Ensuring Persistent Content in Opportunistic Networks via Stochastic Stability Analysis

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The emerging device-to-device communication solutions and the abundance of mobile applications and services make opportunistic networking not only a feasible solution, but also an important component of future wireless networks. Specifically, the distribution of locally relevant content could be based on the community of mobile users visiting an area, if long term content survival can be ensured this way. In this paper we establish the conditions of content survival in such opportunistic networks, considering the user mobility patterns, as well as the time users keep forwarding the content, as the controllable system parameter.

We model the content spreading with an epidemic process, and derive a stochastic differential equations based approximation. By means of stability analysis we determine the necessary user contribution to ensure content survival. We show that the required contribution from the users depends significantly on the size of the population, that users need to redistribute content only in a short period within their stay, and that they can decrease their contribution significantly in crowded areas. Hence, with the appropriate control of the system parameters, opportunistic content sharing can be both reliable and sustainable.

CCS Concepts: • **Mathematics of computing** → *Stochastic differential equations*; *Markov processes*; • **Networks** → *Network performance modeling*; *Network performance analysis*; *Mobile ad hoc networks*;

Additional Key Words and Phrases: Opportunistic networks, content sharing, mobility, stochastic epidemic modeling

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1 INTRODUCTION

Smart mobile hand-held devices with high quality screens and various sensors lead to the extreme popularity of mobile applications. Traditionally, these mobile applications use the cellular infrastructure, however, the emergence of standardized device-to-device communication solutions in licensed [4] as well as in unlicensed spectrum [7], the cost of accessing the fixed infrastructure [46], and the expected spectrum scarcity [10] open up the way towards opportunistic networks, where mobile services are provided without infrastructure support [45]. While opportunistic communication has the clear advantage of spectrum efficiency and independence from the fixed infrastructure,

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the feasibility and viability of the solution depends on two fundamental questions: can the community of the mobile users ensure the required application performance (e.g., connectivity or content spreading), and is the required contribution from the users low enough, so that they are willing to participate?

In this paper we address a specific kind of opportunistic networks where content is shared for location based services. The service is targeted to mobile users in urban areas, and can aim for spreading information, such as of local news, tourist information, transportation schedule or traffic alerts [45], or for forming opportunistic social networks in transient communities [21]. In these systems the dynamically changing user community provides a virtual storage system, and the scheme is feasible if content can survive in the area for a long time. We approach the question of feasibility utilizing the framework of stochastic epidemic modeling [44]. For epidemic models, the main concern is to find conditions under which an infection introduced into the community will develop into a large outbreak, and if it does, to find the conditions under which it will become endemic—able to persist in the population: a direct analogy of the content survival.

We introduced the epidemic model of content sharing in [36, 37], including the Markovian system model and its stochastic differential equations (SDE) based approximation. We demonstrated that this stochastic framework is flexible enough to capture the dynamics of content sharing in a changing user population across diverse mobility scenarios, while abstracting away the intricate heterogeneity of user contact patterns. In this paper we extend the modeling, analysis and validation of the content spreading scheme by focusing on the scheme's operation at the limit of feasibility.

Our results show that the tractable epidemic model approximates well the main operating regions of the content sharing system, and can be used to tune the system parameters such that long-time content sharing becomes feasible. The results demonstrate the viability of opportunistic content sharing, showing that the contribution required from the individual mobile users is rather limited, but also prove the importance of adaptive schemes that can minimize the user contribution under changing population size.

The remainder of this paper is structured as follows. Section 2 reviews related work with respect to both applications and modeling. In Section 3 we describe the opportunistic content sharing scheme and present preliminaries on the system model. Section 4 gives detailed analysis of the system stability, with the conditions for content extinction and persistence. We validate the proposed model and the analytical results by comparing the results of trace-based simulation of content spreading with the corresponding model. Simulation results and discussion are provided in Section 5. Finally Section 6 provides our final remarks.

2 RELATED WORK

This paper addresses the feasibility of location-aware opportunistic content sharing. For assessment, we utilize epidemic models of information spreading and therefore position our work relative to both the application and the modeling approach.

A number of platform and system proposals for sharing ephemeral, localized content have recently emerged, mainly in the vehicular networking field. Most relevant to our work, as they consider information sharing in the wireless domain, are the *hovering information* [8, 49], and *floating content* [20, 35] concepts, and the vehicular ad hoc mesh network specific solutions [26, 30]. In [8], the authors define resource efficient algorithms for information sharing and evaluate information survivability and availability via simulations, [30] investigates, by means of simulations, content longevity in a distributed vehicular network, while [26] implements an adaptive algorithm to ensure content survival in a changing environment.

The studies on floating content model the feasibility of information spreading utilizing percolation theory, considering both static [18] and dynamic networks [19], and [33] follows a similar approach to model disease propagation in social networks. Percolation theory provides tractable models, however, also has some limitations. While being able to accurately predict the final-state—the expected size of the epidemic [33], it does not capture the temporal dynamics of the content spreading process and requires the knowledge of contact patterns between nodes.

In static computer networks, graph-based models of information spreading are proposed to study propagation of computer viruses over static graph structures [11, 48]. Representing the spread of epidemics with a continuous time markov chain (CTMC), these works show that the epidemic spread can be characterized through the spectral radius of the adjacency matrix. There is less understanding on how to model networks where connectivity changes in time [17], like in the case of mobile opportunistic communication. Network models with time-varying adjacency matrix are introduced for example in [38, 41]. This solution allows the derivation of theoretic results for basic graph structures, but becomes untractable for large systems, and requires the exact knowledge of the changes of node connectivity.

An alternative approach, which is the starting point of our modeling procedure, is to model the evolution of information spreading via compartment based models, where each individual belongs to one of the classes—compartments—of susceptible, infected or recovered, and individuals within a compartment are indistinguishable from each other [6], assuming so-called homogeneous mixing. Extensions to include some heterogeneity in the compartment model are suggested in [5, 22, 40]. In compartment models, the system state transitions are as well governed by CTMCs, and the state space grows quickly as the population increases. Therefore, approximate solutions are proposed to evaluate the system performance, based on branching processes [6], ordinary differential equations [5], or fluid approximation [12].

In this paper we follow a different path, and model the information spreading with a diffusion process, described by first order SDEs, using the methodology proposed in [1, 13]. SDEs model short term system randomness, and therefore can capture the behaviour when the population of some of the compartments is small—a necessity for the accurate modeling of the survival of an epidemic. They are also tractable under large populations, and flexible enough to handle the system parameters of the considered opportunistic networking. We assume homogeneous mixing, which has been validated for random, homogeneous networks in [5, 11, 22], and for the specific case of opportunistic networking in [15].

The utilization of stochastic SDE models present a novel contribution in the networking area. In epidemiological studies, on the other hand, there is a plethora of works employing this approach. Stochastic SIR models have been studied extensively and in various settings, e.g. with delayed infectious periods [44], taking into account population churn [9] or modeling non-linear diffusion coefficients [28, 43]. As SIR models are defined by stochastic SDEs, Lyapunov stability theory [31] is a common tool for exploring the behavior of epidemic spreading.

3 LOCATION-BASED OPPORTUNISTIC CONTENT SHARING

3.1 Content sharing scheme operation

We consider an opportunistic service for sharing geographically localized contents, where in addition to obtaining the content, all users are willing to support further content spreading by contributing some amount of their resources for a limited time. The content is geographically tagged to the region of relevance, and it is considered irrelevant and will not be replicated outside of the boundaries of the locale.

The area of interest is characterized by frequent arrivals and departures of users equipped with mobile devices. Applications on user devices use publish/subscribe services [16] to publish new contents, or to find peers within communication range and download or forward contents. That is, initially, a single user publishes a content item which is then shared relying exclusively on opportunistic mechanisms, without the support of an infrastructure. The opportunistic service is feasible if the content persist in the area for a very long time.

As the basic forwarding principle, opportunistic communication schemes usually assume some flavor of epidemic routing [47], where any node that has a content item forwards it to any encountered one that has not obtained the content yet. While this principle guarantees the largest spread of content in the network, it also imposes a high resource overhead. In this study, we consider a modified scheme with time-limited forwarding [50], to save the resources of the nodes and to facilitate the propagation of multiple contents. Under time-limited forwarding each node starts a timer when it receives the content item and continues forwarding the content until the timer expires. While the mobility of the users, the resulting population size and contact pattern is determined by the environment, the forwarding time is a system parameter that can be tuned and optimized to ensure content survival with minimum user contribution.

3.2 The epidemic model of localized content sharing

We assume that content items are shared independently from each other, and therefore we restrict our analysis to a single content item. To keep the model tractable even under large population, we utilize a stochastic *susceptible-infected-recovered* (SIR) epidemic model to describe the content sharing process. In the SIR model, nodes in the area of content sharing can be in one of three classes: *susceptible* (S) are the nodes that have not obtained the content item, *infected* (I) are the nodes currently holding (we will also use the term *carrying*) and forwarding the content, and *recovered* (R) are the nodes that have the content but already stopped forwarding. This system model is commonly referred to as a *compartmental* model, since each node in the population belongs to one of the three classes—compartments; nodes in one compartment are indistinguishable from one another with respect to their infectious status and contact pattern.

In [36, 37] we have validated that location based content sharing can be modeled by such an SIR model, by abstracting away the exact mobility and contact patterns of the nodes. The result is a black-box system model where nodes are fed into the area of content sharing and are released after some time, characterized by the arrival process and by the sojourn time distribution. Inside the area nodes interact according to the homogeneous-mixing model [5], which implies that the probability of two nodes establishing contact is equal for any two nodes. Consequently, interactions can be characterized by the contact rate representing the number of peers each node meets in a unit time.

As demonstrated in [15, 37], a wide range of human mobility scenarios allow a Markovian SIR model of location based content sharing. Susceptible nodes arrive to the considered area according to a Poisson process with rate λ, and stay in the area for an exponentially distributed sojourn time with mean $T_S = 1/\mu$. Once infected, nodes forward the content during an exponentially distributed forwarding time with mean value $T_F = 1/\gamma$. Following a basic epidemic routing scheme, all nodes fully participate in the spreading process; thus each node forwards the content to any susceptible node it encounters during its infectious period. We assume that the contact time is long enough for forwarding the entire content. Contacts happen according to the homogeneous-mixing model, [5], where the inter-contact times that a node experiences are independent and exponentially distributed, and are characterized by the mean contact rate c_N for an average population size N. Note that $N = \frac{\lambda}{\mu}$ by Little's formula. The contact rate c_N depends on the node mobility inside the area, and is one of the input parameters of the model.

Denote by $S(t)$, $I(t)$ and $R(t)$ the number of susceptible, infected and recovered nodes, respectively, at time t, and the total number of nodes by $N(t) = S(t) + I(t) + R(t)$. The system state is represented by the vector $X(t) = [S(t), I(t), R(t)]$, and the state changes can be described by the Markovian

SIR model shown on Fig. 1. The initial system state is $X(0) = [S_0, 1, 0]$, since at the time when the content is first published there is a single infected node in the area and $S(0) = S_0$ susceptible nodes. Since every new arrival is initially susceptible, compartment S observes arrivals with intensity λ. As the infection spreads, nodes move from S to I with the state dependent intensity $\beta(X(t))$, and upon the completion of their infectious period, nodes recover with a rate $\gamma I(t)$, moving to and staying in compartment R. Finally, nodes may leave the system from any compartment with intensity $\mu S(t)$, $\mu I(t)$ and $\mu R(t)$.

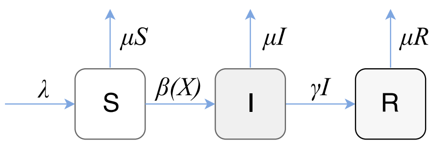


Fig. 1. Diagram of SIR compartments with transition rates.

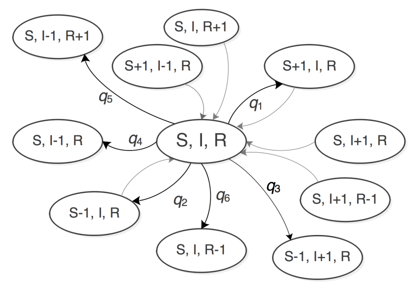


Fig. 2. A section of the Markov Chain with transitions from the state $X = [S, I, R]$.

To complete the definition of the compartmental model in Fig. 1, we next derive the state-dependent infection rate $\beta(X(t))$, building on the contact rate c_N. As it was demonstrated in [15], the instantaneous contact rate $c(t)$ is a linear function of the instantaneous population for a large range of population sizes, that is, $c(t) = c_N \frac{N(t)}{N}$. Given that for the infection to happen a susceptible node has to meet an infected one, the infection rate depends on the number of infected and susceptible nodes at time t as well as a time dependent contact rate $c(t)$ via $\beta(X(t)) = \beta(t) = S(t)\frac{I(t)}{N(t)}c(t)$. Let us introduce the normalized contact rate $\beta = \frac{c_N}{N}$, and we arrive at $\beta(t) = \beta S(t)I(t)$. That is, the infection rate at time t depends on the current number of infected and susceptible nodes and a scenario-dependent constant parameter β.

The spreading process is described by a three-dimensional CTMC, where the system state is defined by the vector of the number of nodes in each compartment, i.e. $X(t) = [S(t), I(t), R(t)]$. Fig. 2 depicts a section of the chain. There are six possible transitions from a state. Each transition is characterized by the state transition rate $q_j = q_j(X(t))$, and by the vector of state changes $\Delta X_j = [\Delta S_j, \Delta I_j, \Delta R_j], j = \{1, ..., 6\}$. With rate q_1 a new arrival occurs, with rates q_2, q_4, q_6 a susceptible, infected, or recovered node, respectively, leaves the system whereas the rate q_3 corresponds to an infection and q_5 to recovery. Consequently, we can define the CTMC by the vector of transition rates as $\mathbf{q} = [q_j]$ and the matrix of state changes $\nu = [\nu_j]$, where the $j-$th column corresponds to the transition j, i.e. $\nu_j = \Delta X_j^T$, as

$$q(X(t)) = [\lambda, \mu S(t), \beta(X(t)), \mu I(t), \gamma I(t), \mu R(t)], \tag{1}$$

$$\nu = [\nu_{ij}] = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}. \tag{2}$$

3.3 SDE model as an approximation of the Markov Chain

The transient behavior of the CTMC given in Fig. 2 is completely described by the transition rates $q(X(t))$ and a set of associated (forward and backward) Kolmogorov equations describing the time-evolving probability distribution of the system state. However, for large systems these equations often become intractable and therefore a common approach of tackling the problem is to approximate the Markov process with a diffusion process. We follow the steps suggested in, for example, [1, 13].

Let $X_i(t), i = \{1, 2, 3\}$, corresponding to $S(t)$, $I(t)$, and $R(t)$, denote the number of nodes in the i–th compartment. Consider the possible state changes $\Delta X(t, \tau)$ in the interval $(t, t + \tau]$. For τ sufficiently small, the state can not change significantly, and the transition rates can be considered constant. Let $K_j(X(t), \tau), j = \{1, ..., 6\}$ be the number of events that occur in the observed interval $(t, t + \tau]$, corresponding to the transition of type j. Since each of these events changes the size of population X_i by v_{ij}, the number of nodes in a compartment i at time $(t + \tau)$ will be

$$X_i(t, t + \tau) = X_i(t) + \sum_{j=1}^{6} v_{ij} K_j(X(t), \tau), \quad i = \{1, 2, 3\}. \tag{3}$$

The population change events occurring in the short interval $(t, t+\tau]$ can be considered independent of each other, and consequently, $K_j(X(t), \tau)$ can be regarded as independent Poisson random variables, associated with distinct events. For sufficiently large expected value, $K_j(X(t), \tau)$ can be further approximated by a normal random variable $\mathcal{N}_j$ with a mean and variance equal to $q_j(X(t))\tau$, inherited from the Poisson distribution. Consequently, the state changes from (3) get the form

$$\begin{aligned} \Delta X_i(t, t + \tau) =& X_i(t, t + \tau) - X_i(t) = \\ =& \sum_{j=1}^{6} v_{ij} q_j(X(t))\tau + \sum_{j=1}^{6} v_{ij} \sqrt{\tau q_j(X(t))}\, \mathcal{N}_j(0, 1), \quad i = \{1, 2, 3\}. \end{aligned} \tag{4}$$

The difference in (4) can be regarded as an Euler-Maruyama approximation of the Itô integral, giving a first order SDE approximation of the Markov process

$$dX_i(t) = \sum_{j=1}^{6} v_{ij} q_j(X(t))dt + \sum_{j=1}^{6} v_{ij} \sqrt{q_j(X(t))} dW(t), \quad i = \{1, 2, 3\}. \tag{5}$$

Finally, by substituting the transition rate and population change values from (1) and (2), we arrive at the system of stochastic differential equation for $X(t) = [S(t), I(t), R(t)] \in \mathbb{R}^3$

$$\begin{cases} dS(t) = [\lambda - \beta(X(t)) - \mu S(t)]dt + g_1 dW(t) \\ dI(t) = [\beta(X(t)) - (\mu + \gamma)I(t)]dt + g_2 dW(t) \\ dR(t) = [\gamma I(t) - \mu R(t)]dt + g_3 dW(t) \\ X(0) = [S(0), I(0), R(0)]^T, \end{cases} \tag{6}$$

where $W(t)$ is a vector of six independent Wiener processes and g_i, $i = \{1, 2, 3\}$ is the i-th row of the matrix

$$G(X(t), t) = \begin{bmatrix} \sqrt{\lambda} & \sqrt{\mu S(t)} & -\sqrt{\beta(X(t))} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\beta(X(t))} & -\sqrt{\gamma I(t)} & \sqrt{\mu I(t)} & 0 \\ 0 & 0 & 0 & \sqrt{\gamma I(t)} & 0 & \sqrt{\mu R(t)} \end{bmatrix}. \tag{7}$$

Denoting by $F = F(X(t), t)$ the function governing the deterministic part of system (6)

$$F(X(t), t) = \begin{bmatrix} \lambda - \mu S(t) - \beta(X(t)) \\ \beta(X(t)) - (\mu + \gamma)I(t) \\ \gamma I(t) - \mu R(t) \end{bmatrix}, \tag{8}$$

we get a compact form of our system model

$$dX(t) = F(X(t), t)dt + G(X(t), t)dW(t). \tag{9}$$

Note that to establish the SDE model (9), we first assumed that the system size is large enough to allow for diffusion approximation. Then, we applied the Euler-Maruyama truncation method on the higher orders of the normal random variable; this method is known to have the lowest order of convergence to the original Markov process [25]. Although more accurate integration schemes are available, multivariate systems with higher order terms quickly become too complex to analyse, thus we restrict ourselves to a stochastic model of the first order. For details on the necessary assumptions and the tightness of the diffusion approximation we refer to [13] and [25].

For a given initial state, the solution of (6) can be found in a form of probability distribution given by Fokker-Planck formula [42] in the case when such distribution exists. However, in this paper we are seeking answer to a more fundamental question, that in turn would help the design of opportunistic content sharing systems: under what circumstances does the system ensure that the content survives in the area and can be shared for a long time.

4 ANALYSIS OF THE SIR MODEL

Consider the deterministic part of the system (6). By solving the system of equations $dX(t)$=$F(X(t), t)dt \equiv \mathbf{0}$, it can be easily seen that the deterministic system yields two solutions: the trivial one $X(t) = [\frac{\lambda}{\mu}, 0, 0]$, when the system exhibits content extinction, and the endemic equilibrium $X^* = [S^*, I^*, R^*] = \left[\frac{\mu+\gamma}{\beta}, \frac{\lambda}{\mu+\gamma} - \frac{\mu}{\beta}, \frac{\lambda\gamma}{\mu(\mu+\gamma)} - \frac{\gamma}{\beta}\right]$, corresponding to content persistence. The necessary condition which the system parameters have to satisfy for content persistence in the deterministic case is given by the epidemic threshold [2]

$$\frac{\lambda\beta}{\mu} > \mu + \gamma. \tag{10}$$

In the followings we derive the conditions of content persistence in the stochastic system.

4.1 Existence of the global positive solution

First, we need to know that the system does have a unique solution for any given initial state. Generally, for proving uniqueness of solutions, the system has to satisfy the Lipschitz continuity (global or local, which is sufficient in some cases) and linear growth condition. Roughly speaking, the Lipschitz condition guarantees uniqueness while the linear growth condition rules out finite escape times. The first condition guarantees that $F(X(t), t)$ and $G(X(t), t)$ do not change faster than $X(t)$, which implies the continuity of these functions. The second condition bounds the growth of the solution, ensuring that the solution cannot "explode"—reach infinite values in a finite time. Such explosion can occur when $F(X(t), t)$ and $G(X(t), t)$ are not bounded [24, 25]. Regarding the system (6), both conditions may get violated when the trajectories of $S(t)$ and $I(t)$ converge to 0.

This prompts us to consider a modified model that fulfills the uniqueness and stability conditions. To obtain the new SDE model, we start from the original stochastic SIR model and substitute its stochastic components with a diffusion that ensures Lipschitz continuity. We choose to focus on the region around the deterministic equilibrium and therefore form the new model by substituting the matrix G with matrix $\widehat{G}$, such that $\widehat{G}(X^*) = G(X^*)$:

$$\widehat{G}(X(t)) = \begin{bmatrix} \sigma_1 & \sigma_2 S(t) & -\sigma_3 S(t)I(t) & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 S(t)I(t) & -\sigma_4 I(t) & \sigma_5 I(t) & 0 \\ 0 & 0 & 0 & \sigma_4 I(t) & 0 & \sigma_6 R(t) \end{bmatrix}, \tag{11}$$

where the positive constant coefficients $\sigma_i, i = \{1, ..., 6\}$ are elements of the matrix:

$$\begin{aligned} \Gamma &= \begin{bmatrix} \sigma_1 & \sigma_2 & -\sigma_3 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & -\sigma_4 & \sigma_5 & 0 \\ 0 & 0 & 0 & \sigma_4 & 0 & \sigma_6 \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{\lambda} & \sqrt{\mu/S^*} & -\sqrt{\beta/(S^* I^*)} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\beta/(S^* I^*)} & -\sqrt{\gamma/I^*} & \sqrt{\mu/I^*} & 0 \\ 0 & 0 & 0 & \sqrt{\gamma/I^*} & 0 & -\sqrt{\mu/R^*} \end{bmatrix}. \end{aligned} \tag{12}$$

The approximate SDE system thus becomes

$$\begin{cases} dS(t) = [\lambda - \beta(X(t)) - \mu S(t)]dt + \sigma_1 dW_1(t) + \sigma_2 S(t) dW_2(t) - \sigma_3 S(t) I(t) dW_3(t) \\ dI(t) = [\beta(X(t)) - (\mu + \gamma) I(t)]dt + \sigma_3 S(t) I(t) dW_3(t) - \sigma_4 I(t) dW_4(t) + \sigma_5 I(t) dW_5(t) \\ dR(t) = [\gamma I(t) - \mu R(t)]dt + \sigma_4 I(t) dW_4(t) + \sigma_6 R(t) dW_6(t)(t), \end{cases} \tag{13}$$

or, in a shorter representation, analogously to (9):

$$dX(t) = F(X(t), t)dt + \widehat{G}(X(t), t) dW(t). \tag{14}$$

The existence of a global positive solution of stochastic SIR models like (14) has been proven in [28] and [43].

4.2 Sufficient conditions for content extinction

The first case we consider is when the content spreading is likely to continue for a limited time only, that is, the content *becomes extinct* shortly after being introduced in the mobile population. Since we are dealing with a stochastic system, the content eventually always dies out. The following theorem gives the conditions under which this happens almost surely in some *finite* time.

THEOREM 4.1. *Consider stochastic system (13) with initial conditions $X(0) = [S(0), I(0), R(0)] \in \mathbb{R}^3$. The solution of this system obeys*

$$\limsup_{t \to \infty} \frac{\log I(t)}{t} \le \frac{\lambda\beta}{\mu} - (\mu + \gamma) - \tfrac{1}{2}\left((\tfrac{\sigma_3 \lambda}{\mu})^2 + \sigma_4^2 + \sigma_5^2 \right). \tag{15}$$

If the following holds

$$\frac{\lambda\beta}{\mu} < (\mu + \gamma) + \tfrac{1}{2}\left((\tfrac{\sigma_3 \lambda}{\mu})^2 + \sigma_4^2 + \sigma_5^2 \right) \tag{16}$$

then $I(t)$ tends to zero exponentially almost surely. In other words, the number of infected nodes tends to zero with probability one, and the content becomes extinct almost surely in a finite time.

PROOF. As shown in [24], a nonlinear stochastic system is always stable in probability if the linearized system is asymptotically stable, where linearization is performed by dropping terms of the second and higher orders. Therefore we prove the theorem utilizing the following two main results.

First we rely on Lyapunov stability theory that introduces a metric called the *Lyapunov exponent*, which characterizes exponential rate of convergence (if negative) or divergence (if positive) of the linearized system's solution [31].

Second, we utilize the multiplicative ergodic theorem of Oseledets [34]. The theorem states that the necessary and sufficient condition for the almost sure asymptotic stability of the trivial solution of the nonlinear system is that the largest Lyapunov exponent of the linearized system is negative. Moreover, according to [32] (p. 119), if the largest Lyapunov exponent is negative, all sample paths of the solution will tend to the equilibrium position exponentially fast. This property is called *almost sure exponential stability.*

To find the largest Lyapunov exponent of the linearized system, we transform the system (13) by the change of variables $y_1 = S - \frac{\lambda}{\mu}$, $y_2 = I$, $y_3 = R$, which centers the system on the disease-free equilibrium:

$$\begin{cases} dy_1(t) = \left[-\beta\left(y_1(t) + \frac{\lambda}{\mu}\right)y_2(t) - \mu y_1(t)\right]dt + \sigma_1 dW_1(t) + \sigma_2\left(y_1(t) + \frac{\lambda}{\mu}\right)dW_2(t) - \sigma_3 S(t)\left(y_1(t) + \frac{\lambda}{\mu}\right)y_2(t)dW_3(t) \\ dy_2(t) = \left[\beta\left(y_1(t) + \frac{\lambda}{\mu}\right)y_2(t) - (\mu+\gamma)y_2(t)\right]dt + \sigma_3\left(y_1(t) + \frac{\lambda}{\mu}\right)y_2(t)dW_3(t) - \sigma_4 y_2(t)dW_4(t) + \sigma_5 y_2(t)dW_5(t) \\ dy_3(t) = [\gamma y_2(t) - \mu y_3(t)]\,dt + \sigma_4 y_2(t)dW_4(t) + \sigma_6 y_3(t)dW_6(t). \end{cases} \tag{17}$$

We consider the linear part of the system (17) and drop the notation of time-dependence (for brevity):

$$\begin{cases} du_1 = \left[-\mu u_1 - \frac{\lambda\beta}{\mu}u_2\right]dt + \sigma_1 dW_1 + \sigma_2\frac{\lambda}{\mu}dW_2 + \sigma_2 u_1 dW_2 - \sigma_3\frac{\lambda}{\mu}u_2 dW_3 \\ du_2 = \left[\frac{\lambda\beta}{\mu} - (\mu+\gamma)\right]u_2 + \sigma_3\frac{\lambda}{\mu}u_2 dW_3 - \sigma_4 u_2 dW_4 + \sigma_5 u_2 dW_5 \\ du_3 = [\gamma u_2 - \mu u_3]dt + \sigma_4 u_2 dW_4 + \sigma_6 u_3 dW_6\ . \end{cases} \tag{18}$$

Let us denote the initial state by $u_0 = [u_{10}, u_{20}, u_{30}]$. Observe that the second equation in system (18) depends only on $u_2(t)$. This is a scalar linear SDE and can be solved explicitly (see, for example, equation (4.59) in Kloeden [25]):

$$u_2(t) = u_{20}e^{-a_2 t + \frac{\sigma_3\lambda}{\mu}W_3(t) - \sigma_4 W_4(t) + \sigma_5 W_5(t)} \tag{19}$$

where $a_2 = -\frac{\lambda\beta}{\mu} + (\mu+\gamma) + \frac{1}{2}\left((\frac{\sigma_3\lambda}{\mu})^2 + \sigma_4^2 + \sigma_5^2\right)$. Next, one can solve $u_1(t)$ and $u_3(t)$ from the first and third equation of system (18). For $u_3(t)$ we find

$$u_3(t) = e^{-\left(\mu+\frac{\sigma_6^2}{2}\right)t+\sigma_6 W_6(t)}\left[u_{30} + \int_0^t \gamma u_2(s)e^{\left(\mu+\frac{\sigma_6^2}{2}\right)s-\sigma_6 W_6(s)}ds + \int_0^t \sigma_4 u_2(s)e^{\left(\mu+\frac{\sigma_6^2}{2}\right)s-\sigma_6 W_6(s)}dW_4\right], \tag{20}$$

see [25] and the general form solutions for d-dimensional linear SDEs.

We cannot obtain explicit solutions in these cases, therefore we provide bounds on the Lyapunov exponents. Let $a_3 = \mu + \sigma_6^2/2$:

$$\begin{aligned} u_3(t) =& e^{-a_3 t+\sigma_6 W_6(t)}\left[u_{30} + \gamma u_{20}\int_0^t e^{(a_3-a_2)s+\zeta W'(s)}ds + \sigma_4 u_{20}\int_0^t e^{(a_3-a_2)s+\zeta W'(s)}dW_4(s)\right] \\ =& u_{30}e^{-a_3 t+\sigma_6 W_6(t)} + \gamma u_{20}e^{-a_3 t+\sigma_6 W_6(t)}\int_0^t e^{(a_3-a_2)s+\zeta W'(s)}ds \\ &+ \sigma_4 u_{20}e^{-a_3 t+\sigma_6 W_6(t)}\int_0^t e^{(a_3-a_2)s+\zeta W'(s)}dW_4(s) \\ \le& C_1^{-a_3 t+\sigma_6 W_6(t)} + C_2 e^{\sigma_6 W_6(t)}\cdot\left[te^{-a_3 t} + e^{-a_2 t} + e^{-a_3 t}\right]e^{c\sqrt{2t\log\log t}(\sum_{i=1}^4|\zeta_i|)}. \end{aligned} \tag{21}$$

Above, $\zeta = [\sigma_3\frac{\lambda}{\mu}, -\sigma_4, \sigma_5, -\sigma_6]$, C_1, C_2 and C_3 are some positive constants, and $W' = [W_3, W_4, W_5, W_6]^T$.

Finally, $u_1(t)$ can be written in the following form:

$$\begin{aligned} u_1(t) &= \Phi_{t,0}\left[u_{10} - \frac{\lambda}{\mu}\int_0^t \Phi_{t,0}^{-1}\left(\beta u_2(s) - \sigma_2^2\right)ds + \sum_{i=1}^{3}\Phi_{t,0}^{-1}b^k dW_k(s)\right], \\ \Phi_{t,0} &= e^{-(\mu+\frac{\sigma_2^2}{2})t+\sigma_2 W_2(t)}, \quad b(t) = [\sigma_1, \sigma_2\frac{\lambda}{\mu}, \sigma_3\frac{\lambda}{\mu}u_2(t)]. \end{aligned} \tag{22}$$

From (19)–(22) it can be observed that the largest Lyapunov exponent of the linearized system (18) belongs to u_2 and equals $-a_2$. To fulfill the almost sure exponential asymptotic stability condition, this exponent needs to be negative, which proves the theorem. □

Comparing the sufficient condition of content extinction from Theorem (4.1) with that of the deterministic system given in (10), we see that the region of content extinction is always larger in the stochastic system. Evaluating the effect of the tunable system parameter γ we see that for given mobility parameters λ, μ and β, a large value of γ, that is, short content forwarding time T_F may mean content extinction, while decreased γ will tighten the condition, and can allow content survival.

4.3 Sufficient condition for content persistence

Having explored conditions for content extinction, the next question we pose is: what happens above the epidemic threshold? We expect that, under certain conditions, the system will converge towards an endemic equilibrium, and upon reaching it, continue to fluctuate around the equilibrium level. In this section we establish the conditions on the existence of a stochastically stable endemic equilibrium.

First we provide some introductory definitions and results on stochastic stability.

Definition 4.2. Stochastic stability.
Consider the d-dimensional SDE system

$$dX(t) = F(X(t), t)dt + G(X(t), t) \cdot dW(t), \ t \geq 0, \ X(0) = X_0. \tag{23}$$

The equilibrium solution X^* of the SDE (23) is *stochastically stable* (stable in probability) if for every $\epsilon > 0$ and $s \geq 0$ holds

$$\lim_{X_0 \to X^*} \Pr\left(\sup_{0 \leq s < \infty} \|X_{s,X_0}(t) - X^*\| \geq \epsilon \right) = 0 \tag{24}$$

where $X_{s,X_0}(t)$ denotes the solution of (23) at time $t \geq s$, satisfying $X(s) = X_0$, and $\|\cdot\|$ denotes the Euclidean norm.

Definition 4.3. Lyapunov function and differential operator.
Denote by $C^{2,1}(\mathbb{R}^d \times [0, \infty]; \mathbb{R})$ the family of all non-negative functions $V(X, t)$ defined on $\mathbb{R}^d \times [0, \infty]$ such that they are twice continuously differentiable in X and once in t. Define $\mathcal{L}$ as the differential operator associated with Eq. (23) by

$$\mathcal{L} = \frac{\partial}{\partial t} + \sum_{i=1}^{d} F_i(X, t) \frac{\partial}{\partial X_i} + \frac{1}{2} \sum_{i,j=1}^{d} [G(X, t) G^T(X, t)]_{ij} \frac{\partial^2}{\partial X_i \partial X_j}. \tag{25}$$

If $\mathcal{L}$ acts on a function $V(X, t)$ then

$$\mathcal{L}V(X, t) = V_t(X, t) + V_X(X, t) F(X, t) + \frac{1}{2} Tr[G^T(X, t) V_{XX}(X, t) G(X, t)] \tag{26}$$

where

$$\begin{gathered} V_t = \frac{\partial V}{\partial t}, \quad V_X = \left(\frac{\partial V}{\partial X_1}, \ldots, \frac{\partial V}{\partial X_d} \right) \\ V_{XX} = \left(\frac{\partial^2 V}{\partial X_i \partial X_j} \right) = \begin{pmatrix} \frac{\partial^2 V}{\partial X_1 \partial X_1} & \cdots & \frac{\partial^2 V}{\partial X_1 \partial X_d} \\ \vdots & & \vdots \\ \frac{\partial^2 V}{\partial X_d \partial X_1} & \cdots & \frac{\partial^2 V}{\partial X_d \partial X_d} \end{pmatrix} \end{gathered} \tag{27}$$

and $Tr[\cdot]$ is the trace of a square matrix, i.e. the sum of its diagonal elements. Function $V(X,t)$ is a *Lyapunov function* used to investigate the stability properties of dynamic systems.

The following theorem relates $V(X,t)$ and stochastic stability.

Theorem 4.4. Conditions of stochastic stability [3].
Assume that F and G satisfy the existence and uniqueness assumptions and that they have continuous coefficients with respect to t. Suppose that there exists a positive definite function $V \in \mathbb{C}^{2,1}(U_h \times [0,\infty))$, where $U_h = \left\{X \in \mathbb{R}^d : \|X - X^*\| < h\right\}$ for $h > 0$, such that for all $t \geq 0$ and $X \in U_h$: $\mathcal{L}V(X,t) \leq 0$. Then X^* is the equilibrium solution of (23) and it is stochastically stable.

Now we can state the theorem on the stability of the endemic equilibrium of the approximate opportunistic content sharing SDE model (13).

Theorem 4.5. *Assume that the opposite of the condition for content extinction from Theorem 4.1 holds, that is*

$$\frac{\lambda\beta}{\mu} \geq \mu + \gamma + \tfrac{1}{2}\left((\tfrac{\sigma_3\lambda}{\mu})^2 + \sigma_4^2 + \sigma_5^2\right) \tag{28}$$

and, in addition, the following are satisfied:

i)

$$\sigma_2^2 < 2\mu - \tfrac{2\mu+\gamma}{\mu+\gamma}\beta, \quad \sigma_6^2 < \mu; \tag{29}$$

ii)

$$0 < \varepsilon < \min\left(\frac{\mu^2}{m_1}S^{*2}, \frac{[2\mu(\mu+\gamma) - \rho\gamma^2]^2}{4\mu^2 m_2}I^{*2}, \frac{\rho\mu^2}{2(\mu-\sigma_6^2)}R^{*2}\right) \tag{30}$$

where

$$\begin{aligned} 0 &< \rho < \frac{2(\mu+\gamma) - (\sigma_4^2+\sigma_5^2)}{\sigma_4^2 + \gamma^2/\mu}, \\ m_1 &= \frac{(2\mu-\sigma_2^2)\beta - (2\mu+\gamma)I^*\sigma_3^2}{2\beta}, \\ m_2 &= \frac{2\mu(\mu+\gamma) - (\sigma_4^2+\sigma_5^2)\mu - \rho(\gamma^2+\mu\sigma_4^2)}{2\mu}, \end{aligned} \tag{31}$$

and

$$\begin{aligned} \varepsilon = {} & \frac{\mu}{2\beta m_1}[\beta\sigma_2^2 + (2\mu+\gamma)\sigma_3^2 I^*]S^{*2} + \frac{[2\mu(\mu+\gamma)-\rho\gamma^2][(\rho+1)\sigma_4^2+\sigma_5^2]}{4\mu m_2}I^{*2} + \frac{\rho\mu\sigma_6^2}{2(\mu-\sigma_6^2)}R^{*2} \\ & + \frac{\sigma_1^2}{2} + \frac{2\mu+\gamma}{\beta}(\sigma_4^2+\sigma_5^2)I^*. \end{aligned} \tag{32}$$

Then there exists an endemic equilibrium of (13), and this equilibrium is stochastically stable.

Proof. Define a C^2-function $V : \mathbb{R}_+^3 \to \mathbb{R}_+$ as

$$V(X) = \frac{1}{2}(S - S^* + I - I^*)^2 + \theta\left(I - I^* - I^*\log\frac{I}{I^*}\right) + \frac{\rho}{2}(R-R^*)^2 \tag{33}$$

where θ and ρ are positive constants to be chosen later.

$V(X)$ is positive definite and decrescent, and therefore we aim to prove that $\mathcal{L}V < 0$.

For simplicity, divide (33) into a sum of three functions: $V(X) = V_1(X) + V_2(X) + V_3(X)$, where

$$\begin{aligned} V_1(X) &= \frac{1}{2}(S - S^* + I - I^*)^2, \\ V_2(X) &= \theta \left(I - I^* - I^* \log \frac{I}{I^*} \right), \\ V_3(X) &= \frac{\rho}{2}(R - R^*)^2. \end{aligned} \tag{34}$$

Let us find $\mathcal{L}V_1$, $\mathcal{L}V_2$ and $\mathcal{L}V_3$ by applying formula (26).

$$\begin{aligned} \mathcal{L}V_1 &= (S - S^* + I - I^*)[\lambda - \mu S - \beta SI + \beta SI - (\mu + \gamma)I] \\ &+ \frac{1}{2}(\sigma_1^2 + \sigma_2^2 S^2 + (\sigma_4^2 + \sigma_5^2)I^2) = -\mu(S - S^*)^2 - (\mu + \gamma)(I - I^*)^2 \\ &- (2\mu + \gamma)(S - S^*)(I - I^*) + \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 S^2 + (\sigma_4^2 + \sigma_5^2)I^2\right), \end{aligned} \tag{35}$$

$$\begin{aligned} \mathcal{L}V_2 &= \theta \left(1 - \frac{I^*}{I} \right) [\beta SI - (\mu + \gamma)I] + \frac{\theta I^*}{2I^2} \left(\sigma_3^2 S^2 + \sigma_4^2 + \sigma_5^2 \right) I^2 \\ &= \theta\beta(S - S^*)(I - I^*) + \frac{\theta I^*}{2} \left(\sigma_3^2 S^2 + \sigma_4^2 + \sigma_5^2 \right), \end{aligned} \tag{36}$$

$$\begin{aligned} \mathcal{L}V_3 &= \rho(R - R^*)(\gamma I - \mu R) + \frac{\rho}{2}(\sigma_4^2 I^2 + \sigma_6^2 R^2) \\ &= \rho\gamma(I - I^*)(R - R^*) - \rho\mu(R - R^*)^2 + \frac{\rho}{2}(\sigma_4^2 I^2 + \sigma_6^2 R^2) \\ &\leq \frac{\rho\gamma^2}{2\mu}(I - I^*)^2 - \frac{\rho\mu}{2}(R - R^*)^2 + \frac{\rho}{2}(\sigma_4^2 I^2 + \sigma_6^2 R^2). \end{aligned} \tag{37}$$

Summing up (35)–(37) we obtain that for $\mathcal{L}V$ holds:

$$\begin{aligned} \mathcal{L}V \leq &- \mu(S - S^*)^2 - (\mu + \gamma)(I - I^*)^2 - (2\mu + \gamma)(S - S^*)(I - I^*) \\ &+ \theta\beta(S - S^*)(I - I^*) + \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2 S^2 + (\sigma_4^2 + \sigma_5^2)I^2\right) \\ &+ \frac{\theta I^*}{2}\left(\sigma_3^2 S^2 + \sigma_4^2 + \sigma_5^2\right) + \frac{\rho\gamma^2}{2\mu}(I - I^*)^2 \\ &- \frac{\rho\mu}{2}(R - R^*)^2 + \frac{\rho}{2}(\sigma_4^2 I^2 + \sigma_6^2 R^2). \end{aligned} \tag{38}$$

It is convenient to eliminate from (38) the multiplicative terms containing $(S - S^*)(I - I^*)$, thus we choose $\theta = \frac{2\mu+\gamma}{\beta}$

$$\begin{aligned} \mathcal{L}V = \mathcal{L}V_1 + \mathcal{L}V_2 + \mathcal{L}V_3 &\leq -\mu(S - S^*)^2 - (\mu + \gamma + \frac{\rho\gamma^2}{2\mu})(I - I^*)^2 \\ &+ \frac{1}{2}(\sigma_1^2 + \sigma_2^2 S^2 + (\sigma_4^2 + \sigma_5^2)I^2) + \frac{2\mu + \gamma}{2\beta} I^*(\sigma_3^2 S^2 + \sigma_4^2 + \sigma_5^2) \\ &- \frac{\rho\mu}{2}(R - R^*)^2 + \frac{\rho}{2}(\sigma_4^2 I^2 + \sigma_6^2 R^2). \end{aligned} \tag{39}$$

After algebraic manipulations, we can write

$$\mathcal{L}V \leq -m_1(S - \kappa_1 S^*)^2 - m_2(I - \kappa_2 I^*)^2 - m_3(R - \kappa_3 R^*)^2 + \varepsilon \tag{40}$$

where $m_i, \kappa_i, i = \{1, 2, 3\}$ are given by

$$\begin{aligned}
m_1 &= \mu - \frac{\sigma_2^2}{2} - \frac{\theta\sigma_3^2 I^*}{2} \\
\kappa_1 &= \frac{\mu}{m_1} \\
m_2 &= \mu + \gamma - \frac{\rho\gamma^2}{2\mu} - \left(\frac{\sigma_4^2}{2}(\rho+1) + \frac{\sigma_5^2}{2}\right) \\
\kappa_2 &= \frac{\mu + \gamma - \frac{\rho\gamma^2}{2\mu}}{m_2} \\
m_3 &= \frac{\rho}{2}(\mu - \sigma_6^2) \\
\kappa_3 &= \frac{\mu}{\mu - \sigma_6^2},
\end{aligned} \tag{41}$$

and $\varepsilon, \xi_1, \xi_2, \xi_3$ by

$$\begin{aligned}
\varepsilon &= \xi_1 S^{*2} + \xi_2 I^{*2} + \xi_3 R^{*2} + \frac{\sigma_1^2}{2} + \theta I^*(\sigma_4^2 + \sigma_5^2) \\
\xi_1 &= \frac{\mu}{2m_1}(\sigma_2^2 + \theta\sigma_3^2 I^*) \\
\xi_2 &= \frac{(\mu + \gamma - \frac{\rho\gamma^2}{2\mu})(\frac{\sigma_4^2}{2}(\rho+1) + \frac{\sigma_5^2}{2})}{m_2} \\
\xi_3 &= \frac{\rho\mu\sigma_6^2}{2(\mu - \sigma_6^2)}.
\end{aligned} \tag{42}$$

According to Theorem 4.4, the system has a stochastically stable equilibrium if $\mathcal{L}V \leq 0$. The conditions

$$0 < \varepsilon < \min\left(m_1\kappa_1^2 S^{*2}, m_2\kappa_2^2 I^{*2}, m_3\kappa_3^2 R^{*2}\right), \qquad m_i > 0, \kappa_i > 0, i = \{1, 2, 3\} \tag{43}$$

ensure that the ellipsoid

$$-m_1(S - \kappa_1 S^*)^2 - m_2(I - \kappa_2 I^*)^2 - m_3(R - \kappa_3 R^*)^2 + \varepsilon = 0 \tag{44}$$

lies entirely in $\mathbb{R}_+^3$. We can then take as U any neighborhood of this ellipsoid such that its closure $\bar{U} \subset \mathbb{R}_+^3$. Thus, we have $\mathcal{L}V(S, I, R) < 0$ for $(S, I, R) \in \mathbb{R}_+^3 \setminus U$.

Notice that m_1 and m_2 in (41) are the same as in (31), while (42) is merely another representation of ε from (32). Therefore, conditions in inequality (43) are equivalent to those in (30). The conditions $m_1 > 0$ and $m_3 > 0$ are ensured directly by (29), and κ_1 and κ_3 are positive by definition. If we select $0 < \rho < \frac{2(\mu+\gamma)-(\sigma_4^2+\sigma_5^2)}{\sigma_4^2+\gamma^2/\mu}$, then even $m_2 > 0$ and $\kappa_2 > 0$ are ensured. This concludes the proof. □

4.4 Implications of the analytic results

To interpret and summarize the results from Theorems 4.1 and 4.5, let us start with the condition for content persistence in a deterministic system, given by inequality (10). Since $\frac{\lambda\beta}{\mu} = c_N$, this condition simply states that any contact rate c_N, infinitesimally larger than the sum of departure and recovery rates, guarantees content persistence. In other words, if an infected node, before leaving the area or becoming recovered, meets on average at least one other node—the content is bound to survive.

According to established results in the theory of deterministic and stochastic models, while the deterministic models guarantee survival of the epidemic above the threshold, in stochastic models this is only a necessary condition: the epidemic is still prone to stochastic variations, which can lead to epidemic extinction [39]. Theorem 4.1 and condition (16) complies this general result, and specifies how the inequality is strengthened by additional terms with σ_i, $i = \{3, 4, 5\}$, coming from the second equation of the system (13). However, even if Theorem 4.1 ensures that if (16) holds content extinction cannot be escaped, the complementary inequality does not ensure the opposite, and Theorem 4.5 gives sufficient additional conditions for the content to persist. Due to these additional conditions, Theorems 4.1 and 4.5 determine three epidemic regions. A region when the content will surely become extinct, a region when it will surely survive, and a middle region, where the outcome can not be determined based on the analytic results. While Theorems 4.1 and 4.5 could be tightened by avoiding the bounds in the proofs, such a middle region seems to be present in stochastic systems, where the epidemics does not extinct exponentially fast, but does not survive for a longer time either [44].

Finally, let us discuss the results from the point of view of system design. The objective is then to find the largest γ value, that is, shortest average forwarding time T_F, that ensures stochastic stability. For this, we need to solve numerically the inequality (43). The preferred value of γ is the maximum value that satisfies the inequality.

5 MODEL VALIDATION AND PERFORMANCE EVALUATION

In this section we validate the system model and the derived conditions of endemic equilibrium by comparing the analytic results with simulations, considering system parameters that we derive from realistic mobility scenarios. We investigate and compare the behavior of the following processes: 1) the content spreading process directly obtained from realistic contact traces, 2) the original SDE system with state-dependent diffusion coefficients, and 3) the approximate SDE system with constant diffusion coefficients.

Fig. 3. The simulation scenario: the Östermalm area.

5.1 Mobility scenarios

For emulating realistic mobility, we use the *KTH walkers* traces [14, 27] generated by *Legion Studio* [29], a multi-agent mobility simulator commonly used for pedestrian traffic planning and public spaces design and dimensioning. The traces contain snapshots of the node positions taken every 0.6 seconds. The mobility scenario represents an outdoor urban space, modeling a part of the downtown Stockholm, which we will further refer to as the Östermalm scenario. The topology is represented by a grid of streets, as shown on Fig. 3. Pedestrians enter the area and depart from the fourteen streets opening at the area boundary. Nodes arrive to each of the entry points according to a Poisson process with rate λ_s, resulting a total arrival rate λ. Each node traverses the area at a constant speed chosen randomly from a truncated normal distribution with a value range $(0.6; 2.0)$ m/s and a mean of 1.3 m/s. At each intersection, the node continues moving straight with probability 0.5, or turns into one of the intersecting streets with equal probability. Since the node mobility inside the area is determined by the topology and the speed distribution, the distribution of the sojourn times are independent from the arrival rate, and are similar in all investigated scenarios. We consider 6 scenarios with different arrival rates and consequently, different average population sizes – ranging from around 40 nodes to 350 nodes. The duration of the first trace is 5 hours, the duration of the second is 3 hours and the other traces are 2 hours long, this ensures at least 2000 arrivals in each scenarios. The mobility traces provide the arrival rate λ and the sojourn time $T_S = \frac{1}{\mu}$ parameters.

From the mobility traces, we generate the trace of contacts using the ONE simulator [23]. The ONE simulator takes mobility traces as an input and generates timestamps of contact events whenever two nodes are within the specified transmission range. Specifically, we consider a transmission range of 10 m. From the timestamps of contact events we obtain the value of the individual contact rate c.

The parameters of the explored scenarios are detailed in Table 1. Note that some of the assumptions that allow us to form the Markovian model, e.g. Poisson arrivals to the area, are readily satisfied in these scenarios. As discussed in [37], the time intervals between the contact events can also be assumed to be exponentially distributed. The distribution of the sojourn times turns out to be non-exponential, but the exact distribution has little impact on the accuracy of the model.

Table 1. Scenario parameters estimated from traces.

Scenario #	**Sojourn time T_S** [s]	**Contact rate c_N** [s^{-1}]	**Arrival rate λ [s^{-1}]**	**Average population size N**	Forwarding time	
					T_F^0 [s]	**T_F^{sd} [s]**
1	322.0	0.0230	0.1176	37	87	183
2	322.1	0.0485	0.2380	77	37	109
3	340.6	0.0742	0.3687	127	23	74
4	328.2	0.0825	0.4610	152	20	67
5	320.3	0.1011	0.5830	195	16	56
6	337.7	0.1865	1.0404	348	8	35

5.2 The boundaries of three epidemic regions

In 4.2 and 4.3 we established the conditions for content survival and existence of the endemic stationary distribution. Given that, the approximate constant coefficient model (13) predicts when: 1) the content vanishes from the population in finite time almost surely and 2) the system reaches the endemic equilibrium, after which its trajectories continue fluctuating around the endemic levels.

Fig. 4 shows the predicted borders for the three epidemic regions. The mobility parameters and the contact rate are set based on the six introduced scenarios, and $T_F = \frac{1}{\gamma}$ is tuned according to Theorems 4.1 and 4.5. The critical T_F values depend on λ and T_S in a more complex way than through their product—average number of nodes in the system—but for the sake of clarity, we choose a representation with the population size on the x-axis. Fig. 4 depicts the three regions where a scenario may fall into. The solid line represents the forwarding time T_F^0 below which the content dies out almost surely within a finite time derived based on Theorem 4.1, whereas the dashed line marks T_F^{sd}, the time necessary for the existence of the endemic stationary distribution, based on Theorem 4.5. The estimated critical values are given in the two last columns of Table 1 as well. Region 1 corresponds to parameter sets which will lead to content extinction within finite time. In Region 3, T_F is sufficient to support indefinite content survival, and finally, Region 2 is the gray area where Theorems 4.1 and 4.5 do not give information about the system behavior.

The figure provides interesting insights about the requirements on an opportunistic content sharing system. First of all, compared to the average sojourn time of the nodes, which is several hundreds of seconds, the time necessary for content forwarding is small, as soon as the population has reasonable size. Already at a population size of 80, $T_F = 100$ seconds achieves content survival, and with a population size of 300, nodes do not need to forward the content for more than 40 seconds. These results show that to keep the energy consumption of the participating devices low, and thus the opportunistic services more popular, the forwarding time T_F should be tuned according to the experienced population size.

Next we turn to validate Theorems 4.1 and 4.5. For this we consider three characteristics mobility scenarios, Östermalm_1, Östermalm_4 and Östermalm_6, with small, medium and large population respectively.

5.3 Validation of the content extinction condition

We first evaluate the results of Theorem 4.1. The objective of the evaluation is twofold: we demonstrate that the established condition really leads to content extinction in the SDE model, and validate the model itself by trace-based simulation.

Fig. 5 compares the population of the S, I and R compartments, $S(t), I(t), R(t)$ as well as the entire population, $N(t)$, where the forwarding time T_F is chosen such that the condition for content survival is not satisfied ($T_F < T_F^0$). We compare the result of trace driven simulation with that of the simplified, constant coefficients SDE model. We examine three cases: Östermalm_1, Östermalm_4, and Östermalm_6 and choose the values $T_F = \{80, 19, 8\}$ seconds, respectively, these values place the three cases in Region 1 in Fig. 4.

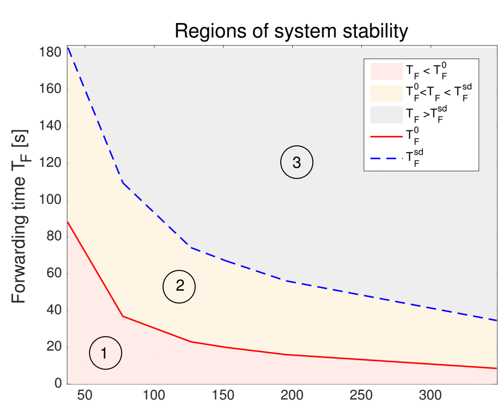
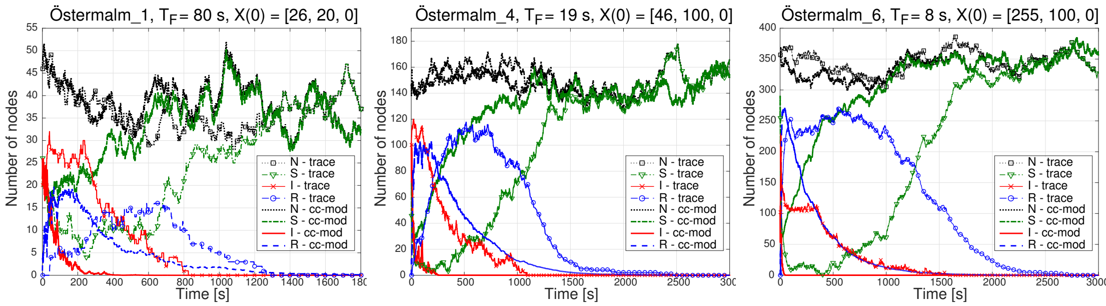


Fig. 4. The stability regions of the SDE model, as a function of the average population size N and the forwarding time T_F.



(a) (b) (c)

Fig. 5. Trajectories of $S(t), I(t)$ and $R(t)$ and total populations $N(t)$ representing results of simulations on the contact traces (*trace*) and of the approximate constant coefficient SDE model (*cc-mod*), at the critical T_F values for content extinction. The number of initially infected nodes are $I(t) = \{20, 100, 100\}$ for (a), (b) and (c), respectively.

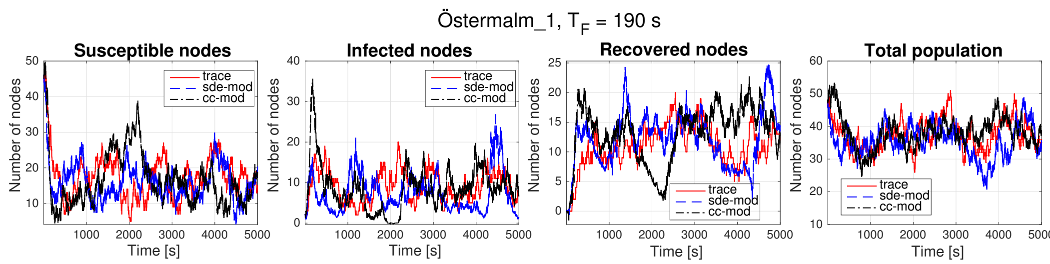
According to Theorem 4.1 $I(t)$ always converges to zero under condition (16). We therefore consider the system after a bootstrapping phase, where many of the nodes in the population are already infected. The initial number of infected nodes is 20 for the first scenario and 100 for the other two scenarios. The figures show single realizations of the content sharing process, thus we do not expect overlapping paths for the model and simulation results, but compare the general behavior. Fig. 5 clearly shows that both the trace-based and modeled paths lead to content extinction: $I(t)$ and $R(t)$ converge to zero exponentially fast, while the size of the entire population $N(t)$, which then equals $S(t)$, reaches $N = \lambda/\mu$ and fluctuates around this value. The system then converges to a *disease-free equilibrium.*

5.4 Achieving content survival by tuning the forwarding time T_F

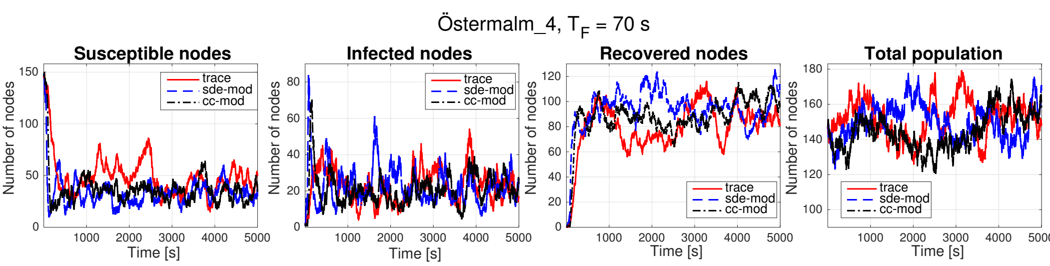
Let us now evaluate the results of Theorem 4.5. We select the T_F values such that $T_F > T_F^{sd}$ to ensure that content survives in all cases and that process has a stable endemic equilibrium. We consider the same three mobility scenarios as in Section 5.3, with T_F^{sd} values given in Table 1. We compare the results of the contact trace driven simulations, of the original SDE model, and of the approximate, constant coefficient SDE model.

Fig. 6 shows a sample path of the number of nodes in the S, I and R compartments and the size of total population. The results verify that the conditions of Theorem 4.5 ensure long term content survival, the number of nodes in the three compartments seem to reach equilibrium and stay positive in the considered 5000 seconds. We evaluate the accuracy of the original and the constant coefficient SDE model with the box diagrams shown on Fig. 7. In each box, the central horizontal (red) line is the median, the edges of the (blue) box are the 25th and 75th percentiles, the (black) whiskers extend to the most extreme data points not considered outliers, while outliers are plotted individually as (red) crosses. Both models are accurate in the low population scenario of Östermalm_1. In the other two scenarios there is some discrepancy in the distributions of the susceptible nodes. This is an artifact of the linear homogeneous mixing model, already identified in [15], which overestimates the infection rate in large instantaneous population, and consequently, predicts greater numbers of infected nodes compared to the simulations.

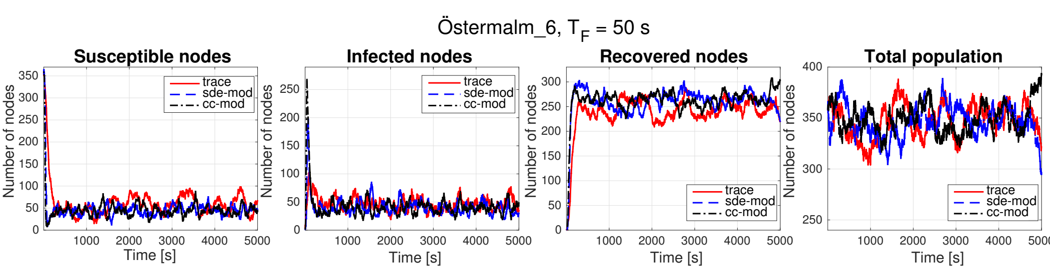
Finally, we investigate how the system behaves under parameter sets belonging to the second region of Fig. 4, where none of the Theorems 4.1 or 4.5 applies. As in Section 5.3, we consider



(a)



(b)



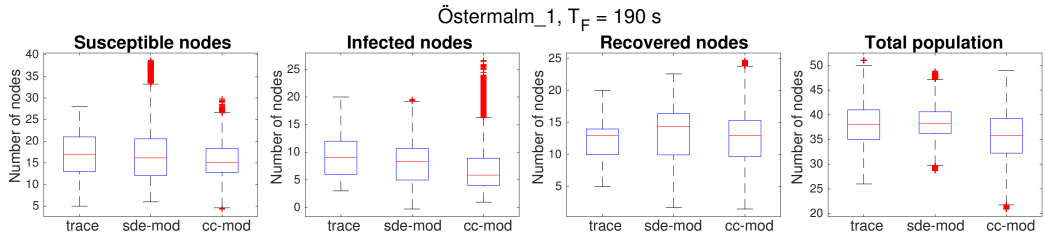
(c)

Fig. 6. Trajectories of $S(t), I(t)$ and $R(t)$ and total populations $N(t)$ representing results of simulations on the contact traces (*trace*), of the original SDE model (*sde-mod*) and of the approximated constant coefficient model (*cc-mod*).

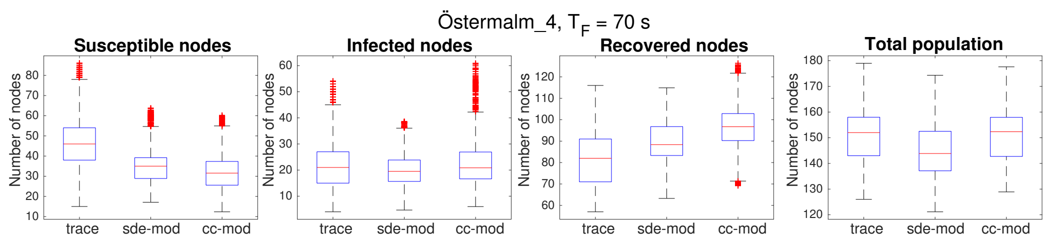
$I(0) > 1$ to avoid immediate content extinction. Fig. 8 shows the usual three mobility scenarios, but now with $T_F^0 < T_F < T_F^{sd}$. As we see, the SDE models can not capture the outcome of the content distribution in this case, and the results are very different in the trace driven simulation, in the original, and in the approximate SDE model. In all scenarios the simulations show that the number of infected nodes $I(t)$ drops to zero after an initial epidemic period, and the content vanishes. The original SDE model (sde-mod) exhibits explosion, due to the dependence of the diffusion coefficients on $\sqrt{I(t)}$, while the approximate constant coefficients SDE model maintains a low number of infected nodes, but the system does not seem to settle around an equilibrium point.

5.5 Comparison of the original and approximated SDE model

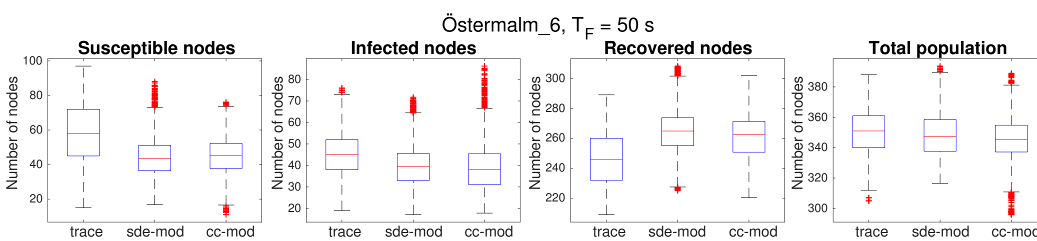
It is convenient at this point to reflect on the Markovian content sharing process we started from, its first SDE representation and the SDE model with constant coefficients used in the previous section



(a)



(b)



(c)

Fig. 7. Comparison of the distributions of nodes in S, I, R compartments and total populations. *trace* – results of simulations from the contact traces; *sde-mod* – original SDE model; *cc-mod* – approximate model. In each box, the central horizontal (red) line is the median, the edges of the (blue) box are the 25th and 75th percentiles, the (black) whiskers extend to the most extreme data points not considered as outliers, while outliers are plotted individually as (red) crosses.

for stability analysis. Clearly, since both SDE models represent approximations of the initial process, it is important that they capture similar process behavior, at least on how the different regions (content extinction or persistence) manifest depending on the system parameters. As we see on Figs. 6 and 7, in cases when the endemic equilibrium exists, the models show, at least qualitatively, the same behavior and similar distributions as the trace-based spreading process. Furthermore, we observe that the original model, compared to the approximate model, gives slightly worse distribution fit to the results obtained from the traces (see *Infected nodes* plots in Fig. 7). However, as we show in Fig. 8, the original SDE model is prone to explosions, which happen when the number of nodes in some compartment approaches zero. This in turn reflects that it is challenging to derive stability conditions for the original model, and the approximate SDE model can provide a good

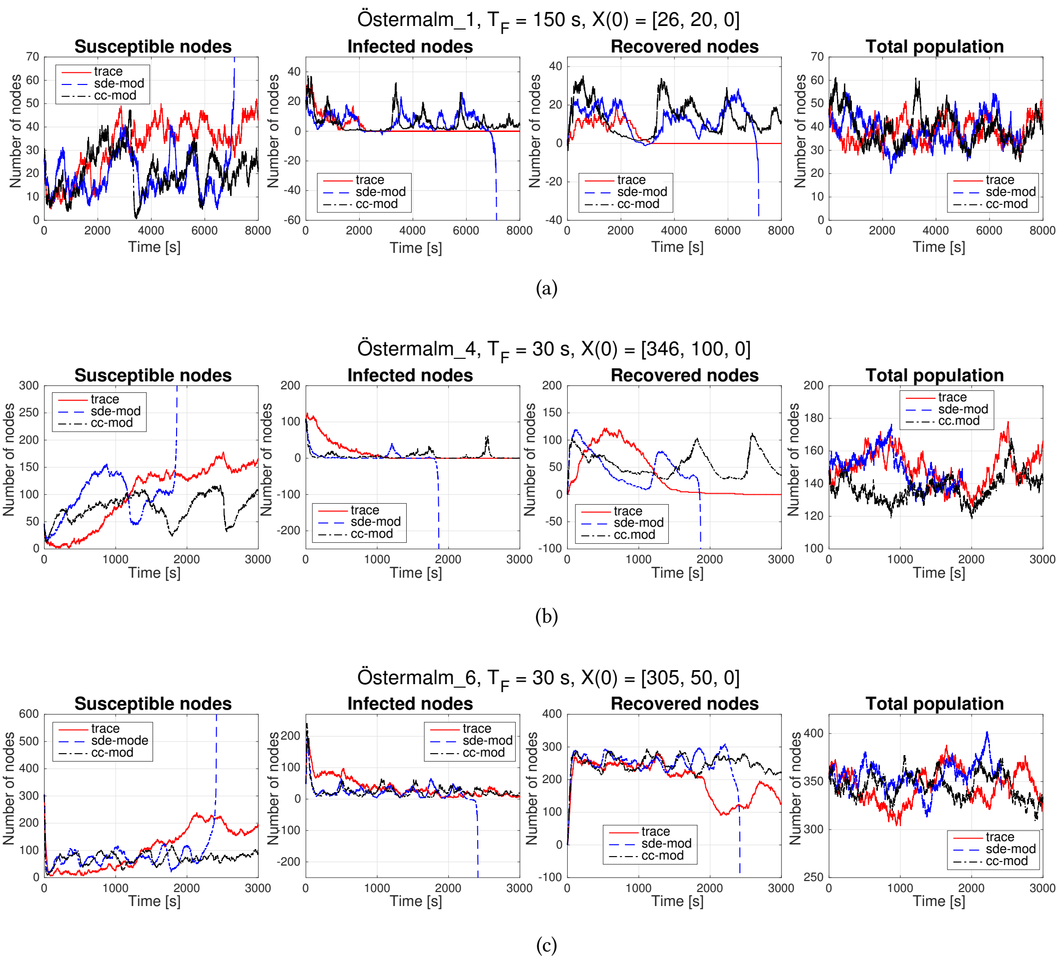


Fig. 8. Sample paths of three processes (trace-based, original SDE model and SDE model with constant coefficients). Forwarding times T_F satisfy the condition for content survival established by the model, but not the condition for the stability of endemic equilibrium. The number of initially infected nodes: $I(t) = \{20, 100, 50\}$ for a), b) and c) respectively.

compromise. These results also demonstrate that there is an operating region of the opportunistic content sharing, where the content may survive for some time, carried only by a few nodes, but in this case the effect of random disturbances becomes significant, and neither our original, nor the constant coefficient model provides adequate prediction for the system behavior.

6 CONCLUSION

We studied the feasibility and the performance of opportunistic location-aware content spreading, where mobile users constantly join and leave an area of interest, but while present they are willing to participate in content spreading by carrying content and forwarding it to other nodes they meet, at least for a limited time. In this way, the content "survives" in the area without infrastructure support.

We have studied under what conditions, that is, the combination of mobility parameters such as node arrival rate, sojourn time and contact rates, and the tunable system parameter, forwarding time, this content spreading scheme is feasible. We modeled the spreading process with a stochastic epidemic model and employed Lyapunov stability theory for stochastic systems to establish conditions when the system reaches the state of persistence, in which the content is likely to survive for a very long time. The validity of the system model as well as the established analytical results are confirmed via simulations using realistic mobility traces.

Our results are valuable for system design, as we provide theoretic tools to predict the success of the opportunistic content sharing, and to tune the system parameters to ensure that the content survives in the area for a long time. The numerical results considering realistic mobility scenarios show that the forwarding times required by the users are relatively short – ranging from less than a minute in dense environments to a couple of minutes in very sparse scenarios. This finding can help to give incentives for the users to participate and share contents since the requirement for user resources is obviously not that large. We also find that the population size and thus the contact rate has a significant impact on the forwarding time required for content survival. This motivates future research on the design of adaptive schemes, that can estimate the required forwarding time. These schemes could then ensure successful content sharing, while minimizing the contribution and thus the energy consumption of the participating users.

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