Question 1. Let $f_j : [0,1] \mapsto [0,\infty]$ be a sequence of $\mu-$measurable functions and $f_j(x) \to f(x)$, for every $x \in [0,1]$, and some measurable function $f : [0,\infty] \mapsto [0,\infty]$. Will

$$\lim_{j\to\infty} \int_0^1 f_j(x) d\mu(x) = \int_0^1 f(x) d\mu(x).$$

Question 2. The statement of Proposition 2.2.2 suggests that the Hausdorff measure and the spherical measure are not necessarily equal. Give an example of a set A in $\mathbb{R}^n$ such that $\mathcal{H}^m(A)$ not equal to $\mathcal{S}^m(A)$.

Question 3. Is there a counter example to the result in Prop. 2.2.2 (p 62) for a non regular measure?

Question 4. It is important that $\mu(A) < \infty$ in Theorem 1.3.3 (Egorov's theorem). What would a counter example be if this constraint was removed? Also, do we need any restrictions on the set X?

Question 5.

Proposition 1. *Let be a regular measure on $\mathbb{R}^N$, and let $0t < \infty$ be fixed. If $\mathcal{H}^m(A) < \infty$ and $\Theta^m(\mu, p)t$ holds for all $p \in A$, then $\mu(A)t \cdot 2^m \mathcal{H}^m(A)$.*

There is a step in the proof that suggests that maybe 2^m is not optimal. Is the constant 2^m optimal? **(Probably difficult:)**If not, what is the optimal one?

Question 6. In the proof of Corollary 2.4.6, we use that for any $C \subset \mathbb{R}^m$ there exists a Borel set B with $C \subset B$ such that $\mathcal{H}^1(B) = \mathcal{H}^1(C)$. In general, if the "test sets" in Carathéodory's construction are Borel sets, then the resulting measure will be Borel regular. But in our construction of the Hausdorff Measures we let the "test sets" be all sets, and so Borel regularity is not obvious. However, on page 55 it is claimed that the same measure would result even if we restricted ourselves to all closed sets or all open sets (which would give us Borel regularity). Why is this true? Also, why do we need to work with Borel sets in the proof of Corollary 2.4.6 in the first place?

Question 7: Is the two dimensional Hausdorff measure $\mathcal{H}^2$ lower semi-continuous with respect to convergence in Hausdorff distance.

Question 8: Hilbert's third problem[1] is

Specify *"two tetrahedra of equal bases and equal altitudeswhich can in no way be split up into congruent tetrahedra, and which cannotbe combined with congruent tetrahedra to form two polyhedra which themself could be split up into congruent tetrahedra."*

How is that related to the problem om determining areas.

[1] This was solved by Max Dehn in 1901.

Question 9: Is Corollary 2.4.5 true for $m = 0$?

Question 10. On page 68; how would one show that the Hausdorff dimension of the Sierpinski triangle is $\log 3/\log 2$? The book also states that the Hausdorff dimension of any compact $\mathcal{C}^1$-curve is 1. What would the details in the proof of this be?

Question 11. For the cone $A = x^2 + y^2 = z^2$ in $\mathbb{R}^3$, how to compute the 2-dimensional density of the Hausdorff measure (restricted to A) at all the points of A. (Allegedly the density is 1 at $p \in A$ $p \neq 0$ and density $\sqrt(2)$ at $p = 0$).

Question 12. Is the Hausdorff distance in Definition 1.6.1. on page 33 a pseudometric? Proof?

KOMMENTAR: *A psedometric is a metric except that there are two different elements with zero distance.*

Question 13. (Probably difficult.) Does it exist a finction $f : [0, 1] \mapsto \mathbb{R}^2$ such that $f \in C^\alpha([0, 1], \mathbb{R}^2)$ and $\mathcal{H}^1(f([0, 1])) = \infty$. Note that Corollary 2.4.5 answers the question for $\alpha = 1$ the question is whether the corollary holds for $\alpha < 1$?

Question 14. In Proposition 1.4.3 Cavalieri's is mentioned offhandedly, but how do you prove it?

Question 15. The most common measures one constructs via the Carathéodory method appears to be the Hausdorff measure and the spherical measure. Is that because they are the most natural (as generalizations of the Lebesgue measure) or are there practical advantages when analyzing geometrical structures. If so, what are the advantages?

Question 16. In example 1.2.17 we construct a non-measurable set. How does this example motivate the theory of measures. **Reformulate.**

Question 17. Why is Theorem 1.5.2. trivial if $N = m$?

Question 18. On p.9, lines -6 and -7 in the proof of Caratheodory's criterion there seems to be typos, I presume some limit is taken here on both lines. However, why is the limit of the right hand side of line -6, as $n \to \infty$, equal to 0?

Question 19. On p. 11, there seems to be a typo in example 1.2.17 (a construction of a measurable subset). It concerns the existence of the element c in the third paragraph. Can you spot it?