Lecture 6
Properties of laser beams*

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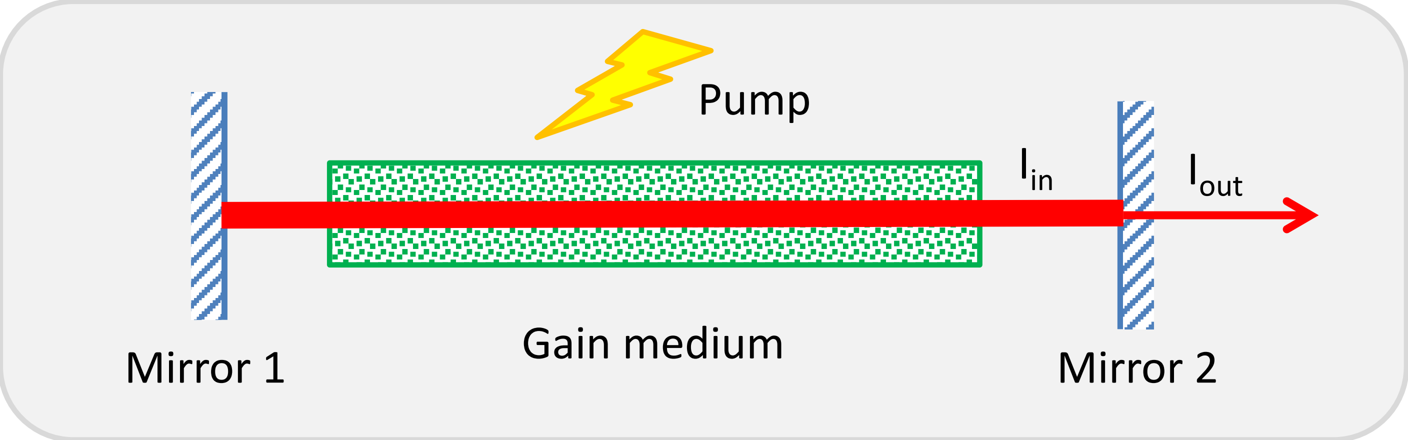
* Some figures and texts belong to: O. Svelto, *Principles of Lasers*, 5th Ed., Springer.

Reading

- *Principles of Lasers* (5th Ed.): Chapter 11.
- Skip: 11.3.4, 11.7.
- Squeeze: 11.3.3, 11.3.5, 11.4.3.

- Web (with video)
  - Mitsubishi CO_2 laser
    http://www.mcmachinery.com/products-and-solutions/ex-plus-series/
  - Mitsubishi fiber laser
    http://www.mcmachinery.com/products-and-solutions/nx-f-series/
  - Amada laser cutting systems
    http://www.amada.com/america/laser-cutting-system
  - ABB Robotic laser systems
    https://www.youtube.com/watch?v=7k20Zp5aPjY

Laser



$$E(\mathbf{r}, t) = A(\mathbf{r}, t) \exp\left[j \langle \omega \rangle t - \phi(\mathbf{r}, t)\right]$$

- Amplitude and phase (and frequency) vary w.r.t. time.
- Always more than one frequency exist.

Contents

Content	Time
1. Monochromaticity	10′
2. Coherence - Spatial - Temporal	30′
3. Directionality	20′
4. Brightness	10′
5. Laser speckle	10′
Total:	**80′**

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Monochromaticity

Pure monochromatic light just does not exist.

Causes for frequency fluctuation:
-Amplitude fluctuation (change in pump or cavity loss, <1%)
-Phase fluctuation (vibration, temp. variation, zero-point fluctuation)

Active stabilization: 10-50kHz → 0.1Hz

Q-switched or mode-locked laser: $\Delta\nu_L$ can be 100MHz even 50THz

Narrow $\Delta\nu_L$ (10-100kHz): metrology, coherent applications
Broad $\Delta\nu_L$ (1MHz): other common applications (e.g. DWDM)

DWDM: channel spacing 50 GHz (0.4 nm)

LIGO: 60 mHz linewidth (https://www.ligo.caltech.edu/page/laser)

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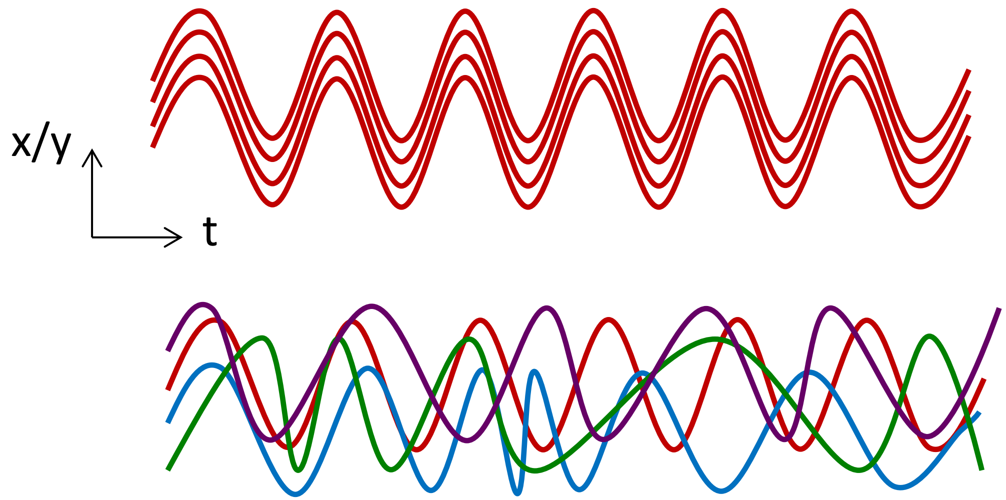
Coherence

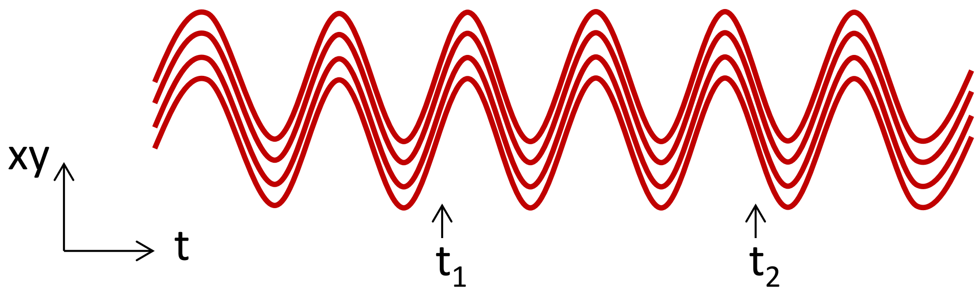
Order, harmony, consistency



Spatial coherence: waves at two lateral points (along x/y direction)
Temporal coherence: waves at two time instances (same position)

Quantification

For stationary beam



r=r$_1$

$\tau = t_2 - t_1 < T$

The 1st-order correlation function:

$$\Gamma^{(1)}(\mathbf{r}_1, \mathbf{r}_1, t_1, t_2) = \Gamma^{(1)}(\mathbf{r}_1, \mathbf{r}_1, \tau) = \langle E(\mathbf{r}_1, t+\tau) E^*(\mathbf{r}_1, t) \rangle$$

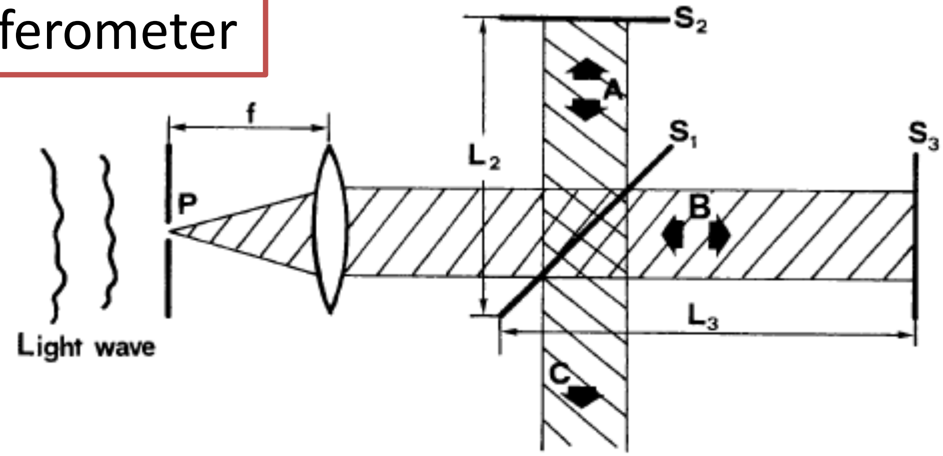
Complex degree of temporal coherence $\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_1,\tau)$:

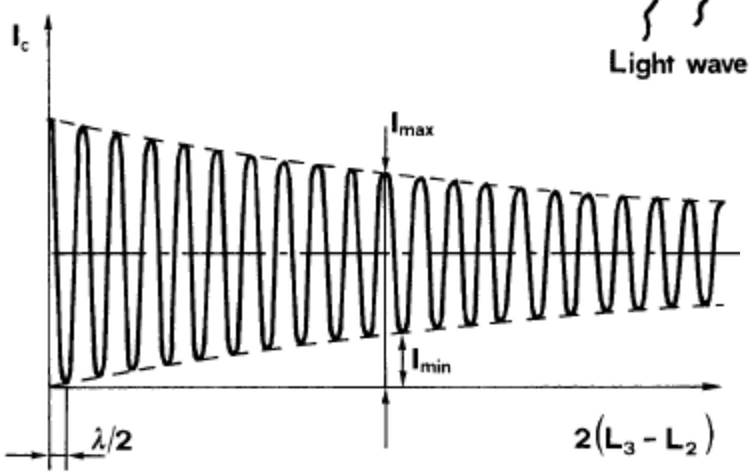
$$\gamma^{(1)} = \frac{\langle E(\mathbf{r}_1,\ t+\tau)\ E^*(\mathbf{r}_1,\ t) \rangle}{\langle E(\mathbf{r}_1,\ t)\ E^*(\mathbf{r}_1,\ t) \rangle^{1/2} \langle E(\mathbf{r}_1,\ t+\tau)\ E^*(\mathbf{r}_1,\ t+\tau) \rangle^{1/2}}$$

- Degree of temporal coherence: $|\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_1,\tau)|$
- Degree of spatial coherence: $|\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_2,0)|$

Measurem’t: Temporal coherence

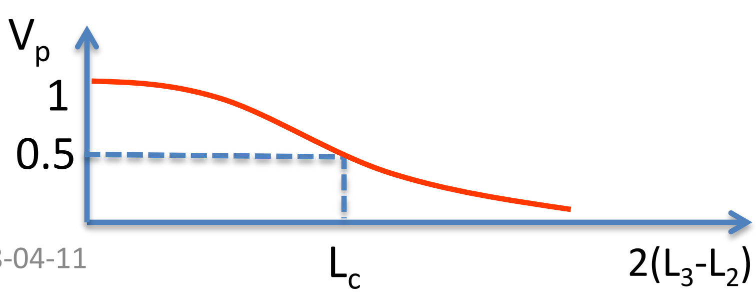
Method: Michelson interferometer





$$V_p(\tau) = \left|\gamma^{(1)}(\mathbf{r}, \mathbf{r}, \tau)\right|$$

where $\tau = \frac{2(L_3 - L_2)}{c}$



$$\tau_{co} = \frac{L_c}{c}$$

Properties: Temporal coherence

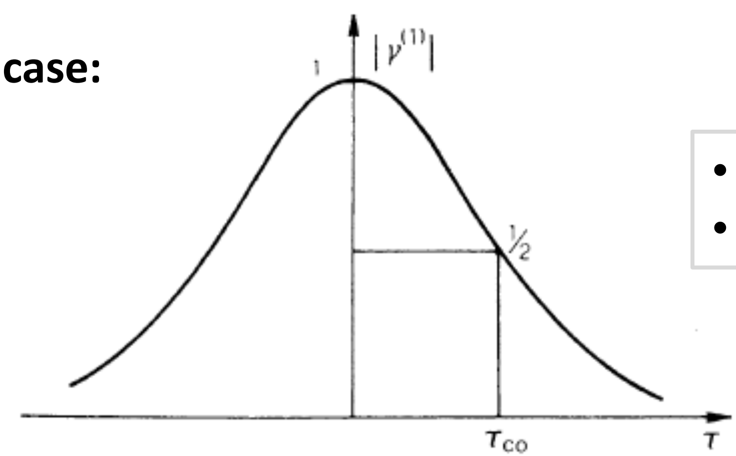
Properties:

1. $\gamma^{(1)}=1$ for $\tau=0$
2. $\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_1,-\tau) = \gamma^{(1)*}(r_1,r_1,\tau)$
3. $|\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_1,\tau)|\leq 1$

Two extremities:

- **Perfect temporal coherence**: $|\gamma^{(1)}|=1$ for $\tau \geq 0$
- **Zero temporal coherence**: $|\gamma^{(1)}|=0$ for any $\tau>0$

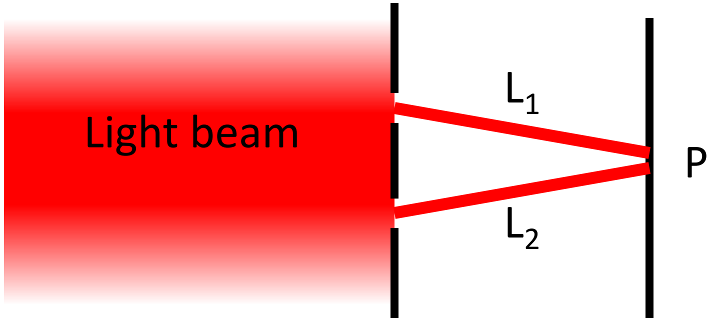
General case:



- Coherence time τ_{co}
- Coherence length $L_c=c\tau_{co}$

Measurem't: Spatial coherence

Method: Young's interferometer



Fringe visibility at P:

$$V_P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$$V_p = \left|\gamma^{(1)}(\mathbf{r}_1, \mathbf{r}_2, 0)\right|$$

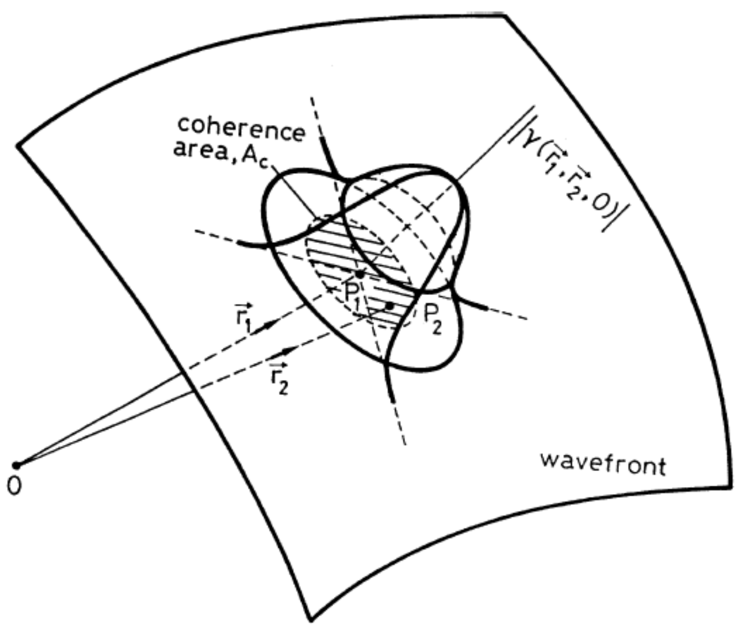
In general $V_p = \left|\gamma^{(1)}(\mathbf{r}_1, \mathbf{r}_2, \tau)\right|$ where $\tau = \frac{L_2 - L_1}{c}$

Properties: Spatial coherence

Complex degree of spatial coherence

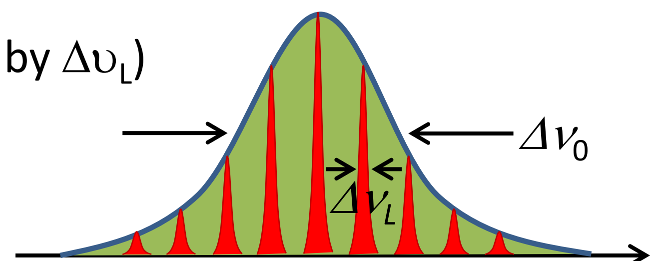
$$\gamma^{(1)} = \frac{\langle E(\mathbf{r}_1,\, t)\, E^*(\mathbf{r}_2,\, t)\rangle}{\langle E(\mathbf{r}_1,\, t)\, E^*(\mathbf{r}_1,\, t)\rangle^{1/2}\, \langle E(\mathbf{r}_2,\, t)\, E^*(\mathbf{r}_2,\, t)\rangle^{1/2}}$$

Again two extremities exist, and in general: $|\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_2,0)| \leq 1$ for $|\mathbf{r}_1\text{-}\mathbf{r}_2|>0$



- Coherence area

More remarks on coherence

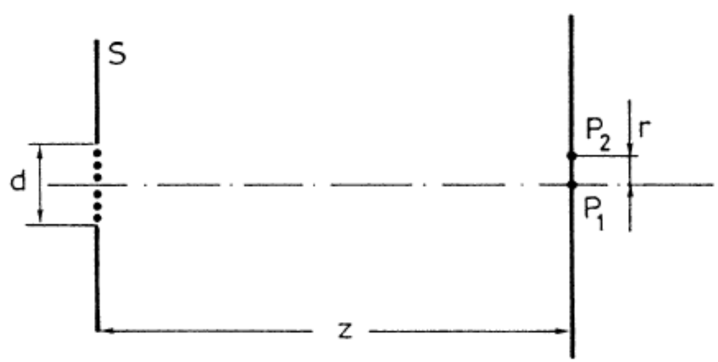


- Temporal coherence and monochromaticity $\tau_{co}\Delta\nu_L \geq \frac{1}{4\pi}$

$\tau_{co}\Delta\nu_L = 1$

Example:
If $\Delta\upsilon_L$=20kHz,
τ_{co}=? L_c=?

- **Single-mode laser**: spatially coherent, τ_{co} limited by $\Delta\upsilon_L$
- **Single-transverse-multi-longitudinal-mode laser**:
  - Spatially coherent, τ_{co} limited by $\Delta\upsilon_0$
  - (if mode-locked: spatially coherent, τ_{co} limited by $\Delta\upsilon_L$)
- **Multi-transverse-multi-longitudinal mode laser**:
  - partial spatial coherence, τ_{co} limited by $\Delta\upsilon_0$
- **Thermal light**: τ_{co}<1ps, spatial correlation ↑ as distance ↑



$$|\gamma^{(1)}| = 0.88 \text{ if } r \approx 0.16\frac{\lambda z}{d}$$

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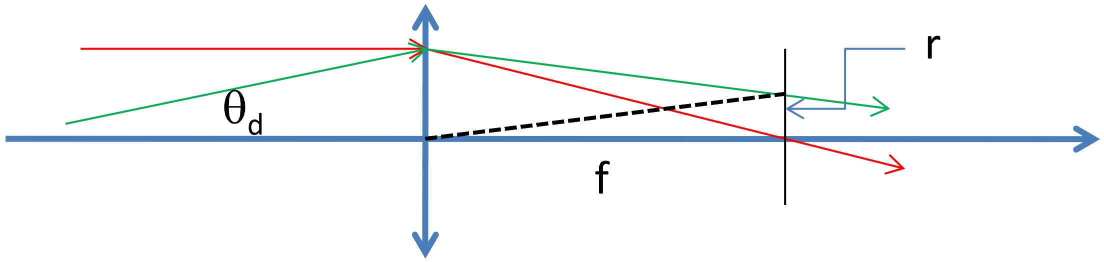
Directionality (divergence)

$\propto$ spatial coherence

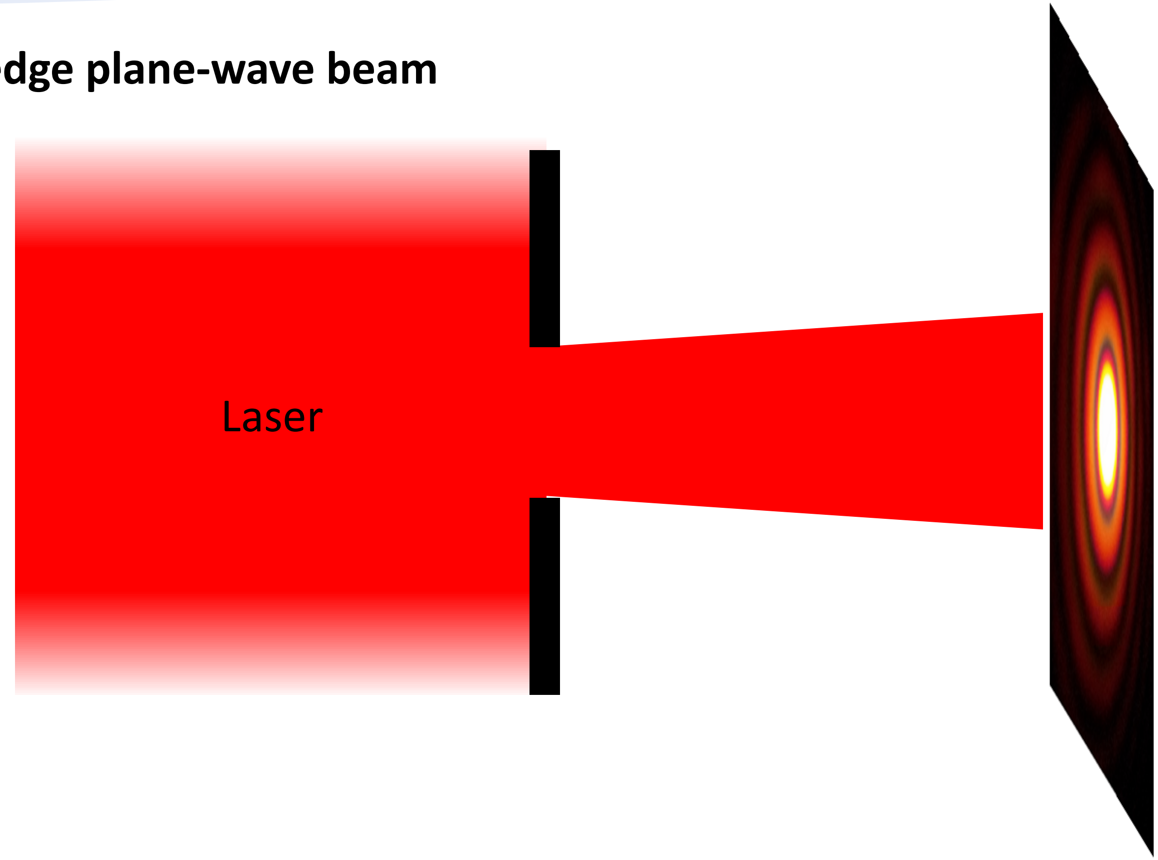
Measurement methods:

- Measure spot size at a very large distance: $\theta_d = W/z$
- Measure beam spot at a lens' focus: $\theta_d = r/f$



θ_d for perfect beam

- **Hard-edge plane-wave beam**



θ_d for perfect beam

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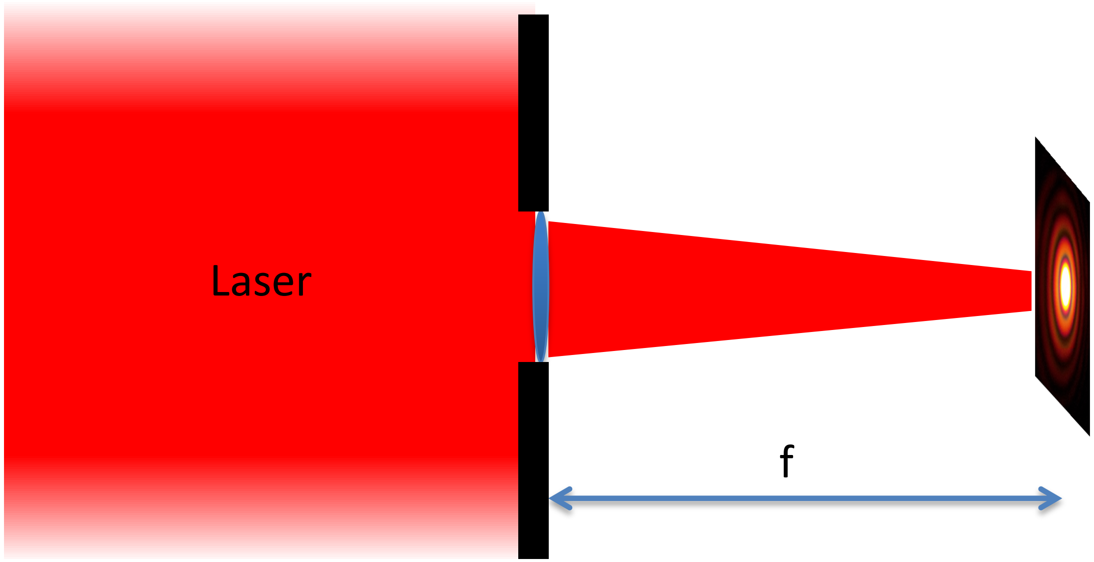
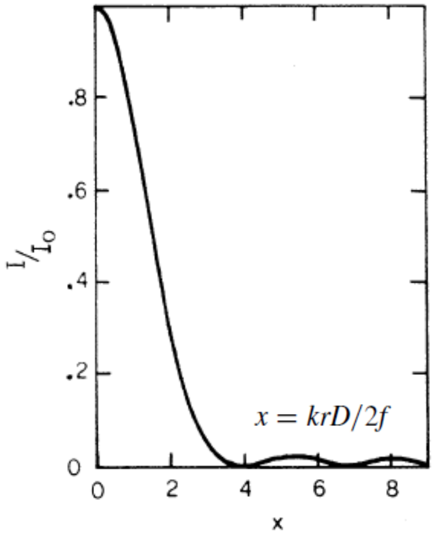


θ_d for perfect beam

- **Hard-edge plane-wave beam**

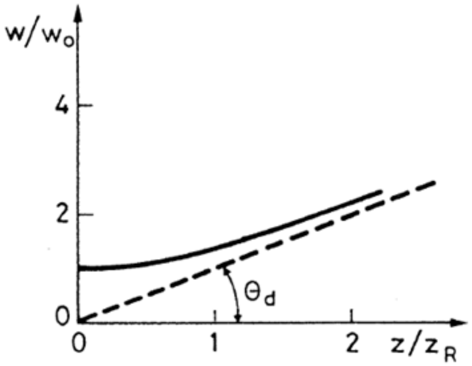
At lens focus



Airy beam: $I = \left[\frac{2 J_0 \left(\frac{krD}{2f} \right)}{\frac{krD}{2f}} \right]^2 I_0$

$$\theta_d = 1.22 \frac{\lambda}{D}$$

D: aperture (lens) diameter



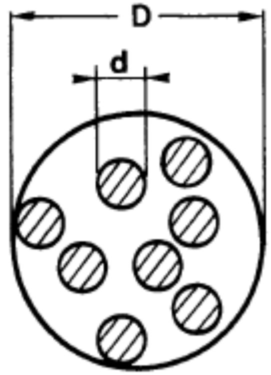
- **Gaussian beam**

$$\theta_d = \frac{w}{z} = \frac{\lambda}{\pi w_0}$$

50% of hard-edge case

Diffraction limited beam: $\theta_d = \beta \frac{\lambda}{D}$ with $\beta \approx 1$

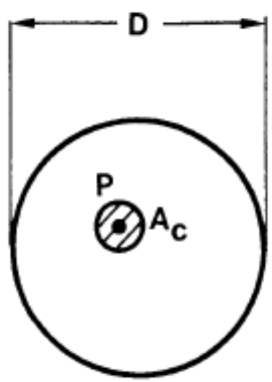
Partial spatial coherence



Un-correlated small beams: $\theta_d = \beta\lambda/d$

Correlated small beams: $\theta_d = \beta\lambda/D$

General case:



Ac: coherence area

$\theta_d = \beta\lambda/D_c$

M^2 factor

Eliminates ambiguity of beam-diameter definition

Simplified definition: Quality of a general beam compared to a Gaussian beam

$M^2 \geq 1$, being 1 for TEM_{00} Gaussian mode.

A **multimode laser** beam propagating along z axis, its beam waist (across x)

$$W^2(z) = W_0^2 + M^4 \frac{\lambda^2}{\pi^2 W_0^2}(z - z_0)^2$$

As $z \to \infty$

$$W(z) = M^2 \frac{\lambda}{\pi W_0}(z - z_0)$$

Hence

$$\theta_d = \frac{W(z)}{z - z_0} = M^2 \frac{\lambda}{\pi W_0}$$

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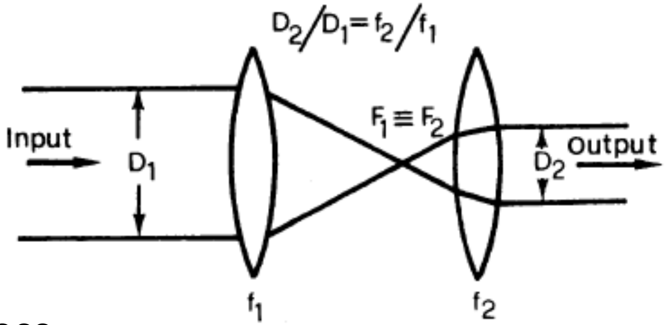
Brightness

- Brightness (power per unit area per solid angle) ≠ Irradiance (power per unit area)
- Proportional to the max peak intensity achievable by focusing a beam

$$B = \frac{P}{A\Omega}$$

P: power
A: area
Ω: Solid angle (steradian)



- **Cause:** Coherence → directionality → brightness

Example:
- Ar laser (TEM_{00}, 1W, λ=514nm): B = 4P/ λ^2 = **1.6×10^9**W/cm^2·sr
- Lamp (10W output, examined at λ=546nm): B = **95**W/cm^2·sr

- **Use:** Material processing

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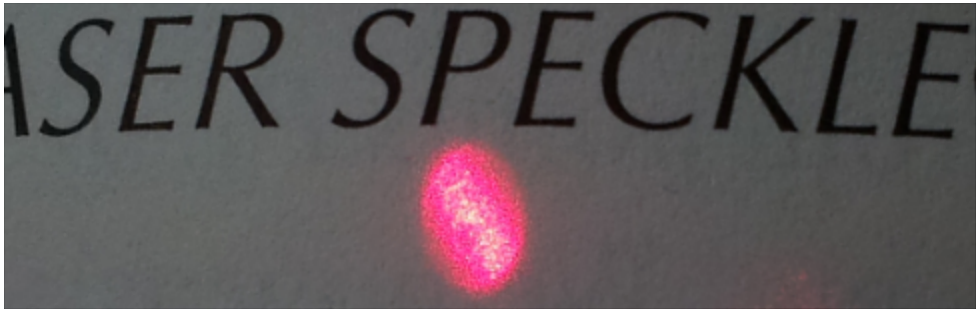
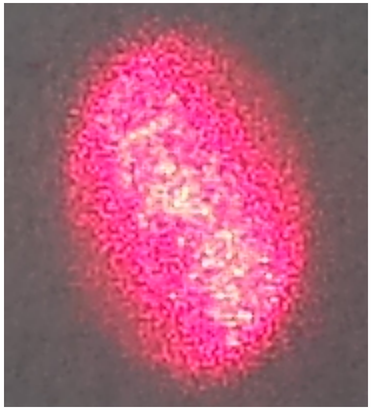
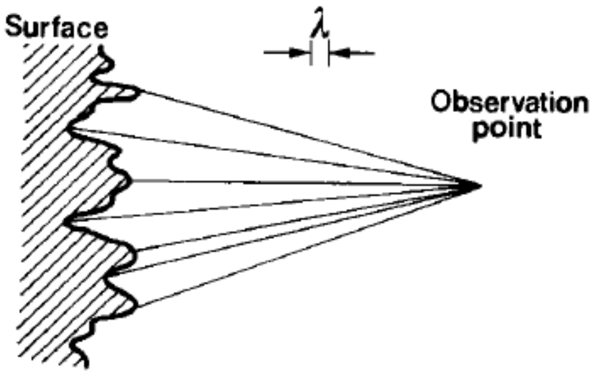
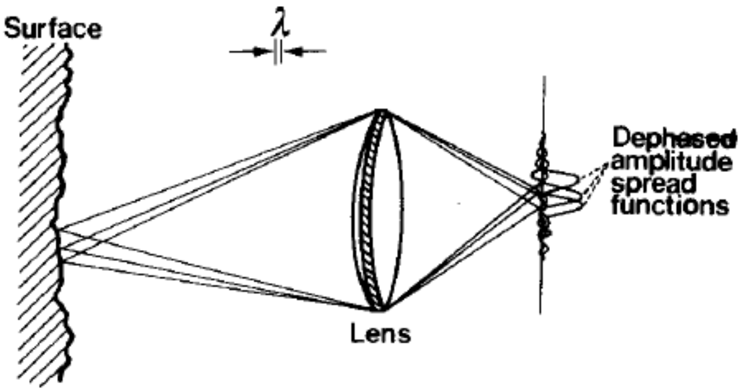
Laser speckles





Cause: high degree of laser light coherence

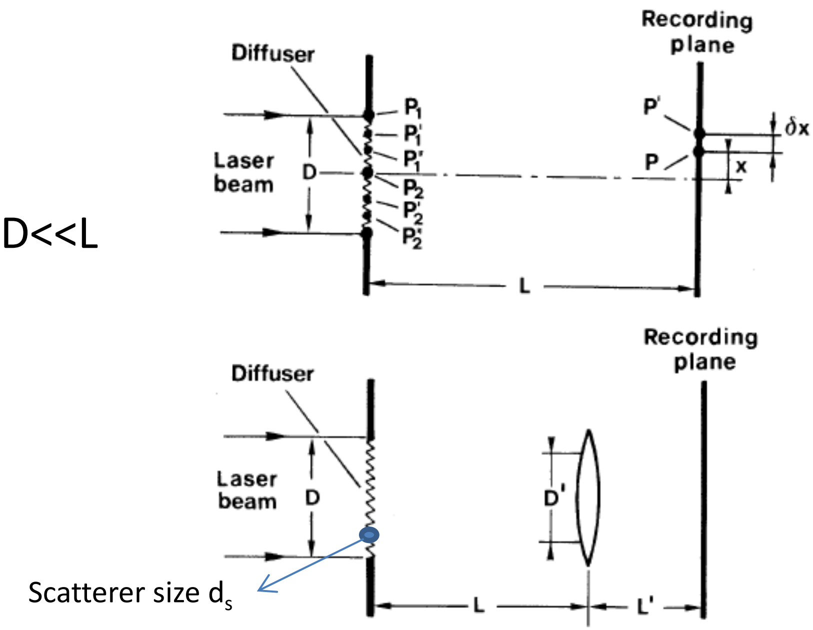




Calculation and impact

- **Grain size d_g**

D<<L



$d_g = 2\lambda L/D$

if $\frac{2\lambda L}{d_s} \geq D'$

$d_g = 2\lambda L'/D'$

- **Impact:** Limits the image resolution of an object illuminated with laser (speckle noise rather than diffraction limit)

- **Use:** Sensors (stress, vibrations)

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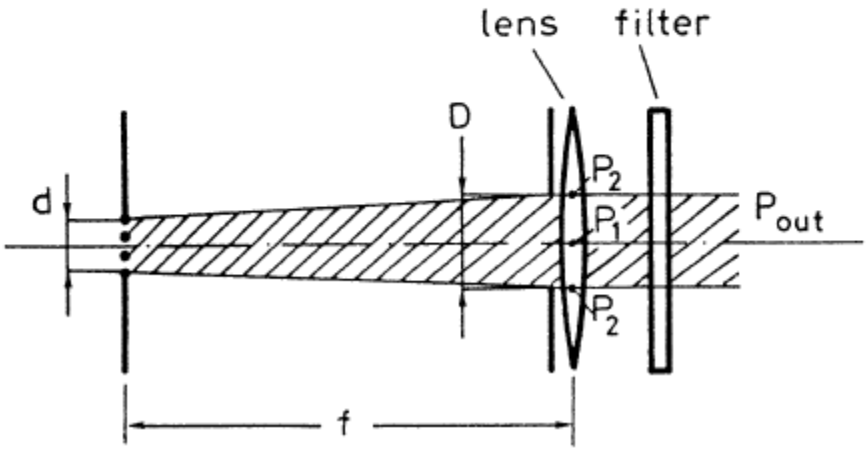
One more slide

- **Laser v.s. Thermal light**



10W → 10^{-18}W

Cobolt 04-01 Series

	Twist™	Blues™	Calypso™	Fandango™	Samba™	Jive™	Mambo™	Flamenco™	Rumba™
Wavelength (nm)	457.0 ± 0.3	473.0 ± 0.3	491.5 ± 0.3	514.4 ± 0.3	532.1 ± 0.3	561.2 ± 0.3	593.6 ± 0.3	659.6 ± 0.3	1064.2±0.6
Available Power Levels (mW)	25 50	25 50	25 50 75 100	25 50 100 150	25 50 100 150 200 300 400	25 50 100 150 200	25 50 100	100	400
Noise, 20 Hz - 20 MHz (pk-pk)	< 2% , typical < 1.5%		< 3%	< 2% , typical < 1.5%			< 3%	< 1 %	
Noise, 20 Hz - 20 MHz (rms)	< 0.25% , typical < 0.15%		< 0.3%	< 0.25% , typical < 0.15%			< 0.3%	< 0.1%	
Long-term power stability (8 hrs ± 3°C)	< 2%		< 3%	< 2%			< 3%	< 2%	
Beam divergence (full angle, mrad)	< 1.2						< 1.3	< 1.5	< 1.6
Spatial mode (TEM_{00})	M² < 1.1								M² < 1.2
Beam diameter at aperture (µm)	700 ± 50								1000 ± 50
Spectral linewidth (FWHM)	< 1 MHz								
Wavelength stability (after warm-up)	2 pm over ± 2 °C and 8 hrs								
Beam symmetry at aperture	>0.95 : 1								
Beam pointing stability (over 10-40°C)	< 10 µrad / °C , typical 5 µrad / °C								
Polarization ratio (linear, vertical)	> 100:1								
Total system power consumption	< 35 W, typical < 15 W								
Operating temperature	10-40°C								
Maximum laser head baseplate temp.	50 °C								
Heat sink thermal resistance Recommended	0.6 K/W or 0.4 K/W *							0.4 K/W	
Laser head dimensions [mm] [inches]	102 x 60 x 40 4.0 x 2.4 x 1.6								
Controller dimensions [mm] [inches]	190 x 72 x 28 7.6 x 2.9 x 1.1								
Communication	RS-232 or USB								
Model number structure			CDRH/CE (key-switch for on/off)				OEM (auto-start mode)		
	RS-232 Controller		wavel-04-xy-pwr-500				wavel-04-xy-pwr-600		
	USB Controller		wavel-04-xy-pwr-700				wavel-04-xy-pwr-800		
Warranty	24 months								