On the gains of deterministic placement and coordinated activation in sensor networks

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Abstract—In this paper we discuss topology design and dimensioning of sensor networks to achieve full sensing coverage. We consider two ways of deploying the sensors, placing them according to some regular pattern or scattering them randomly, and two ways of activating the sensors, optimally according to some predefined schedule or randomly, when each sensor follows a wake-up schedule independently from the other sensors. We provide analytic expressions for the necessary and sufficient number of sensors that guarantee coverage in these scenarios and determine the cases when deterministic sensor placement or optimal sensor activation can achieve significant gains. We consider sensing with bounded delay and show that the number of sensors to be deployed can be decreased significantly even at low sensing delays.

I. INTRODUCTION

Sensor network dimensioning involves determining the required sensor density according to some objectives, like sensing coverage, connectivity of the sensor network, given throughput, limited data fusion delay, or robustness against errors. In this paper we consider full sensing coverage as objective, that is, the sensor network should detect any event in an area. The number of sensors needed for sensing coverage depends on the sensing radius of the sensors, on the ratio of time the sensors are active and on the sensor placement and activation method. If sensors are scattered randomly, the position of a sensor is independent of the other sensor positions, thus many sensors may sense the same area. If sensors are placed according to a regular pattern, the areas sensed by many sensors and thus the number of sensors needed for coverage can be minimized. Similarly, if wake-up cycles are not coordinated across the network, sensors close to each other may be active at the same time. The optimal scheduling of wake-up cycles minimizes the areas sensed by several active sensors and thus the number of sensors required for coverage. Deterministic sensor placement is, however, not possible in all applications, and coordinated wake-up schedules require the exchange of control messages in the sensor network, using transmission capacity and energy, both being scarce resources in typical sensor networks. Our aim is to derive the possible gains of deterministic sensor placement and coordinated sensor activation, given the sensing radius of the sensors and the fraction of the time they are active.

Network dimensioning for coverage has been addressed

by several research papers before. In [1] the deterministic placement of always active sensors is considered, and the necessary number of sensors needed for coverage and connectivity is derived for different deployment patterns. Network coverage with connectivity is addressed in [2] for the case of deterministic placement and random node failure. The main theorems of this paper are starting points for our analysis. In [3] conditions for k-coverage with probability one are derived, as the number of nodes approaches infinity, considering deterministic, uniform and Poisson deployment and random wake-up schedules. A large number of proposals addresses the problem of coordinating wake-up schedules in order to minimize the time sensors need to be active, usually assuming random sensor placement, for example [4], [5] and more recently [6], [7].

Our work extends these previous results by providing a systematic comparison of all possible scenarios combining deterministic or random sensor placement and optimally coordinated or random wake-up scheduling. As a result, our analysis defines the regions where coordination can achieve significant gains considering the number of sensors deployed or their energy consumption. While we only consider sensing coverage, conditions for coverage with connectivity can be derived as in [1].

The paper is organized as follows. In section II we describe the addressed scenarios. In sections III and IV we derive sufficient and necessary conditions to achieve spatial coverage. Section V discusses the case of event detection with bounded delay. Performance evaluation of the different scenarios is presented in section VI. We conclude the paper in section VII.

II. SYSTEM DESCRIPTION

We consider a unit square area that has to be covered by identical, static sensors. To describe the sensing capability of the sensors we apply the disc model, generally used in previous works. That is, the coverage area of a sensor is a disc with radius r and the sensors can detect all events within this area. We will address probabilistic detection in future work [8]. To extend network lifetime by saving sensor energy, each sensor follows a wake-up cycle, being active in p fraction of time and asleep otherwise. A sensor can detect an event within its sensing area iff it is active. We derive necessary and sufficient

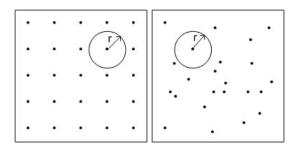


Fig. 1. Sensors a) deployed in a square grid b) scattered randomly

conditions on N, the number of sensors to be deployed, to achieve full coverage at any random point of time t with probability P_c . We consider four scenarios: (DO) sensors are placed deterministically according to some regular pattern and the wake-up cycles follow an optimal schedule across the network; (DR) sensors are placed deterministically according to a regular pattern, but wake-up cycles are not coordinated; (RR) sensors are scattered randomly, and wake-up cycles are not coordinated; and (RO) sensors are scattered randomly, but the wake-up cycles follow an optimal schedule. By optimal schedule we mean that the areas sensed by more than one active sensor are minimized for all points of time.

The DO scenario does not include any randomness and we can calculate the exact number of sensors required to provide full coverage. In [1] N_a , the required number of sensors is derived, assuming that sensors stay always active. If sensors are active in p portion of time, we need at least $\lceil \frac{1}{p} \rceil N_a$ sensors to cover the same area at any random point in time. To simplify notation we write $\frac{1}{p}$ instead of $\lceil \frac{1}{p} \rceil$ in the rest of the paper. Using the results of [1] for N_a , the number of sensors in the DO scenario is:

$$N = 0.38 \frac{1}{m^2}. (1$$

III. SUFFICIENT CONDITIONS FOR COVERAGE

In this section of the paper we study the sufficient conditions for coverage for all the different placement and wake-up scenarios presented above. We first consider the deterministic placement scenario, with random sensor wake-up schedule. Then we continue deriving the sufficient conditions for the random sensor deployment cases, with random or deterministic wake-up schedule.

A. Deterministic Placement - Random Wake-up Schedule (DR)

We consider a set of sensors deployed according to some regular pattern in a unit square area as shown in Figure 1.a. We follow the argumentation in [2] to derive a sufficient condition on the number of sensors, N, in the DR case. We introduce auxiliary variables α and β and cover the unit area with a set of rectangular areas in such a way that the distance between their centers is equal to αr , $\alpha \geq 0$. Around these centers we draw circles with radii $r_c = \beta r$, $\beta \in (0,1)$, as shown in Figure 2. The number of sensors in a circle is $\pi r_c^2 N$ for regular deployment patterns and the total number of circles is $K = \lfloor 1/(\alpha r)^2 \rfloor$. Notice that the circles will

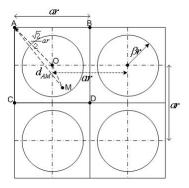


Fig. 2. Drawn circles inside the squares that split the unit area

overlap in case $\alpha < 2\beta$. We ignore possible edge effects, as rounding down the number of drawn circles to the nearest integer might leave uncovered space at the edges of the unit area. We also neglect approximation errors originating from the assumption that the number of sensors in each of the circles is exactly $\pi r_c^2 N$. Errors resulting from these approximations are negligible if the deployment density is high, thus for sufficiently large N. For the rest of the analysis we will always ignore these approximation errors. Also, to ease reading we omit the rounding notations throughout the paper.

The entire area will be covered at time t, if there is at least one active sensor node in each circle and this sensor is able to sense the rectangular area, around the circle. As it is shown in Figure 2, a sensor deployed at an arbitrary point M inside the circle with center O is able to cover the whole square ABCD, if it can sense the points A,B,C and D. Without loss of generality, we assume that distance d_{AM} is larger than d_{BM},d_{CM} or d_{DM} . Applying the triangular inequality, we can write: $d_{AM} \leq d_{MO} + d_{OA} \leq \beta r + \frac{\alpha r}{\sqrt{2}}$. So, the condition that should hold for the whole rectangle to be covered by any sensor inside the circle is:

$$d_{AM} \le r \Rightarrow \beta + \frac{\alpha}{\sqrt{2}} \le 1.$$
 (2)

Let us define event A_i as:

 $A_i(t) := \{ \text{There is at least 1 active sensor in circle } i \text{ at time } t \},$

i=1,2,...,K. The probability that a sensor is active at time t is p, independent of t. Therefore we can omit the time parameter in our derivations. Then, if P_c expresses the probability that the unit area is fully covered by the sensor network, we can write:

$$P_{c} \geq P(\bigcap_{i}^{K} A_{i})$$

$$= 1 - P(\bigcup_{i}^{K} \bar{A}_{i})$$

$$\geq 1 - \sum_{i=1}^{1/\alpha r^{2}} (1 - p)^{\pi r_{c}^{2} N}$$

$$= 1 - \frac{1}{\alpha^{2} r^{2}} (1 - p)^{\pi \beta^{2} r^{2} N}$$

$$\geq 1 - \frac{1}{\alpha^{2} r^{2}} (1 - p)^{\pi (1 - a/\sqrt{2})^{2} r^{2} N}.$$
(3)

Solving (3) with respect to N we get:

$$N \ge \frac{\log(\alpha^2 r^2 (1 - P_c))}{\pi r^2 (1 - \frac{\alpha}{\sqrt{2}})^2 \log(1 - p)}.$$
 (4)

The minimum of (4) with respect to α is found by setting the partial derivative to zero:

$$\frac{\partial}{\partial \alpha} \left(\frac{\log(\alpha^2 r^2 (1 - P_c))}{\pi r^2 (1 - \frac{\alpha}{\sqrt{2}})^2 \log(1 - p)} \right) = 0,$$

which is equivalent to

$$2log(\alpha) + \sqrt{2}/\alpha = 1 + log(1/(r^2(1 - P_c))).$$
 (5)

Thus, (5) gives the value of α that minimizes the sufficient condition derived in (4) for given r and P_c .

B. Random Placement - Random Wake-up Schedule (RR)

According to this scenario the sensors are uniformly distributed, independently from each other inside the unit square area, as shown in Figure 1.b. Thus, the number of sensors inside a specific drawn circle follows a binomial distribution:

$$P(k) = \binom{N}{k} (\pi r^2 \beta^2)^k (1 - \pi r^2 \beta^2)^{N-k}.$$

The probability of the complementary event

$$\bar{A}_i := \{ \text{There is no active sensor in circle } i \}$$

is equal to:

$$P(\bar{A}_i) = \sum_{k=0}^{N} (1-p)^k \binom{N}{k} (\pi r^2 \beta^2)^k (1 - \pi r^2 \beta^2)^{N-k}.$$
 (6)

 $P(\bar{A}_i)$ is equal to the Z-Transform of the binomial distribution: $P_{A_i} = G(1-p)$. For N sufficiently large, we can write:

$$G(z) = e^{(z-1)E[X]},$$
 (7

where E[X] is the mean of the binomial distribution: $E[X] = N\pi r^2 \beta^2$.

From (6) and (7) we get: $P(\bar{A}_i) = e^{-pN\pi r^2\beta^2}$. Replacing this result to (3), we derive a sufficient condition for N similar to that in (4):

$$N \ge -\frac{\log(\alpha^2 r^2 (1 - P_c))}{\pi r^2 (1 - \frac{\alpha}{\sqrt{2}})^2 p}.$$
 (8)

Again, (5) gives the value of α that minimizes N in (8).

C. Random Placement - Optimal Wake-up Schedule (RO)

In this case we still consider that the sensors are randomly deployed inside the unit area, however their wake-up schedule is coordinated optimally, that is, an arbitrary point in the unit area requires to be covered by 1/p sensors. Complying with the argumentation developed in the previous cases, the sufficient condition for coverage is the existence of at least 1/p sensors in each of the drawn circles. The number of sensors inside each circle still follows a binomial distribution. If we define event $A_i^{(p)}$ as:

$$A_i^{(p)} := \{ \text{At least } 1/p \text{ sensors in circle } i \},$$

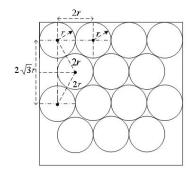


Fig. 3. Non overlapping circles in the unit area

we obtain the following sufficient condition:

$$P_{c} \geq P(\bigcap_{i} A_{i}^{(p)})$$

$$= 1 - P(\bigcup_{i} \bar{A}_{i}^{(p)}) \geq 1 - \sum_{i=1}^{\frac{1}{\alpha^{2}r^{2}}} P(\bar{A}_{i}^{(p)})$$

$$= 1 - \sum_{i=1}^{\frac{1}{\alpha^{2}r^{2}}} \sum_{j=0}^{\frac{1}{p}-1} {N \choose j} (\pi r^{2} \beta^{2})^{j} (1 - \pi r^{2} \beta^{2})^{N-j}$$

$$= 1 - \frac{1}{\alpha^{2}r^{2}} \sum_{j=0}^{\frac{1}{p}-1} {N \choose j} (\pi r^{2} \beta^{2})^{j} (1 - \pi r^{2} \beta^{2})^{N-j}.$$
(9)

The sufficient number of sensors that guarantees coverage at least with probability P_c is the lowest N that satisfies (9) with α, β satisfying (2).

IV. NECESSARY CONDITIONS FOR COVERAGE

In this section we derive the lower bounds on the number of sensors to be deployed to guarantee coverage. Again, we start with the case of a grid network and then we extend our study to random deployment of sensors.

A. Deterministic Placement - Random Wake-up Schedule (DR)

We consider again the grid sensor network of N sensors shown in Figure 1.a. The derivation of the necessary condition for the DR case is based on the idea discussed in [2]. We place in the unit area a number of non-overlapping circles with radius equal to the sensing radius r, as shown in Figure 3. To fully cover the unit area it is necessary to cover the centers of the circles. That is, there has to be at least one active sensor in each circle:

$$P_c \le P(\text{At least 1 active sensor in each circle})$$

= $[1 - (1 - p)^{\pi r^2 N}]^K$, (10)

where $K = \frac{1}{2\sqrt{3}r^2}$ is the number of circles and $\pi r^2 N$ is the number of sensors in each circle.

Solving (10) with respect to N we get:

$$N \ge \frac{\log(-2\sqrt{3}r^2\log(P_c))}{\pi r^2\log(1-p)}.$$
(11)

B. Random Placement - Random Wake-up Schedule (RR)

The derivation of the necessary condition in the case of grid deployment of sensors was based on the fact that the number of active sensors in each of the non-overlapping circles does not depend on the number of sensors inside the rest of the circles. However, this assumption does not hold in the case of random deployment. Therefore, for the derivation of a necessary bound in the RR case we have to follow a slightly different approach.

Again, it is necessary, that there is at least one active sensor in each of the K circles in Figure 3. We define event C_i as:

 $C_i = \{\text{There are } i \text{ circles without any active sensors}\}.$

Since the sensors are uniformly distributed inside the unit area, the probability of the event C_i will be:

$$P(C_i) = {K \choose i} (1 - i\pi r^2 p)^N.$$
 (12)

As the events C_i , i=1,2,...,K are mutually exclusive, we can write:

 $P_c \leq P(\text{at least 1 active sensor in each circle})$

$$= 1 - P(\text{at least 1 empty circle})$$

$$= 1 - P(\bigcup_{i} C_{i}) = 1 - \sum_{i=1}^{K} P(C_{i})$$

$$= 1 - \sum_{i=1}^{K} {K \choose i} (1 - ip\pi r^{2})^{N},$$
(13)

that gives the necessary number of sensors to cover the area in the RR scenario.

C. Random Placement - Optimal Wake-up Schedule (RO)

As in the derivation of the sufficient condition for the RO case, a necessary condition for coverage requires that each of the drawn circles has at least 1/p sensors. We define:

$$C_i^{(p)} := \{ \text{There are } i \text{ circles with less than } 1/p \text{ sensors} \}.$$

Considering uniform distribution of sensors, we determine the probability of this event:

$$P(C_i^{(p)}) = {K \choose i} \sum_{x_1=0}^{1/p-1} \sum_{x_2=0}^{1/p-1} \dots \\ \dots \sum_{x_i=0}^{1/p-1} f^{(N)}(x_1) \dots f^{(N)}(x_{i-1}) f^{(N-x_1-\dots-x_{i-1})}(x_i),$$
(14)

where x_j is the number of sensors in circle j and $f^{(N)}(x_j)$ is the binomial distribution:

$$f^{(N)}(x_j) = \binom{N}{x_j} (\pi r^2)^{x_j} (1 - \pi r^2)^{N - x_j}.$$
 (15)

Extending the argumentation in (13) the necessary number of sensors in the RO case is given by:

 $P_c \le P(\text{at least } 1/p \text{ active sensors in each circle})$

$$= 1 - P(\text{at least 1 circle with less then } 1/p \text{ sensors})$$

$$=1-P(\bigcup_{i} C_{i}^{(p)})=1-\sum_{i=1}^{K} P(C_{i}^{(p)}).$$

The derivation of the necessary bound may require exhaustive calculation, in case K, the number of circles, is large. We



Fig. 4. Event detection with limited delay and optimal scheduling

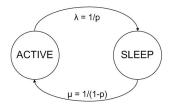


Fig. 5. Markovian wake-up cycle model

obtain a slightly looser bound, if we limit the summation in (16) up to an integer $k \ll K$. For high P_c it is very unlikely that more than a few circles will contain less than 1/p sensors, so we reduce the computation overhead, without distorting the analytical result significantly.

V. NECESSARY AND SUFFICIENT CONDITIONS INTRODUCING DELAY FLEXIBILITY

In this section we study how the necessary and sufficient conditions derived in the previous sections are affected if delay flexibility is introduced in the sensing process. We assume that events that take place at random time instants do not have to be sensed immediately. Instead, a bounded sensing delay d is allowed. We examine both the scenarios of optimal and random wake-up schedule.

A. Delay with Optimal Wake-up Schedule

We consider the sensors synchronized with each other, allocating their active times optimally. As discussed above, an arbitrary point in the unit area should be covered by 1/p sensors, where p is defined as the portion of the cycle period a sensor is active. We consider now a deterministic wake-up cycle. The cycle period has a unit length, starting with an active period of time p. We express the delay flexibility d as a percentage of the cycle length. An event is detected by a sensor if it happens during its active period or within d time-interval before the sensor becomes active. Then, as shown in Figure 4, a random point should be covered by max(1, 1/(p+d)) sensors. This value replaces 1/p in the sufficient or necessary bounds derived in (1), (9) and (16).

B. Delay with Random Wake-up Schedule

Now we model the effect of delay flexibility when the wake-up cycles of the sensors are not coordinated across the network. We would like to evaluate whether the distribution of the active and sleep times of the sensors affects the achievable gain. Therefore we consider both deterministic and Markovian wake-up cycles.

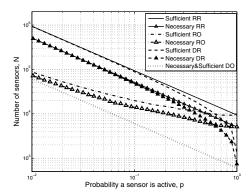


Fig. 6. Number of sensors versus the probability that a sensor is active, $r=0.025\,$

Deterministic wake-up cycle: The sensor follows a deterministic wake-up cycle of unit length, being active in p portion of the cycle. The probability that an event will be sensed by the sensor within the delay limit d is equal to

$$\hat{p} = min(1, p + d),$$

where both p and d express percentages of the sensor cycle period.

Markovian wake-up cycle: As shown on Figure 5 a 2-state continuous time Markovian model gives the behavior of the sensor, that is active and sleep periods have exponential distribution [9]. Parameters λ and μ are chosen such that $1/\lambda = p$ and $1/\mu = 1 - p$ gives the average time the sensor spends in active and sleeping states respectively. The probability that a random event will not be sensed by the sensor within the delay limit d can be expressed as the probability that the sensor is sleeping at the time of the event and does not wake up within time d. That is, \hat{p} , the probability that the event will be sensed by the sensor is equal to:

$$\hat{p} = 1 - (1 - p)e^{-d/(1-p)}.$$

The above expressions of \hat{p} replace parameter p in all necessary or sufficient conditions under random wake-up schedule given by (4), (8), (10) and (13).

VI. PERFORMANCE EVALUATION

In this section we compare the number of sensors required to provide full coverage with high probability using the deterministic or random sensor placement and optimal or random wake-up time scheduling. We base our comparison on the necessary and sufficient conditions derived in sections III - V. In all cases we consider $P_c=0.99$. Most of the figures are shown with a log-log scale.

Figure 6 shows the necessary and sufficient conditions on the number of sensors required to provide full connectivity with P_c for the randomized scenarios, and the number of sensors required for full connectivity in the DO case, as a function of p, the probability that a sensor is active. We can conclude that there is one order of magnitude gain between

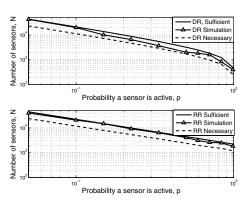


Fig. 7. Necessary and Sufficient conditions along with the simulation results for DR and RR Cases, r = 0.05.

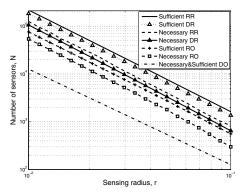


Fig. 8. Number of sensors versus sensing radius, p = 0.3.

DO and RR, independently of the value of p, a gain that sensor placement and wake-up coordination schemes can utilize. The gain due to deterministic placement diminishes though as p decreases, that is, the performance of DO and RO solutions and the performance of DR and RR solutions converge at small p. Specifically, the gain due to deterministic placement diminishes around p = 0.1 under random wake-up. This convergence can be predicted from (4), (8). Similarly, the gain of optimal wake-up scheduling decreases as p increases. and the performance of the RO solution converges to the performance of RR. The ratio of the necessary and sufficient conditions on N (that is the gap between the curves with logarithmic scale) does not depend on p in the case of random wake-up scheduling, and the conditions converge in the case of optimal wake-up schedule. To evaluate the tightness of the conditions we show simulation results for the DR and RR cases on Figure 7. The results show that the sufficient condition on N is rather tight. The necessary conditions are looser due to the uncovered area on Figure 3.

Next we evaluate the tightness of the derived bounds as a function of the sensing radius r on Figure 8, considering p=0.3. The figure shows that the gap between the necessary and sufficient conditions remains constant as r changes. The

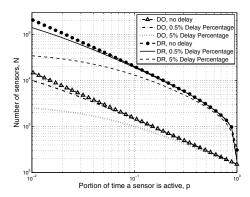


Fig. 9. Number of sensors with sensing delay (DO and DR), r=0.05. (Only Sufficient Conditions plotted)

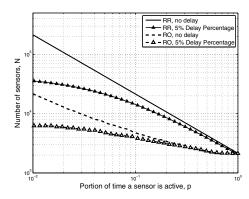


Fig. 10. Number of sensors with sensing delay (RO and RR), r=0.05. (Only Sufficient Conditions plotted)

gaps are very similar for DR, RO and RR. The curves are nearly linear in the log-log graph, reflecting that N increases with $1/r^2$.

Figures 9-11 evaluate the possible gains with limited detection delay based on the results of section V. We show results for d=0.5% and 5%. The possible gain due to delayed detection is related to the wake-up schedules, therefore we compare the DO and DR cases in Figure 9 and the RO and RR cases on Figure 10. As we can see, delayed detection can lead to nearly one order of magnitude gain in the number of sensors at low values of p. Already at 5% delay the number of sensors needed with random wake-up approaches the number of sensors needed with optimal wake-up scheduling but without any detection delay. Finally, we compare delayed detection under deterministic and Markovian wake-up cycles. Figure 11 shows the sufficient conditions for p = 0.5 - 1. For p close to 1 the sufficient condition under the deterministic wake-up cycle converges to its minimum (1 sensor in each drawn circle) much faster than the one under Markovian cycles. The gain diminishes rapidly as p decreases. That is, the distribution of the active and sleeping times does not affect the performance of delayed detection significantly.

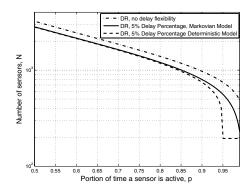


Fig. 11. Number of sensors with sensing delay, deterministic and exponential active times, r=0.05. (Only Sufficient Conditions plotted)

VII. CONCLUSIONS

In this paper we derived analytic models to bound the number of sensors required for full coverage with high probability under deterministic and random sensor placement and optimal and random wake-up schedules. We concluded that there is an order of magnitude gain between the deterministic placement - optimal schedule and random placement - random schedule solutions that sensor placement and wake-up schedule coordination methods can utilize. We showed that the gain of deterministic placement diminishes when sensors are only active in a small fraction of time, while the gain of optimal scheduling decreases with the probability of the sensors being active. Finally, we have shown that the number of sensors required or the need of coordinated wake-up can be relaxed if a detection delay of a couple of percent of the wake-up cycle length is allowed.

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