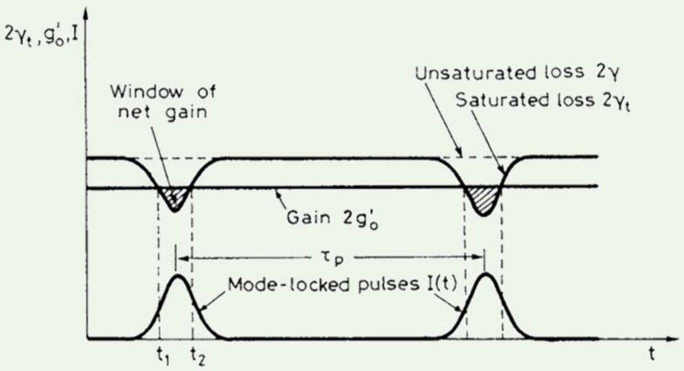
  Method: Pockels cell, acoutsto-optic modulator (AOM)

  The minimum pulse duration is limited by the speed of the active element.

  $\Delta\tau_p > 100\ ps$

- How to Mode-lock? : **Passive**

Fast saturable absorber (response time τ < pulse duration $\Delta\tau_p$)

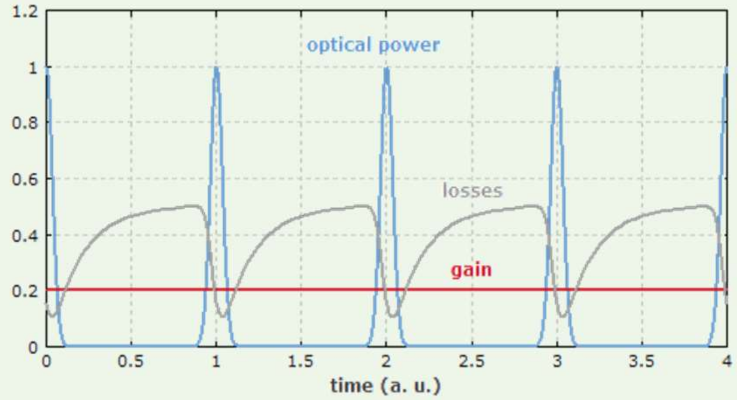


$\tau <$ ps

$\Delta\tau_p \sim$ fs, ps

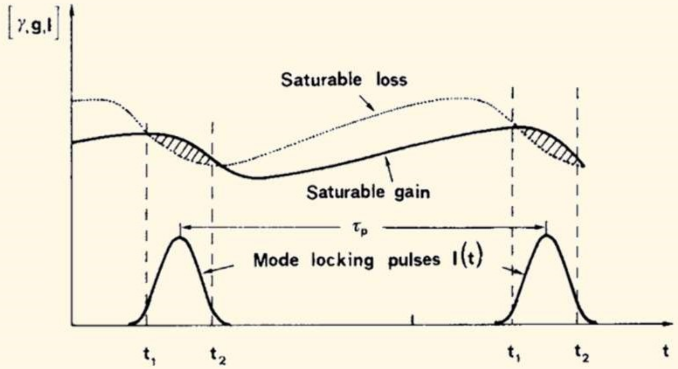
e.g. Semiconductor QW

or Kerr Lens Modelocking



Fast (Intraband) and Slow (Interband)

Slow saturable absorber (response time τ > pulse duration $\Delta\tau_p$)

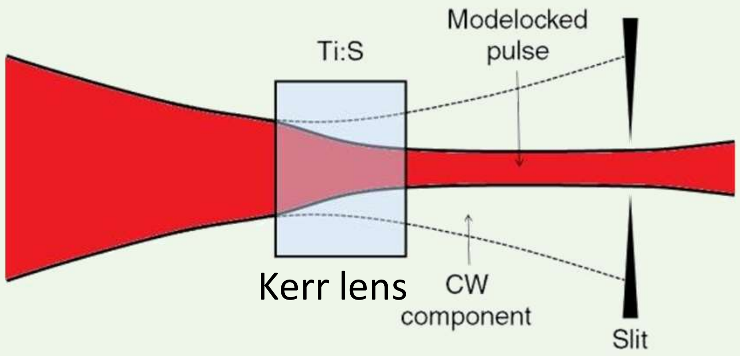


$\tau \sim$ ps, ns

$\Delta\tau_p \sim$ fs, ps

e.g. Saturable dyes

$n = n_0 + n_2 I$, self-focusing



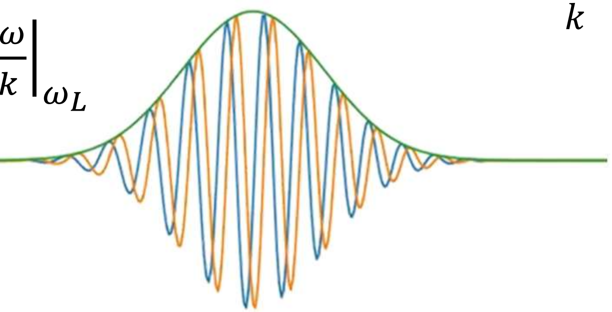
Non-resonant, very fast (fs)

TABLE 8.1. Most common media providing picosecond and femtosecond laser pulses together with the corresponding values of: (a) gain linewidth, $\Delta\nu_0$; (b) peak stimulated emission cross-section, σ; (c) upper state lifetime, τ; (d) shortest pulse duration so far reported, $\Delta\tau_p$; (e) shortest pulse duration, $\Delta\tau_{mp}$, achievable from the same laser

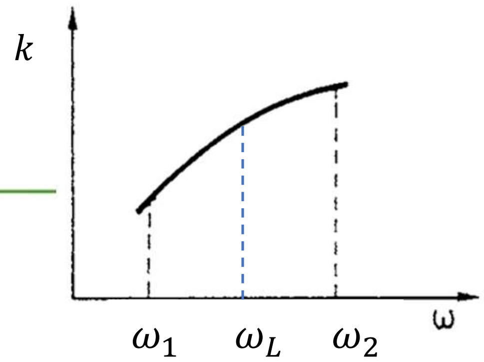
Laser medium	$\Delta\nu_0$	$\sigma\,[10^{-20}\ \mathrm{cm}^2]$	$\tau\,[\mu\mathrm{s}]$	$\Delta\tau_p$	$\Delta\tau_{mp}$
Nd:YAG $\lambda = 1.064\ \mu\mathrm{m}$	135 GHz	28	230	5 ps	3.3 ps
Nd:YLF $\lambda = 1.047\ \mu\mathrm{m}$	390 GHz	19	450	2 ps	1.1 ps
Nd:YVO_4 $\lambda = 1.064\ \mu\mathrm{m}$	338 GHz	76	98	<10 ps	1.3 ps
Nd:glass $\lambda = 1.054\ \mu\mathrm{m}$	8 THz	4.1	350	60 fs	55 fs
Rhodamine 6G $\lambda = 570$ nm	45 THz	2×10^4	5×10^{-3}	27 fs	10 fs
Cr:LISAF $\lambda = 850$ nm	57 THz	4.8	67	18 fs	8 fs
Ti:sapphire	100 THz	38	3.9	6–8 fs	4.4 fs

▪ Phase velocity, Group Velocity, GDD, GVD

Phase velocity: $v_{ph} = \frac{\omega}{k}$ Group velocity: $v_g = \left.\frac{d\omega}{dk}\right|_{\omega_L}$



v_g = slope of dispersion curve



Time delay $\tau_g = \frac{l}{v_g} = l \left.\frac{dk}{d\omega}\right|_{\omega_L} = \left.\frac{d\phi}{d\omega}\right|_{\omega_L} = \phi'(\omega_L)$

Difference in time delay at ω_1 and ω_2 $\Delta\tau_g = \phi'(\omega_2) - \phi'(\omega_1) \cong \phi''(\omega_L) \times (\omega_2 - \omega_1)$

Broadening of the pulse (chirped): $\Delta\tau_d \cong |\phi''(\omega_L)|\Delta\omega_L$

$\phi''(\omega_L)$: Group- Delay Dispersion (GDD) $\phi''(\omega_L) = l \left.\frac{d^2k}{d\omega^2}\right|_{\omega_L}$

$\left.\frac{d^2k}{d\omega^2}\right|_{\omega_L}$: Group- Velocity Dispersion (GVD) $= \left.\frac{d}{d\omega}\left(\frac{1}{v_g}\right)\right|_{\omega_L}$

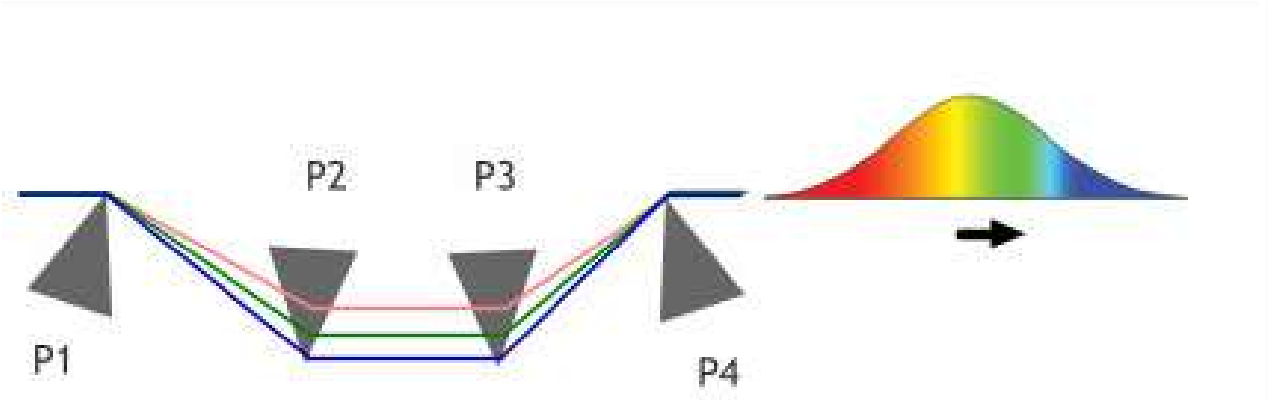
▪ Phase velocity, Group Velocity, GDD, GVD

Broadening of the pulse: $\Delta\tau_d \cong |\phi''(\omega_L)|\Delta\omega_L$ $\phi''(\omega_L)$: Group- Delay Dispersion (GDD)

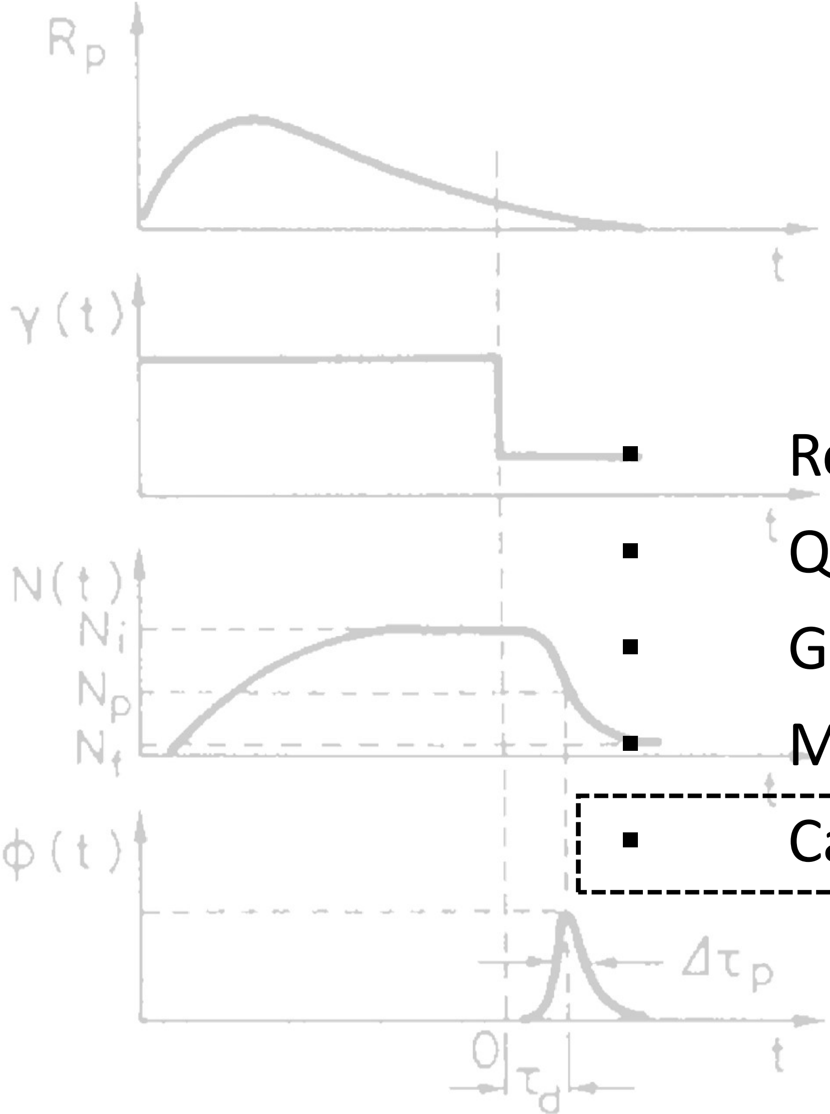
Gain bandwidth $\Delta\nu_0 = 100\,THz$ $\Delta\tau_p \approx \frac{1}{\Delta\nu_0} \approx 10$ fs in an ideal non-dispersive medium

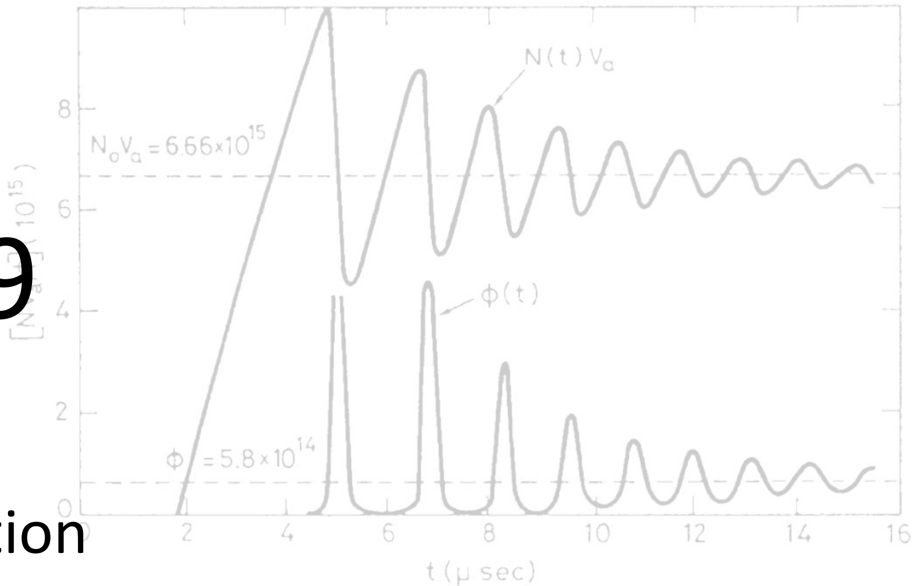
Limitation on Pulse Duration due to GDD: $\Delta\tau_p \cong \left(\frac{27.4}{g_0}\right)|\phi''|\Delta\nu_0 \approx 162$ fs

Dispersion compensation with prism pairs (GDD<0) :



Lecture 9





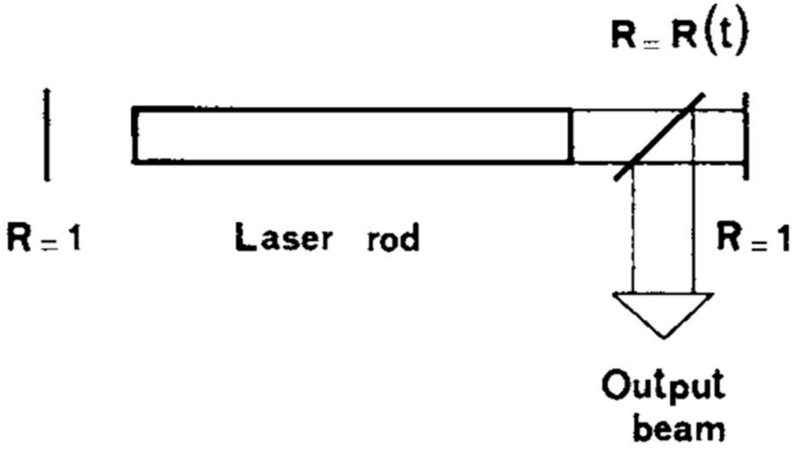
- Relaxation Oscillation
- Q-switching
- Gain-switching
- Mode-locking
- Cavity dumping

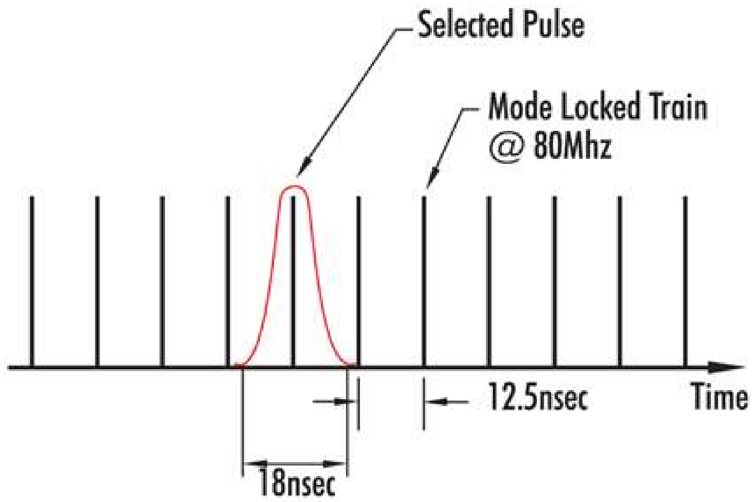
- Cavity Dumping:

Applicable to CW, Q-switched and mode-locked lasers

Emptying out intracavity Power

pulse-picking





Sudden increase of Loss into output coupling

Pockels cell, Acousto-optics grating

Lecture 9

- Relaxation Oscillation
- Q-switching
- Gain-switching
- Mode-locking
- Cavity dumping